Physics 202. Review Session for Exam3
Quizzes/Exercises: Determine Direction Of emf

- Indicate the direction of emf in the following cases:

  - |B| increases
  - |B| decreases
  - |B| decreases
  - |B| increases
  - |B| decreases
  - Path outside B
Faraday’s Law (Reminder)

- The emf induced in a “circuit” is proportional to the time rate of change of magnetic flux through the “circuit”.

\[ \mathcal{E} = -\frac{d\Phi_B}{dt} \]

- Notes:
  - “Circuit”: any closed path
    → does not have to be real conducting circuit
  - The path/circuit does not have to be circular, or even planar

\[ \Phi_B = \int \mathbf{B} \cdot d\mathbf{A} \]
Methods to Change Electric Flux

\[ \mathcal{E} = - \frac{d\Phi_B}{dt} \quad \Rightarrow \quad - \frac{d(BA\cos\theta)}{dt} \]

- Change of \( \Phi_B \rightarrow \text{emf} \)
  - To change \( \Phi_B \):
    - Change \( B \rightarrow \text{emf produced by an induced E field} \)
    - Change \( A \rightarrow \text{motional emf} \)
    - Change \( \theta \rightarrow \text{motional emf} \)
    - Combination of above

Electric Generators
Problem 1: (25 points)
A conducting loop of width \( w \) and length \( d \) is moved with constant speed \( v \) to the right. It passes through a uniform magnetic field \( B \) as illustrated.

a) Indicate the direction of the induced current (if any) in the figure above for all three positions.

b) Sketch the following:
   - the magnetic flux through the area enclosed by the loop versus \( x \) (sign convention: positive flux into the page, negative flux out of the page). Take the right edge of the loop as reference (see the dotted line; i.e., take \( x=0 \) when the loop enters the field):

\[
\Phi_B = B(\omega d - d\nu t)
\]

\[
\Phi_B = B d\nu t
\]
the induced motional emf versus $x$:

\[ \varepsilon = -\frac{\Delta \Phi}{\Delta t} \]

\[ \varepsilon = RI; \; I = \frac{\varepsilon}{R}; \; F \propto I \propto \varepsilon \]

the external force applied necessary to keep the speed $v$ constant:
In the example above, assume that the resistor is 6.00 \( \Omega \), and that the magnetic field has the magnitude 2.50 T and is directed perpendicularly into the paper. Let \( d = 1.20 \) m. The loop is pulled at a constant speed of 2 m/s.

c) What is the induced current when the loop enters the field?

\[
\mathcal{E} = -\frac{d\Phi}{dt} \quad \Phi = A \cdot B = B \cdot d \cdot x
\]

\[
\mathcal{E} = -B \cdot d \cdot v
\]

\[
I = \frac{\mathcal{E}}{R} = -\frac{Bdv}{R} = -1A
\]

\[
I = -1A
\]
d) Calculate the applied force required to move the bar to the right at this speed.

\[ F = B \cdot d \cdot I \]

\[ F = 3 \, N \]

e) If the resistor is only 0.0006 \, \Omega, what would be the resulting speed, if everything else is kept the same.

\[ F = B \cdot d \cdot I = \frac{B^2 d^2 v}{R} \]

\( F, B, d \) same

\[ \rightarrow v \sim R \]

\[ R_2 = 10^{-4} \, R \]

\[ v = 2 \cdot 10^{-4} \, \frac{\text{m}}{s} \]
Exercise: Calculate Inductance of a Solenoid

-show that for an ideal solenoid:

\[ L = \frac{\mu_0 N^2 A}{l} \]

(see board)

Reminder: magnetic field inside the solenoid (Ch 28)

\[ B = \mu_0 \frac{N}{l} I \]
Energy in an Inductor

- Energy stored in an inductor is $U = \frac{1}{2} LI^2$

- This energy is stored in the form of magnetic field:
  energy density: $u_B = \frac{1}{2} \frac{B^2}{\mu_0}$  (recall: $u_E = \frac{1}{2} \varepsilon_0 E^2$)

- Compare:
  Inductor: energy stored $U = \frac{1}{2} LI^2$
  Capacitor: energy stored $U = \frac{1}{2} C(\Delta V)^2$
  Resistor: no energy stored, (all energy converted to heat)
## Basic Circuit Components

<table>
<thead>
<tr>
<th>Component</th>
<th>Symbol</th>
<th>Behavior in circuit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ideal battery, emf</td>
<td><img src="image" alt="battery" /></td>
<td>$\Delta V = V_+ - V_- = \varepsilon$</td>
</tr>
<tr>
<td>Resistor</td>
<td><img src="image" alt="resistor" /></td>
<td>$\Delta V = -IR$</td>
</tr>
<tr>
<td>Realistic Battery</td>
<td><img src="image" alt="battery" /> <img src="image" alt="resistor" /></td>
<td>$\Delta V = 0$ ($\rightarrow R=0, L=0, C=0$)</td>
</tr>
<tr>
<td>(Ideal) wire</td>
<td><img src="image" alt="wire" /></td>
<td>$\Delta V = 0$ ($\rightarrow R=0, L=0, C=0$)</td>
</tr>
<tr>
<td>Capacitor</td>
<td><img src="image" alt="capacitor" /></td>
<td>$\Delta V = V_- - V_+ = -\frac{q}{C}$, $\frac{dq}{dt} = I$</td>
</tr>
<tr>
<td>Inductor</td>
<td><img src="image" alt="inductor" /></td>
<td>$\Delta V = -L\frac{dl}{dt}$</td>
</tr>
<tr>
<td>(Ideal) Switch</td>
<td><img src="image" alt="switch" /></td>
<td>$L=0, C=0, R=0$ (on), $R=\infty$ (off)</td>
</tr>
<tr>
<td>Transformer</td>
<td><img src="image" alt="transformer" /></td>
<td>Future Topics</td>
</tr>
<tr>
<td>Diodes, Transistors,...</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Turn on LR Circuit

\[ I = \frac{V_0}{R} \left(1 - e^{-\frac{t}{\tau}}\right) \]

Note: the time constant is \( \tau = \frac{L}{R} \)
Turn off LR Circuit

\[ I = I_0 e^{-\frac{t}{L/R}} \]

Note: the time constant is \( \tau = \frac{L}{R} \)
Charging a Capacitor in RC Circuit

\[ q(t) = \varepsilon C (1 - e^{-t/RC}) \]

\[ I(t) = \frac{\varepsilon}{R} e^{-t/RC} \]

Note: \( \tau \equiv RC \) is called time constant
Discharging a Capacitor in RC Circuit

\[ q(t) = Qe^{-t/RC} \]

\[ I(t) = -\frac{Q}{RC}e^{-t/RC} \]

Note the time constant \( \tau = RC \)
Problem 2:

Consider the circuit shown in the figure. Let \( L = 10.00 \text{ H} \), \( R = 10.00 \Omega \), and \( \varepsilon = 100 \text{ V} \).

a) What is the time constant of the circuit?

\[ \tau = \frac{L}{R} \]

b) What is the current at the time \( t=0 \), immediately after the switch is closed?

- What is the current after a long time?

- Provide a sketch of the current versus time.

\[ I(t) = \frac{\varepsilon}{R} \left( 1 - e^{-\frac{t}{\tau}} \right) \]

\[ I_{\text{max}} = \frac{\varepsilon}{R} = 10 \text{ A} \]
c) Provide a sketch of the voltage across R and across L versus time.

\[ \Delta V_R = \varepsilon \left(1 - e^{-\frac{t}{\tau}}\right) \]

\[ \Delta V_L = \varepsilon e^{-\frac{t}{\tau}} \]

---

d) At what time is the voltage across the resistor exactly 50V?

\[ \Delta V_R(t) = \varepsilon \left(1 - e^{-\frac{t}{\tau}}\right) = 50 \, V \]

\[ 1 - e^{-\frac{t}{\tau}} = 0.5 \]

\[ e^{-\frac{t}{\tau}} = 0.5 \]

\[ -\frac{t}{\tau} = \ln 0.5 \]

\[ t = -\tau \cdot \ln 0.5 = 0.693 \, s \]
Energy Stored in a LC Circuit

Reminder Energy stored in:
Inductor: energy stored $U_L = \frac{1}{2} LI^2$
Capacitor: energy stored $U_C = \frac{1}{2} C(\Delta V)^2 = \frac{1}{2} \frac{Q^2}{C}$

\[
U = U_L + U_C
\]

\[
U_C = \frac{1}{2C} Q_m^2 \cos^2 (\omega t + \phi)
\]

\[
U_L = \frac{1}{2} L\omega^2 Q_m^2 \sin^2 (\omega t + \phi)
\]

\[
U = \frac{Q_m^2}{2C} \left[ \cos^2 (\omega t + \phi) + \sin^2 (\omega t + \phi) \right] = \frac{Q_m^2}{2C}
\]
Current And Voltages in a Series RLC Circuit

\[ \Delta v_R = (\Delta V_R)_{\text{max}} \sin(\omega t) \]
\[ \Delta v_L = (\Delta V_L)_{\text{max}} \sin(\omega t + \pi/2) \]
\[ \Delta v_C = (\Delta V_C)_{\text{max}} \sin(\omega t - \pi/2) \]

\[ \Delta V_{\text{max}} = I_{\text{max}} \sqrt{R^2 + (X_L - X_C)^2} \]
\[ \phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) \]

\[ \Delta V = \Delta V_{\text{max}} \sin(\omega t + \phi) \]

Quiz: Can the voltage amplitudes across each components \((\Delta V_R)_{\text{max}}, (\Delta V_L)_{\text{max}}, (\Delta V_C)_{\text{max}}\) larger than the overall voltage amplitude \(\Delta V_{\text{max}}\) ?
# Summary of Impedances and Phases

<table>
<thead>
<tr>
<th>Circuit Elements</th>
<th>Impedance $Z$</th>
<th>Phase Angle $\phi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R$</td>
<td>$R$</td>
<td>$0^\circ$</td>
</tr>
<tr>
<td>$\frac{1}{C}$</td>
<td>$X_C$</td>
<td>$-90^\circ$</td>
</tr>
<tr>
<td>$\frac{1}{L}$</td>
<td>$X_L$</td>
<td>$+90^\circ$</td>
</tr>
<tr>
<td>$\frac{1}{R} \parallel C$</td>
<td>$\sqrt{R^2 + X_C^2}$</td>
<td>Negative, between $-90^\circ$ and $0^\circ$</td>
</tr>
<tr>
<td>$\frac{1}{R} \parallel L$</td>
<td>$\sqrt{R^2 + X_L^2}$</td>
<td>Positive, between $0^\circ$ and $90^\circ$</td>
</tr>
</tbody>
</table>
| $\frac{1}{R} \parallel L \parallel C$ | $\sqrt{R^2 + (X_L - X_C)^2}$ | Negative if $X_C > X_L$ 
Positive if $X_C < X_L$ |

---

*a* In each case, an AC voltage (not shown) is applied across the elements.

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The following 4 questions refer to the situation described below:

A resistor, a capacitor, an inductor, and an AC generator are connected in series.

21. For what frequency \( f \) (in Hz) of the AC voltage generator is the current flow maximized?
   
   a. \( f = 79.6 \text{ Hz} \)
   b. \( f = 884 \text{ Hz} \)
   c. \( f = 3.14 \text{ kHz} \)

22. If the peak voltage supplied by the AC generator in the circuit is \( E_{\text{max}} = 5 \text{ V} \)
    and the generator frequency is set to \( \omega = 650 \text{ rad/sec} \), what is the peak voltage \( V_{\text{C}_{\text{max}}} \) across the capacitor?
    
   a. \( V_{\text{C}_{\text{max}}} = 0 \text{ V} \)
   b. \( V_{\text{C}_{\text{max}}} = 0.37 \text{ V} \)
   c. \( V_{\text{C}_{\text{max}}} = 1.08 \text{ V} \)
   d. \( V_{\text{C}_{\text{max}}} = 1.72 \text{ V} \)
   e. \( V_{\text{C}_{\text{max}}} = 2.83 \text{ V} \)
22. If the peak voltage supplied by the AC generator in the circuit is \(E_{\text{max}} = 5\, \text{V}\) and the generator frequency is set to \(\omega = 650\, \text{rad/sec}\), what is the peak voltage \(V_{\text{c, max}}\) across the capacitor?

a. \(V_{\text{c, max}} = 0\, \text{V}\)
b. \(V_{\text{c, max}} = 0.37\, \text{V}\)
c. \(V_{\text{c, max}} = 1.08\, \text{V}\)
d. \(V_{\text{c, max}} = 1.72\, \text{V}\)
e. \(V_{\text{c, max}} = 2.83\, \text{V}\)

23. At the generator frequency \(\omega = 650\, \text{rad/sec}\), the EMF of the generator leads the current in the circuit.

a. True
b. False
Electromagnetic Waves

- **EM wave equations:**
  \[
  \frac{\partial^2 E_y}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E_y}{\partial t^2} \quad \frac{\partial^2 B_z}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 B_z}{\partial t^2}
  \]

- **Plane wave solutions:**
  \[
  E = E_{\text{max}} \sin(kx - \omega t + \phi) \quad B = B_{\text{max}} \sin(kx - \omega t + \phi)
  \]

- **Properties:**
  - No medium is necessary.
  - E and B are normal to each other
  - E and B are in phase
  - Direction of wave is normal to both E and B
    (EM waves are transverse waves)
  - Speed of EM wave:
    \[
    c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 2.9972 \times 10^8 \text{ m/s}
    \]
  - \(E/B = E_{\text{max}}/B_{\text{max}} = c\)
  - Transverse wave: two polarizations possible

We’ll have quizzes about EM waves, including the Poynting vector.
Momentum Carried By EM Waves

- **EM waves**: momentum = energy/c

- **Radiation Pressure (P)**:
  \[ P = u = \frac{S}{c} \]

  - 100% absorption
    \[ \Delta p = p \rightarrow P = \frac{S}{c} \]
  - 100% reflection
    \[ \Delta p = 2p \rightarrow P = \frac{2S}{c} \]
Wavelength and Frequency

Because of the wave equation the wavelength of and frequency of a EM wave in vacuum are related by:

$$\lambda f = c = 3 \cdot 10^8 \text{ m/s}$$

Example: Determine the wavelength of an EM wave of frequency 50 MHz in free space

$$\lambda = \frac{c}{f} = \frac{3 \cdot 10^8 \text{ m/s}}{50 \text{ MHz}} = \frac{3 \cdot 10^8 \text{ m/s}}{5 \cdot 10^7 \text{ s}^{-1}} = 6 \text{ m}$$
Example: Solar Energy

- The average intensity of the EM radiation from the Sun on Earth is $S \sim 10^3 \text{ W/m}^2$
  
  - What is the average radiation pressure for 100% absorption:
    
    $$P = \frac{S}{c} = \frac{10^3 \text{ W/m}^2}{3 \cdot 10^8 \text{ m/s}} = 3.3 \cdot 10^{-6} \text{ N/m}^2$$

  - What is the force exerted by EM radiation by the Sun on a surface of 1 m$^2$
    
    $$F = PA = 3.3 \cdot 10^{-6} \text{ N/m}^2 \cdot 1\text{m}^2 = 3.3 \cdot 10^{-6} \text{ N}$$
Example: Solar Energy (cont)

- The average intensity of the EM radiation from the Sun on Earth is $S \sim 10^3 \text{ W/m}^2$

  - What is the average radiation pressure for 50% absorption and 50% reflection:

$$P = P_A + P_R = \frac{S_A}{c} + 2 \frac{S_R}{c} = \frac{S}{2c} + 2 \frac{S}{2c} = \frac{3}{2} \frac{S}{c}$$

$$P = \frac{3}{2} \frac{S}{c} = \frac{3}{2} \frac{10^3 \text{ W/m}^2}{c} = \frac{3 \cdot 10^3 \text{ W/m}^2}{2 \cdot 3 \cdot 10^8 \text{ m/s}} = 5 \cdot 10^{-6} \text{ N/m}^2$$