PHYS208, SPRING 2008

**From last time**

- Electric field is $\vec{E} = \frac{\vec{F}}{q}$ ($\Rightarrow$ superposition principle applies both to force and to $\vec{E}$).
- Electric field from a point charge: $\vec{E} = \frac{kQ}{r^2} \hat{r}$.
- From a set of point charges: $\vec{E}_{\text{tot}} = \sum_i \vec{E}_i$.

Calculating E-field for a continuous distribution requires some math...

**Quick Quiz**

What does an electric dipole in a uniform $E$-field do?

A) accelerates left

B) accelerates right

C) rotates CW until aligned to $E$

D) rotates CCW until aligned to $-E$

E) accelerates up

**Electric torque on dipoles**

Net force zero

$F = q\vec{E}$, $r = s'/2$

$F = q\vec{E}$, $r = s/2$

but not aligned $\Rightarrow$ dipole rotates

Torque is a vector and RH rule tells direction of rotation. For a couple of forces ($L$ = distance between lines along which forces act)

$|\vec{F}| = \lambda \sin \theta = qE \sin \theta = p \sin \theta$

$\vec{T} = \vec{p} \times \vec{E}$

$U = -pE \cos \theta = -p \cdot E$

When the dipole is aligned to the field $U$ is minimum: $U = qE$ equilibrium!

**An example**

$\vec{E} = E \hat{u}_y = 2k \int \frac{\lambda dx}{\sqrt{x^2 + r^2}} \cos \theta = 2k \frac{\lambda}{r}$

$\cos \theta = \frac{r}{\sqrt{r^2 + x^2}}$

$\vec{dE} = \hat{u}_x \cdot \frac{\lambda dx}{\sqrt{x^2 + r^2}}$
**Symmetry**

But in situations of high symmetry we can use a smarter approach!

- Cylindrical symmetry
- Spherical symmetry
- Planar symmetry

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**Electric Flux**

Electric flux $\Phi_E$ through a surface:
(component of E-field $\perp$ to surface) $\times$ (surface area)

$\Phi_E \propto$ proportional to number of E-field lines penetrating surface

**Why perpendicular component?**
Which is the mathematical expression?

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**Flux equation**

- Suppose surface make angle $\theta$ surface normal

\[ \vec{A} = \hat{A} \hat{n} \]

\[ \vec{E} = \vec{E}_n \hat{n} + \vec{E}_\parallel \]

Component $\parallel$ surface
Component $\perp$ surface

Only $\perp$ component 'goes through' surface

$\Phi_E = E \cdot A$

This is equivalent to the scalar product:

E uniform on $A$

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**Total Flux**

- Surface not flat
- add up small areas where it is $\sim$ flat

- E not constant
- add up small areas where it is constant

\[ \Phi_E = \lim_{\Delta A \to 0} \sum E_i \cdot \Delta A = \int E \cdot dA \]

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**Flux through a spherical surface**

- Closed spherical surface, positive point charge at center
- E-field $\perp$ surface, directed outward
- E-field magnitude on sphere:
  \[ E = \frac{1}{4\pi \epsilon_0} \frac{q}{r^2} \]

- Net flux through surface:
  - $E$ constant, everywhere $\perp$ surface
  \[ \Phi_E = \oint E \cdot d\vec{A} = \oint E dA = EA - \left( \frac{1}{4\pi \epsilon_0} \frac{q}{r^2} \right) \left( \frac{4\pi r^2}{2} \right) = \frac{q}{\epsilon_0} \]

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**GAUSS’ LAW**

- net electric flux through closed surface = charge enclosed $/ \epsilon_0$

\[ \Phi_E = \oint E \cdot d\vec{A} = \frac{Q \text{enclosed}}{\epsilon_0} \]

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Quick Quiz

Two spheres of radius R and 2R are centered on 2 positive charges of the same value q. The flux through the 2 spheres compares as:

A) Flux(R) = Flux(2R)
B) Flux(R) = 2Flux(2R)
C) Flux(R) = 1/2 Flux(2R)
D) Flux(R) = 1/4 Flux(2R)
E) Flux(R) = 4 Flux(2R)

\[
E(R) \times 4\pi R^2 = E(2R) \times 4\pi (2R)^2
\]

GAUSS’ LAW for any closed surface

- net electric flux through closed surface
  = charge enclosed / \(\epsilon_0\)

\[
\Phi_E = \oint E \cdot d\vec{A} = \frac{Q_{\text{enclosed}}}{\epsilon_0}
\]

The flux depends ONLY on the charge enclosed by the surface!!

How to use Gauss’ law?

Apply to a uniformly charged sphere

What is the field outside (r>a)?

1) Select a sphere as the gaussian surface thru which \(E \cdot dA = 0\)
Given the symmetry, sphere of radius r is a natural choice
2) Important: know E-field direction. Radially out.
3) How its it oriented respect to dA? Parallel to E
4) Flux thru surface: \(E4\pi r^2\)
5) Enclosed charge + Q

\[
\Phi_E = \oint E \cdot d\vec{A} = E4\pi r^2 = \frac{Q}{\epsilon_0}
\]

Same reasoning as for point charge!

Where does it matter that the sphere is not point-like?

- Field out of sphere and parallel to dA
- \(\rho = \frac{Q}{V}\) volume charge density \(dq = \rho dV\)
- \(\rho\) in the gaussian surface (sphere with r<a) is the same as the charge sphere one

\[
q_n = Q (r^3/a^3) \Rightarrow E = \frac{q_n}{4\pi \epsilon_0 r^2} = k\epsilon_0 \frac{Q}{\epsilon_0 r^2}
\]

Quick Quiz

- Which gaussian surface do you have to use to calculate E for an infinite line of charge?

None of these

\[
dq = \lambda d\ell
\]

Flux of E through the gaussian surface:

\[
\Phi_E = E2\pi r\ell
\]

Apply Gauss’ law to calculate E!

\[
E = \frac{\lambda}{2\pi \epsilon_0 \ell}
\]
**Plane of charge**
Which Gaussian surface could be used to calculate $E$-field from infinite sheet of charge?

- $E$ perpendicular to plane and same magnitude at all points equidistant from the plane
- $S = \text{Area of gaussian surface where}$

$$\mathbf{E} \cdot d\mathbf{A} \neq 0$$

Flux $2ES$
Gauss' law: $2ES = \frac{Q}{\varepsilon_0} \Rightarrow E = \frac{\varepsilon_0 \cdot Q}{2S}$

This is a uniform field

**Conductors in equilibrium**
Electrostatic equilibrium

In a conductor in electrostatic equilibrium there is no net motion of charge

**Property 1:** $E=0$ everywhere inside the conductor

- Conductor slab in an external field $E:
- If $E$-field not null inside the conductor, free electrons would be accelerated
- These electrons would not be in equilibrium.
- When the external field is applied, the electrons redistribute until the magnitude of the internal field equals the magnitude of the external field
- The total field inside the conductor is zero

$$E_{\text{tot}} = E + E_{\text{in}} = 0$$

**Property 2:** Charge on only conductor surface

- Chose a gaussian surface inside the conductor (as close to the surface as desired)
- There is no net flux through the gaussian surface (since $E=0$)
- Any net charge must reside on the surface (cannot be inside!)

**Questions for you**

- What is the field in all regions of space inside and outside a charged metal shell?
- Can a uniformly charged sphere as the one considered in this lecture be made of metal?