Physics 202, Lecture 12

Today’s Topics

- Magnetic Field (Ch 29, part II)
- Force on Current Carrying Wires
- For and Torque on a Current Loop In Uniform B Field
- Magnetic dipoles
- Motion of a Charged Particle In a Uniform B Field
- Applications:
  - Magnetic Confinement
  - First Modern Particle Accelerator: Cyclotron
  - Mass Selector (q/m)
  - Hall Effect.

Magnetic Force On Current Carrying Wire
Segment (Review of Last Lecture)

- Magnetic force on a current segment of length L in uniform field B:
  \[ \mathbf{F}_B = \sum q \mathbf{v} \times \mathbf{B} = I \mathbf{L} \times \mathbf{B} \]

  - Key steps to derive:
    - Current: moving charges. \( I = q \mathbf{v} A \)
    - Magnetic force on charge: \( q \mathbf{v} \times \mathbf{B} \)
    - \( \mathbf{F}_B = q \mathbf{v} \times \mathbf{B} \) (ALn) = \( I \mathbf{L} \times \mathbf{B} \)

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Demo: Magnetic Force On A Current Carrying Wire

Top View

Also demo: Bending electron beam

Magnetic Force On Current Carrying Wire

- Magnetic force on a curved wire in uniform field \( \mathbf{B} \):
  \[ \mathbf{F}_B = I \mathbf{L} \times \mathbf{B} \]

  - Derivation: See board

Note: Net force on a current loop in uniform B field is zero.
**Review Exercise: Forces On A Current Loop**

- For a current loop in a uniform magnetic field as shown, what is the direction of the force on each side?
  
  (Lecture 11 review: \( \mathbf{F}_B = I \mathbf{L} \times \mathbf{B} \))

Recall: \( \sum \mathbf{F}_B = 0 \)

**Torque on a Current Loop In Uniform B Field**

- Exercise: For a current loop in a uniform B field, show that the torque on the loop is: \( \tau = I \mathbf{A} \times \mathbf{B} \)
  
  (Quiz: In what direction?)

  \( \Rightarrow \) Conveniently, the result can be rewritten as: \( \mathbf{F}_B = I \mathbf{A} \times \mathbf{B} \)

**Magnetic Dipole Moments**

- Magnetic dipole moment \( \mu \).
  
  Macroscopic: \( \mu = I \mathbf{A} \)
  
  Microscopic: \( \mu = \mathbf{L} \) (angular momentum of orbiting or spin)

**Review and Compare: Electric Dipole Moments**

- Electric dipole moment \( \mathbf{p} \).
  
  \( \sum \mathbf{F} = 0 \)
  
  \( \mathbf{\tau} = \mathbf{\mu} \times \mathbf{B} \)
  
  \( U = -\mathbf{\mu} \cdot \mathbf{B} \) (\( \mu \) in B Field)

\( \sum \mathbf{F} = 0 \)

\( \mathbf{\tau} = \mathbf{p} \times \mathbf{E} \)

\( U = -\mathbf{p} \cdot \mathbf{E} \)
Review of Chapter 8: Stable and Unstable Equilibrium

\[
\begin{align*}
F_B &= 0 \\
\tau_B &= 0 \\
U &= \mp \mu B \text{ (low)} \\
\frac{d^2U}{d\theta^2} &> 0
\end{align*}
\]

\[\Rightarrow\text{Stable Equilibrium}\]

\[
\begin{align*}
F_B &= 0 \\
\tau_B &= 0 \\
U &= \mp \mu B \text{ (high)} \\
\frac{d^2U}{d\theta^2} &< 0
\end{align*}
\]

\[\Rightarrow\text{Unstable Equilibrium}\]

The picture is also true for electric dipole moment

Quick Quiz 1

A magnetic dipole moment initially points at 45°. When a uniform horizontal B field is applied, which of the followings will happen?

1. No change
2. Points towards the B field
3. Points against the B field
4. Points normal to the B field

Quick Quiz 2

A magnetic dipole moment initially points at 135°. When a uniform horizontal B field is applied, which of the followings will happen?

1. No change
2. Points towards the B field
3. Points against the B field
4. Points normal to the B field

Motion Of Charged Particle in a Uniform B Field

Exercise: Show that if a charged particle q of mass m in a uniform B field has an initial velocity \( \mathbf{v} \) in the plane perpendicular to B, its motion is a uniform circular motion in that plane with

- radius \( r = \frac{mv}{qB} \)
- period: \( T = \frac{2\pi}{\nu} = \frac{2\pi m}{qB} \)

Note: \( T \) is independent of \( v \)

Solution: see board.
(recall: uniform circular motion)
Motion Of Charged Particle in a Uniform $\mathbf{B}$ Field – General 3D Case

- On the plane perpendicular to $\mathbf{B}$:
  - $r = \frac{mv_{\perp}}{qB}$
  - $T = \frac{2\pi m}{qB}$
- Parallel to $\mathbf{B}$:
  - spacing between helix $d = \frac{v_{\parallel} T}{v_{\perp}} = \frac{v_{\perp} 2\pi m}{qB}$

Application: Magnetic Confinement

- Magnetic Bottle
- Van Allen Belts
- Tokamak

MST: Madison Symmetric Torus

Application: Cyclotron (First Modern Particle Accelerator)

- Explain how a cyclotron works (see board)

First Cyclotron (1934)
Lawrence & Livinston
Application: Mass Selector

- Explain how a mass selector work (next week’s lab)
  - Speed selected: \( v = \frac{E}{B} \)

Mass selected:
\[
\frac{m}{q} = \frac{rB_0}{v} = rB_0/(E/B)
\]

Mass Selector: J.J Thomson Apparatus (1897)

- This is your lab next week: measuring e/m

Application: The Hall Effect

- Explain how Hall Effect works
  \[
  \Delta V_H = \frac{IB}{nqt}
  \]

See demo