Maxwell’s equations and EM waves

This Lecture

- More on Motional EMF and Faraday’s law
- Displacement currents
- Maxwell’s equations
- EM Waves

From previous Lecture

- Time dependent fields and Faraday’s Law

Calculation of Electric/Magnetic Field for a moving charge

- Coulomb’s Law:

\[ \text{d}E = \frac{1}{4\pi\varepsilon_0} \frac{dq}{r^2} \hat{u}_r \]

- Biot-Savart

\[ \text{d}B = \frac{\mu_0}{4\pi} \frac{ds \times \hat{u}_r}{r^2} \]

Now charge moves inside page

Gauss’ law in magnetostatics

Net magnetic flux through any closed surface is always zero (the number of lines that exit and enter the closed surface is equal)

\[ \oint \mathbf{B} \cdot d\mathbf{A} = 0 \]

No magnetic ‘charge’, so right-hand side=0 for B-field

Basic magnetic element is the dipole

\[ \oint E \cdot dA = \frac{Q_{\text{enclosed}}}{\varepsilon_0} \]

Compare to Gauss’ law for E-field

Lorentz force in E and B-fields

- If a charge moves in the presence of E-field and B-field

\[ \mathbf{F} = q\mathbf{E} + q\mathbf{v} \times \mathbf{B} \]

If we switch on an E-field parallel to B it will accelerate q and loops become more stretched as q gains speed

Summary on static E- and B-fields

- Ampere’s law true for static (no time dependence) electric fields.
- What happens to the Ampere’s law if fields depend on time?

\[ \begin{align*}
\text{Magnetostatics:} & & \oint \mathbf{B} \cdot d\mathbf{A} &= 0 & \oint \mathbf{B} \cdot ds &= \mu_0 I \\
\text{Electrostatics:} & & \oint \mathbf{E} \cdot d\mathbf{A} &= \frac{Q_{\text{enclosed}}}{\varepsilon_0} & \oint \mathbf{E} \cdot ds &= 0
\end{align*} \]
**Time-dependent B-field**

Faraday's law

\[ \oint E \cdot ds = 0 \]

becomes

\[ \oint E \cdot ds = 0 - \frac{d\Phi_B}{dt} \]

\[ \Rightarrow E = -\nabla V \]

Not only charges produce E-field

- a changing B-field also produces an E-field. This is not like a Coulomb field produced by a charge starting or stopping in the charge but it is a ‘circular’ E-field

\[ E \cdot ds = 0 \]

becomes

\[ E \cdot ds = 0 = dBdt \]

**Faraday’s law**

Magnetic flux change: the magnitude of B changes in time

2) the area crossed by B lines changes with t
(motional emf)

3) The angle \( \theta \) between B and normal to loop changes with t

\[ \Phi_B = BA \cos \theta \]

**Quick Quiz**

A rectangular loop of wire is pulled at a constant velocity from a region with \( B = 0 \) into a region of a uniform B-field. The induced current;

- A) will be zero
- B) will be some constant value that is not zero
- C) will increase linearly with time

What happens when the loop exits the B-field region?

**Other ways to change B-flux**

- B flux increases when magnet moves since B from N to S
- B flux decreases when magnet moves, B\( _{\text{ind}} \) parallel to B and I changes direction

**Quick Quiz: motional emf**

A conducting bar moves with velocity v. Which statement is true?

A) + accumulates at the top, and - at the bottom
B) no complete circuit, therefore, no charge accumulation
C) - accumulates at the top, and + at the bottom

**Faraday's and Lenz's law**

B flux increases when magnet moves since B from N to S

B\( _{\text{ind}} \) opposite to B

B flux decreases when magnet moves, B\( _{\text{ind}} \) parallel to B and I changes direction

Magnetic flux change: the magnitude of B changes in time

**Quick Quiz**

Remember last lecture Faraday’s Law

\[ E = \oint E \cdot ds = \frac{d\Phi_B}{dt} = \frac{d}{dt} \int B \cdot dA \]

**Quick Quiz**

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**Other ways to change B-flux**

- 2) the area crossed by B lines changes with t
  (motional emf)
- 3) The angle \( \theta \) between B and normal to loop changes with t

\[ \Phi_B = BA \cos \theta \]
The 4 Maxwell’s Equations

\[ \oint E \cdot dA = \frac{\partial \Phi_B}{\partial t} \quad \text{(Gauss' Law)} \]
\[ \oint B \cdot dA = 0 \quad \text{(Faraday-Henry)} \]
\[ \oint E \cdot ds = \frac{dq}{dt} \quad \text{(Faraday’s law)} \]
\[ \oint B \cdot ds = \mu_0 I + \frac{\partial \Phi_E}{\partial t} \quad \text{(Ampere-Maxwell law)} \]

Consequence: induced current
Lorentz force
\[ \mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \]

Ampere’s - Maxwell Law

Tot current is conduction + displacement
\[ \oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I \quad \text{becomes} \quad \oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I + \mu_0 \frac{d\Phi_E}{dt} \]

Magnetic fields are produced both by conduction currents and by time-varying electric fields

James Clerk Maxwell and the theory of electromagnetism

- Electricity and magnetism are deeply related
- 1865 Maxwell: mathematical theory showing relationship between electric and magnetic phenomena
- Maxwell’s equations predict existence of electromagnetic waves propagating through space at velocity of light
\[ c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} = 2.99792458 \times 10^8 \text{ m/s} \]

Permittivity of free space:
\[ \varepsilon_0 = 8.85 \times 10^{-12} \text{ F/m} \]

Permeability of free space:
\[ \mu_0 = 4\pi \times 10^{-7} \text{ H/m} \]

There are no magnetic monopoles
An E-field can be created by a changing B-field
Currents create a B-field
A changing E-field can create a B-field
**Reminders: classification of waves**

A mechanical wave is a disturbance created by a vibrating object that travels through a medium from one location to another.

- **Longitudinal wave**: medium particles move in direction parallel to direction of propagation of wave (e.g., sound waves)

**Sound wave**: sinusoidal variation of pressure in air

**Transverse wave**: medium particles move in direction perpendicular to direction of wave (waves on a pond, EM waves)

**Velocity of waves**: $v = \lambda f = \frac{\lambda f}{T}$

**Fields exist even with q=0 and I=0!**

- Gauss’ Law: $\oint E \cdot dA = 0$
- Faraday’s Law: $\oint B \cdot ds = \frac{\partial \Phi}{\partial t}$
- Ampere-Maxwell Law: $\oint E \cdot ds = \oint B \cdot dl$

From these equations we get EM wave equations traveling in vacuum!

- Transverse wave composed of E and B fields propagating in empty space with velocity $c$ emerges from Maxwell equations!

**Solutions of Maxwell’s equations**

- The simplest solution to partial differential equations is sinusoidal wave propagating along x:
  - $E = E_{\text{max}} \cos (kx - \omega t)$
  - $B = B_{\text{max}} \cos (kx - \omega t)$

- The speed of the electromagnetic wave is $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$

**Hertz confirms Maxwell’s predictions**

- Heinrich Hertz was the first to generate and detect electromagnetic waves in a laboratory setting
- The most important discoveries were in 1887
- He also showed other wave aspects of light
Hertz’s Experiment (1887)

- Transmitter T: two spherical electrodes separated by narrow gap charged until air gap ionized => sparks
- The discharge has oscillatory behavior of frequency $f \approx 4 \times 10^7$ Hz
- This ‘oscillating dipole’ emits E and B plane waves
- When resonance frequency of T and R match sparks also in R = receiver
- Hertz’s hypothesis: the energy transmitted from T to R is carried by waves

measured $\lambda$ by having standing waves using a metal screen: nodes at distance $\lambda/2$

knowing $f$ he measured $c!$

Radiation has wave properties: interference, diffraction, reflection, refraction and polarization

Quick Quiz on EM Waves

Shown below is the E-field of an EM wave broadcast at 30 MHz and traveling to the right.

What is the direction of the magnetic field during the first $\lambda/2$?

1) Into the page   2) out of the page

What is the wave length?

1) 10 m   2) 5 m

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8 \text{m/s}}{3 \times 10^7 \text{Hz}} = 10 \text{m}$$