Exam 2 covers Ch. 27-32, Lecture, Discussion, HW, Lab

Exam 2 is Wed. Mar. 26, 5:30-7 pm, 2103 Ch: Adam(301,310), Eli(302,311), Stephen(303,306), 180 Science Hall: Amanda(305,307), Mike(304,309), Ye(308)

- Chapter 27: Electric flux & Gauss’ law
- Chapter 29: Electric potential & work
- Chapter 30: Electric potential & field
- Chapter 28: Current & Conductivity
- Chapter 31: Circuits
- Chapter 32: Magnetic fields & forces
  - (exclude 32.6,32.8,32.10)

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Electric current produces magnetic field

- Current (flow of electric charges) in wire produces magnetic field.
- That magnetic field aligns compass needle

Law of Biot-Savart

- Each element of current produces a contribution to the magnetic field.

Magnetic field from long straight wire:
Direction

- What direction is the magnetic field from an infinitely-long straight wire?

Current dependence

How does the magnitude of the B-field change if the current is doubled?

A) Is halved
B) Quadruples
C) Stays same
D) Doubles
E) Is quartered

Distance dependence

How does the magnitude of the B-field at 2 compare to that at 1?

A) \( B_2 = B_1 \)
B) \( B_2 = 2B_1 \)
C) \( B_2 = B_1/2 \)
D) \( B_2 = 4B_1 \)
E) \( B_2 = B_1/4 \)
### Why?

- Biot-Savart says \( \overrightarrow{dB} = \frac{\mu_0 I d\vec{s} \times \hat{r}}{4\pi r^2} \)
- Why \( B(r) \propto \frac{1}{r} \) instead of \( \frac{1}{r^2} \)?

Small contribution from this current element.

- \(~\) independent of \( r \)

Large contribution from this current element.

- Decreases as \( 1/r^2 \)

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### Long straight wire

- All current elements produce \( B \) out of page

\[
\begin{align*}
\overrightarrow{dB} &= \frac{\mu_0 I d\vec{s} \times \hat{r}}{4\pi r} \\
&= \frac{\mu_0 I}{4\pi} \left[ \frac{a}{r^3} \sin \theta \right] \\
&= \frac{\mu_0 I}{4\pi} \left[ \frac{a}{r^3} \left( x^2 + a^2 \right)^{3/2} \right]
\end{align*}
\]

Add them all up:

\[
B = \frac{\mu_0 I}{4\pi} \int \frac{dx}{r^3} = \frac{\mu_0 I}{2\pi a}
\]

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### Forces between currents

Which of these pairs of currents will attract each other?

- A. A
- B. A & C
- C. B
- D. A & B

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### Force between current-carrying wires

- Attractive for parallel currents.
- Repulsive for antiparallel currents

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### Field from a circular loop

- Each current element produces \( dB \)
- All contributions add as vectors
- Along axis, all components cancel except for \( x \)-comp

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### Magnetic field from loop

Which of these graphs best represents the magnetic field on the axis of the loop?

- A.
- B.
- C.
- D.
Magnetic field from a current loop

- One loop: field still loops around the wire.
- Many loops: same effect

Solenoid electromagnet

- Sequence of current loops can produce strong magnetic fields.
- This is an electromagnet

Comparing Electric, Magnetic

- Biot-Savart: calculate B-field from current distribution.
  - Resulting B-field is a vector, and...
  - complication: current (source) is a vector!
- Coulomb: calculate E-field from charge distribution
  - Resulting E is a vector but charge (source) is not a vector

A shortcut: Ampere’s law

- Integral around closed path proportional to current passing through any surface bounded by path.
- Ampere’s law
  \[ \oint B \cdot ds = \mu_0 I \]
- Right-hand ‘rule’:
  - Thumb in direction of positive current
  - Curled fingers show direction of integration

‘Testing’ Ampere’s law

- Long straight wire \( B(r) = \frac{\mu_0 I}{2\pi r} \), \( B \perp r \)
  \[ \oint B \cdot ds = \oint \frac{\mu_0 I}{2\pi r} ds = \frac{\mu_0 I}{2\pi} \]
- Circular path
  - path has constant \( r \)
  - path length = \( 2\pi r \)
- Surface bounded by path

Using Ampere’s law

- Could have used Ampere’s law to calculate B
  \[ \oint B \cdot ds = \oint = B 2\pi r = \mu_0 I \Rightarrow B = \frac{\mu_0 I}{2\pi r} \]
Quick Quiz

Suppose the wire has uniform current density. How does the magnetic field change inside the wire?

A. Increases with $r$
B. Decreases with $r$
C. Independent of $r$
D. None of the above

\[
\oint B \cdot ds = 2\pi r B = \mu_0 J_{\text{avg}} = \mu_0 \frac{\pi r^2}{\pi R^2} \Rightarrow B(r) = \frac{\mu_0 J}{2\pi R^2} r
\]

Building a solenoid

Ampere's law for the solenoid