The Description of the microscopic world

Previous Lecture:
Quantization of light, photons
Photoelectric effect
Particle-Wave dualism

This Lecture:
Matter waves
Uncertainty Principle
Wave functions
Start the atom
MTE3 on Wed 23, 5:30-7:00 pm, Ch 2103 and SH 180, same sessions in each room

Alternate Exams on Wed
2:30-4:00 pm
6:00-7:30 pm
in the Lab room

Send your requests for Alternate Exams by email specifying the class conflict
MTE 3 Contents

- Ampere’s Law and force between wires with current (32.6, 32.8)
- Faraday’s Law and Induction (ch 33, no inductance 33.8-10)
- Maxwell equations, EM waves and Polarization (ch 34, no 34.2)
- Photoelectric effect (38.1-2-3)
- Matter waves and De Broglie wavelength (38.4)
- Atom (37.6, 37.8-9, 38.5-7)
- Wave function and Uncertainty (39)
Quantization of light

- Light is made of ‘quanta’ of energy called photons

- **quantum’ of energy:** \( E = hf \) \( f = \) frequency of light

- Photon is a particle, but moves at speed of light!
  - This is possible because it has zero mass.

- Zero mass, but it does have momentum:
  - Photon momentum \( p = E/c \) (remember \( U/c \) for radiation)

- Photoelectric effect (1905)

\[
K_{\text{max}} = hf - \phi
\]

*No matter how intense is the light: until the light wavelength passes a certain threshold, no electrons are ejected.*
How much is a quantum of green light?

One quantum of energy for 500 nm light (green):

\[
E = hf = \frac{hc}{\lambda} = \left(6.634 \times 10^{-34} \text{ Js}\right) \times \left(3 \times 10^8 \text{ m/s}\right) \div \left(500 \times 10^{-9} \text{ m}\right) = 4 \times 10^{-19} \text{ J}
\]

We need a convenient unit for such a small energy!

1 electron-volt = 1 eV = |charge on electron| \times (1 volt) = 1.602 \times 10^{-19} \text{ J}

Energy of an electron accelerated in a potential difference of 1 V

In these units,

\[
\text{E(1 green photon)} = (4 \times 10^{-19} \text{ J}) \times (1 \text{ eV} / 1.602 \times 10^{-19} \text{ J}) = 2.5 \text{ eV}
\]

\[
hc = 1240 \text{ eV nm}
\]

so you can use 1240 eV nm/500 nm = 2.5 eV
Quiz on photoelectric effect

Which of the following is not true of photoelectric emission?

A. increasing the light intensity causes no change in the kinetic energy of photoelectrons
B. the max energy of photoelectrons depends on the frequency of light illuminating the metal
C. increasing the intensity of the light will increase the KE of photoelectrons
D. Doubling the light intensity doubles the number of photoelectrons emitted

A. is true because the intensity is connected to the number of electrons not to the energy of each ones
B. is true because $K_{\text{max}} \propto f$
C. is false because $K_{\text{max}}$ depends on $f$
D. Intensity = Power/Area = energy/(time x A) = (Nhf/time x Area)
Is Light a Wave or a Particle?

- **Wave**
  - Electric and Magnetic fields act like waves
  - Superposition, Interference and Diffraction

- **Particle**
  - Photons
  - Photo-electric effect
  - Compton Effect

**Sometimes Particle Sometimes Wave depending on the experiment!**
Wave properties of particles

- 1924 graduate student
- Nobel Prize in 1929

\[ p = \frac{h}{\lambda} \quad \Rightarrow \quad \lambda = \frac{h}{p} \]

**de Broglie wavelength**

Should be able to see interference and diffraction for any material particle!!

De Broglie postulated that it holds for any object with momentum - an electron, a nucleus, an atom, a baseball,......
Wavelength of an electron of 1 eV

\[ p = \frac{h}{\lambda} \]

\text{de Broglie wavelength}

\[ \text{KE} = \frac{1}{2}mv^2 \quad \text{and} \quad p = mv, \quad \text{so} \quad \text{KE} = \frac{p^2}{2m} \]

Solve for \( p = \sqrt{2m(\text{KE})} \)

\[ \lambda = \frac{h}{\sqrt{2m(\text{KE})}} = \frac{hc}{\sqrt{2mc^2(\text{KE})}} = \frac{1240 \text{ eV nm}}{\sqrt{2(511,000 \text{ eV})(1 \text{ eV})}} \]

Result: \( \lambda = 1.23 \text{ nm} \)

\( m_e = 9.1 \times 10^{-31} = 0.511 \text{ MeV/c}^2 \)
De Broglie question

Compare the wavelength of a bowling ball with the wavelength of a golf ball, if each have 10 Joules of kinetic energy.

A) $\lambda_{\text{bowling}} > \lambda_{\text{golf}}$

B) $\lambda_{\text{bowling}} = \lambda_{\text{golf}}$

C) $\lambda_{\text{bowling}} < \lambda_{\text{golf}}$

The largest the mass of the object the less noticeable are the quantistic effects!

Football launched by Brett Favre can go at 30m/s and m = 0.4kg

$$\lambda = \frac{h}{p} = \frac{6.6 \times 10^{-34} \text{ Js}}{0.4 \text{ kg} \times 30 \text{ m/s}} = 5.5 \times 10^{-35} \text{ m}$$
**Light properties as a wave**

- Light has wavelength, frequency, speed
  - $f\lambda = \text{speed}$
- Light shows interference and diffraction phenomena

\[ d \sin \theta = m\lambda \]
\[ d \sin \theta = (m + \frac{1}{2})\lambda \]

\[ \sin \theta = \frac{2\lambda}{a} \]
\[ \sin \theta = \frac{\lambda}{a} \]
\[ \sin \theta = 0 \]
\[ \sin \theta = -\frac{\lambda}{a} \]
\[ \sin \theta = -\frac{2\lambda}{a} \]
Is an Electron a Particle or a Wave?

- Particle: you can “bounce” things off them.
- How would know if electron was a wave?

Look for interference!

This is what we observe like if 1 electron passes through both slits!!

We destroy interference illuminating to determine the slit they go through
Electron Diffraction: the experiment

- Parallel beams of mono-energetic electrons on a double slit
- Slit widths $<< \lambda_{\text{electron}}$
- Electron detector far from slits

![Computer simulation](a) After 28 electrons
(b) After 1000 electrons
(c) After 10000 electrons
(d) Two-slit electron pattern

![Photograph](©2004 Thomson - Brooks/Cole)
Davisson-Germer experiment

- Diffraction of electrons from a nickel single crystal.
- Found pattern by heating just by chance. Nickel formed a crystalline structure.
- Established that electrons are waves.

54 eV electrons ($\lambda=0.17\text{nm}$)

Bright spot: constructive interference

Davisson: Nobel Prize 1937
What’s your view of atoms?
Hystory of Atoms

• Thompson’s classical model - raisin-cake (1897): cloud of + charge with embedded e⁻
• Problem: charges cannot be in equilibrium

Planetary model

- Positive charge concentrated in the nucleus (∼10⁻¹⁵ m)
- Electrons orbit the nucleus (r~10⁻¹⁰ m)

Problem1: emission and absorption at specific frequencies
Problem2: electrons on circular orbits radiate

(Atractive) Coulomb force plays role of gravity
Most of the alpha-particles (ionized He nuclei made of 2p+2n) not deflected much but a few bounced back.
Bohr’s Model of Hydrogen Atom (1913)

• Postulate 1: Electron moves in circular orbits where it does not radiate (stationary states)

\[ r_n = n^2 a_0 \]

\[ a_0 = 0.053 \text{ nm} \]

• Postulate 2: radiation emitted in transitions between stationary states

\[ E_i - E_f = hf \]

• Orbital angular momentum quantized

\[ L = mvr = n \frac{h}{2\pi} \]

Notice: Sometimes f is indicated by \( \nu \)

\[ E_n = -13.6 \text{ eV/n}^2 \]

E\(_i\)  

E\(_f\)  

Emitted photon has energy hf
Emitting and absorbing light

Photon emitted

\( hf = E_2 - E_1 \)

Photon absorbed

\( hf = E_2 - E_1 \)

Energy axis

\[ E_1 = -\frac{13.6}{1^2} \text{ eV} \]

\[ E_2 = -\frac{13.6}{2^2} \text{ eV} \]

\[ E_3 = -\frac{13.6}{3^2} \text{ eV} \]
Question

This quantum system has equally-spaced energy levels as shown. Which photon could possibly be absorbed by this system?

A. 1240 nm
B. 413 nm
C. 310 nm
D. 248 nm

\[ E_{\text{photon}} = \frac{hc}{\lambda} = \frac{1240 \text{ eV} \cdot \text{nm}}{\lambda} \]

- \( E_3 = 7 \text{ eV} \)
- \( E_2 = 5 \text{ eV} \)
- \( E_1 = 1 \text{ eV} \)

\[ 2eV \neq \frac{1240}{413} \]
\[ 2eV \neq \frac{1240}{310} \]
\[ 2eV \neq \frac{1240}{248} \]
\[ 4eV \neq \frac{1240}{1240} \]
\[ 4eV \neq \frac{1240}{413} \]
\[ 4eV = \frac{1240}{310} \]
Why “quantized” orbits?

- Electron is a wave.
- Its ‘propagation direction’ is around circumference of orbit.
- Wavelength = $\frac{h}{p}$
- Waves on a circular orbit?

- Incorporating wave nature of electron gives an intuitive understanding of ‘quantized orbits’
Standing waves on a string

A guitar vibrates at frequency that depends on string length

Standing wave displacement:

Boundary conditions:

\[ \lambda_n = \frac{2L}{n} \]

Fundamental, wavelength \( 2L/1 = 2L \), frequency \( f \)

1st harmonic, wavelength \( 2L/2 = L \), frequency \( 2f \)

2nd harmonic, wavelength \( 2L/3 \), frequency \( 3f \)

\[ D(x,t) = 2A \sin kx \cos \omega t \]

\[ \sin kx = 0 \]

\[ \sin kL = 0 \]

Vibrational modes equally spaced in frequency

\[ n=1 \]

\[ n=2 \]

\[ n=3 \]

\[ n=4 \]
Waves in a circular “Donut flute”

Wind instrument with particular fingering plays a particular pitch (or wavelength)

- Integer number of wavelengths around circumference.
- Otherwise destructive interference occurs when wave travels around ring and interferes with itself.
Quantization from de Broglie

- Integer number wavelengths along circumference of radius $r_n$

- Circumference \( = 2\pi r_n = n\lambda \)

- deBroglie: \( \lambda = \frac{h}{p} = \frac{h}{m_e v} \)

- Result: \( 2\pi r_n = \frac{nh}{m_e v} \)

...giving

\[ L = \text{angular momentum} = m_e v r = n\frac{h}{2\pi} = n\hbar, \text{quantized!} \]
Bohr H atom question

- Peter Flanary’s sculpture ‘Wave’ outside Chamberlin
- What quantum state of H?
  
  A. \( n = 2 \)
  B. \( n = 3 \)
  C. \( n = 4 \)

- Integer number of wavelengths around circumference.

\[
L = pr = n \frac{h}{2\pi} \Rightarrow \frac{h}{\lambda} = n \frac{h}{2\pi r} \Rightarrow 2\pi r = n\lambda
\]
Let’s demonstrate $E = -13.6Z^2 \text{ eV } /n^2$

\[ F = k \frac{Ze^2}{r^2} = m \frac{v^2}{r} \quad \Rightarrow \quad k \frac{Ze^2}{r} = mv^2 \quad \Rightarrow \quad r = k \frac{Ze^2}{mv^2} \]

\[ mvr = n\hbar \quad \Rightarrow \quad v = \frac{n\hbar}{mr} \quad (2) \]

Z=n. of protons in nucleus of H-like atom

Substitute (2) in (1):

\[ r = k \frac{Ze^2 m^2 r^2}{mn^2\hbar^2} \Rightarrow r = \frac{n^2}{Z} \frac{\hbar^2}{me^2 k} = \frac{n^2}{Z} \frac{(1.055 \times 10^{-34})^2}{9.11 \times 10^{-31} \times (1.6 \times 10^{-19})^2 \times 9 \times 10^9} = \frac{n^2}{Z} \frac{0.53 \times 10^{-10} m}{Z} \]

\[ r = \frac{n^2}{Z} a_0 \quad a_0 = \text{Bohr radius} = 0.53 \times 10^{-10} \text{ m} = 0.53 \text{ Å} \]

Total energy = kinetic energy + potential energy = kinetic energy + eV

\[ E = \frac{p^2}{2m} - k \frac{Ze^2}{r} = \frac{mv^2}{2} - k \frac{Ze^2}{2r} = -k \frac{Ze^2}{2} \times \frac{Zme^2 k}{n^2\hbar^2} = -\frac{Z^2 k^2 me^4}{2\hbar^2} = -13.6eV \frac{Z^2}{n^2} \]

\[ E = -\frac{Z^2 (9 \times 10^9)^2 \times 9.11 \times 10^{-31} \times (1.6 \times 10^{-19})^4}{2 \times (1.055 \times 10^{-34})^2} = -\frac{Z^2}{n^2} 2.2 \times 10^{-18} J = -\frac{Z^2}{n^2} 13.6eV \]