Physics 202 Midterm Exam 3
April 13th 2009

Name: ........................................... ID#: .........................................

Section: .................................

TA and section (please circle):

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Instructions:

1. Be sure to write down your name and ID number, and indicate your section number above.
2. **Show your work. A reasonable amount of work is required to receive full credit.**
3. Intermediate steps earn points even if the final answer is incorrect. Erase (or cross out) any mistakes or you will be marked down.
4. For a vector quantity, both the magnitude and direction must be specified for full credit.
5. Express units in standard SI conventions for the final answer for each quantity you are asked to obtain. Points will be deducted if units are not reduced properly.

**Fundamental Constants and Conversions:**

\[ k = 8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2 \quad \varepsilon_0 = (4\pi k)^{-1} = 8.85 \times 10^{-12} \text{ C}^2 / (\text{N} \cdot \text{m}^2) \]
\[ m_p = 1.67 \times 10^{-27} \text{ kg} \quad m_e = 9.11 \times 10^{-31} \text{ kg} \quad \mu_0 = 4\pi \times 10^{-7} \text{ T} \cdot \text{m} / \text{A} \]
\[ q_p = -q_e = e = 1.6 \times 10^{-19} \text{ C} \quad 1 \text{ eV} = 1.6 \times 10^{-19} \text{ J} \]

Scores:

Problem 1  ................. Problem 2  ................. Problem 3  .................

Problem 4  ................. TOTAL  .................
Problem 1 (25 points)

A coil with 7 turns and a radius of 5cm rotates in a magnetic field $B$, of 3 Tesla, with an angular velocity, $\omega = \pi/2$ rad/sec. At time $t=0$, the angle between the vector normal to the coil and the B field is equal to 0. A resistor, $R$ of 50 $\Omega$ is attached to the coil.

(a) Sketch the flux of the magnetic field through the coil as a function of time. Label the x axis. 
$$\Phi_B = NAB \cos(\omega t)$$

(b) Sketch the motional EMF as a function of time. Label the x axis. 
$$\mathcal{E} = -\frac{d\Phi_B}{dt} = NABw \sin(\omega t)$$

(c) Sketch the current that goes through the resistor. Label the x axis. 
$$I = \frac{\mathcal{E}}{R} = \frac{NABw}{R} \sin(\omega t)$$
Problem 2 (30 points)

A solenoid with 50 turns, 10cm of radius and 15cm of length has an iron core. The permeability of iron is $\mu_i=4000\mu_0$.

\[ B = \mu \frac{N}{l} I \quad \mu = 4000 \mu_0 \]

a) Derive the formula of the self-inductance of the solenoid and calculate it.

\[ P_C = N B A = N \frac{\mu N I}{l} A = \frac{\mu N^2 I}{l} A \quad I = L I \]

\[ L = \frac{\mu N^2}{l} A \]

b) What is the energy stored in the solenoid if a current of 10A runs through it?

\[ E = \frac{1}{2} L I^2 = \frac{1}{2} \left( \frac{\mu N^2}{l} A \right) I^2 \]

\[ (J) \]

c) What is the energy density in the solenoid if a current of 10A runs through it?

\[ \rho = \frac{E}{V} = \frac{1}{2} \sqrt{\left( \frac{\mu N^2}{l} A \right)} I^2 \]

\[ (\frac{J}{m^2}) \]
d) What is the maximum current through the resistor?

\[ I_{\text{max}} = \frac{E_{\text{max}}}{R} = \frac{N A B \omega}{R} \quad (A) \]

e) What is the average power dissipated by the resistor?

\[ P_{\text{ave}} = \frac{RI_{\text{max}}^2}{2} = \frac{(N A B \omega)^2}{2 R} \quad (W) \]
Problem 3 (25 points)

Given this LRC circuit in series:

\[ R = 200 \, \Omega \]
\[ L = 50 \, \text{mH} \]
\[ C = 100 \, \mu\text{F} \]
\[ V = V_0 \sin \omega t \]
\[ V_0 = 10 \, \text{V} \]
\[ \omega / 2\pi = 100 \, \text{Hz} \]

a) What is the phase shift \( \varphi \) between the driving voltage \( V \) and current \( I \)?

\[ \varphi = \tan^{-1} \left( \frac{L \omega - \frac{1}{R \omega}}{R} \right) > 0 \] (rad)

b) Which of the following phasor diagrams qualitatively represents this circuit?

![Phasor Diagrams]

(a)  
(b)  
(c)  

d) The generator frequency is now changed to the resonant frequency of the circuit (and all other given parameters are left the same). What is the peak voltage across the inductor after this change?

\[ Z = R, \quad I_{\text{max}} = \frac{V_0}{R}, \quad \Delta V_L = I_{\text{max}} \frac{L \omega}{R} = \frac{V_0}{R} L \frac{1}{\sqrt{L C}} \] (V)

e) With the circuit in resonance, what is the average power \( P_{AV} \) delivered to the circuit by the generator?

\[ P_{AV} = \frac{R \overline{I_{\text{max}}}^2}{2} = \frac{V_0^2}{2R} \] (W)
d) Assuming that no current is flowing at \( t=0 \), calculate the EMF induced by a homogeneous magnetic field that is gradually turned on inside the solenoid such that \( B = \alpha t \), \( \alpha = 10 \ T/s \). The magnetic field is parallel to the axis of the solenoid.

\[
\Phi_B = NBA,
\]

\[
E = - \frac{d \Phi_B}{dt} = - \frac{d}{dt} \left( NBA \right) = -NA \frac{dB}{dt} = -NA \frac{\alpha t}{t} = -N A \alpha (V)
\]
Problem 4 (20 points)

Electro-magnetic waves

a) Which of the following statements are correct? Circle the correct answers

i. EM waves are longitudinal waves \[ \text{YES/NO} \]
ii. EM waves cannot propagate in solids \[ \text{YES/NO} \]
iii. EM waves cannot propagate in liquids \[ \text{YES/NO} \]
iv. EM waves can be decomposed into two polarizations \[ \text{YES/NO} \]
v. The vectors E and B have a constant relative phase of 90° \[ \text{YES/NO} \]
vi. EM waves carry momentum because they carry energy \[ \text{YES/NO} \]
vii. The Poynting vector is parallel to the direction of propagation \[ \text{YES/NO} \]
viii. The Poynting vector is expressed in units of pressure \[ \text{YES/NO} \]
ix. EM waves can produce standing waves \[ \text{YES/NO} \]
x. EM waves produce mechanical pressure on surfaces \[ \text{YES/NO} \]

b) Calculate the wavelength of an EM wave with a frequency of 500 GHz

\[
\lambda = \frac{C}{f} = \frac{3 \times 10^8 \text{ m/s}}{500 \times 10^9 \text{ Hz}} (\text{ m})
\]

c) Calculate the speed of an EM wave in a medium with permittivity \( \varepsilon = 2\varepsilon_0 \) and

\( \mu = 2\mu_0 \).

\[
V = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = \frac{1}{\sqrt{2\varepsilon_0 \cdot 2\mu_0}} = \frac{1}{2 \sqrt{\varepsilon_0 \mu_0}} = \frac{1}{2} C (\text{ m/s})
\]

d) The intensity of an EM wave is \( 10^5 \text{ W/m}^2 \). What is the force exerted on a surface of 10 m\(^2\) when 25% of the wave gets absorbed and 75% gets reflected?

\[
P = P_A + P_R = \frac{SA}{C} + 2 \frac{SR}{C} = \frac{1}{4} \frac{S}{C} + 2 \cdot \frac{3}{4} \frac{S}{C} = \frac{7}{8} \frac{S}{C} (\text{ N})
\]

\[
F = PS (\text{ N})
\]