Physics 202, Lecture 19

Today’s Topics

- AC Circuits with AC Source
- Resistors, Capacitors and Inductors in AC Circuit
- RLC Series in AC Circuit
- Impedance
- Resonances in Series RLC Circuit

AC Circuit

- Find out current $i$ and voltage difference $\Delta V_R$, $\Delta V_L$, $\Delta V_C$.

Notes:
- Kirchhoff’s rules still apply!
- A technique called phasor analysis is convenient.

Resistors in an AC Circuit

- $\Delta V - IR = 0$ at any time

$\Delta V = \Delta V_{max} \sin \omega t$

$i_R = \Delta V_{max}/R$

- The current through a resistor is in phase with the voltage across it

Inductors in an AC Circuit

- $\Delta V - L di/dt = 0$

$\Delta V = \Delta V_{max} \sin (\omega t - \pi/2)$

$i_L = \Delta V_{max}/X_L$

$X_L = \omega L$ → Inductive reactance

- The current through an inductor is 90° behind the voltage across it
Capacitors in an AC Circuit

- \( \Delta V - q/C = 0 \), \( dq/dt = i \)
- \( i = I_{\text{max}} \sin(\omega t + \pi/2) \)
- \( I_{\text{max}} = \Delta V_{\text{max}} / X_C \)
- \( X_C = 1/(\omega C) \) → capacitive reactance
- The current through a capacitor is 90° ahead of the voltage across it.

Summary of Phasor Relationship

Note that we set \( \Delta V \) w.r.t. \( I \)

RLC Series In AC Circuit

- The current at all point in a series circuit has the same amplitude and phase (set it be \( I_{\text{max}} \sin(\omega t) \))
- \( \Delta V_R = I_{\text{max}} R \sin(\omega t) \)
- \( \Delta V_L = I_{\text{max}} X_L \sin(\omega t + \pi/2) \)
- \( \Delta V_C = I_{\text{max}} X_C \sin(\omega t - \pi/2) \)

Voltage across RLC:
\( \Delta V_{\text{RLC}} = \Delta V_R + \Delta V_L + \Delta V_C \)
\( = I_{\text{max}} R \sin(\omega t) + I_{\text{max}} X_L \sin(\omega t + \pi/2) + I_{\text{max}} X_C \sin(\omega t - \pi/2) \)

Phasor Technique

- The phasor of \( \Delta V_{\text{RLC}} \) = vector sum of phasors for \( \Delta V_R \), \( \Delta V_L \), \( \Delta V_C \)
- \( \Delta V_{\text{max}} = \sqrt{\Delta V_R^2 + (\Delta V_L - \Delta V_C)^2} \)
- \( \phi = \tan^{-1} \left( \frac{X_L - X_C}{R} \right) \)
- \( \Delta V = \Delta V_{\text{max}} \sin(\omega t + \phi) \)

Note: \( X_L = \omega L \), \( X_C = 1/(\omega C) \)
Current And Voltages in a Series RLC Circuit

\[ \Delta V_{\text{max}} = \Delta V_{\text{max}} \sin(\omega t + \phi) \]

\[ \Delta v_R = (\Delta V_R)_{\text{max}} \sin(\omega t) \]

\[ \Delta v_L = (\Delta V_L)_{\text{max}} \sin(\omega t + \pi/2) \]

\[ \Delta v_C = (\Delta V_C)_{\text{max}} \sin(\omega t - \pi/2) \]

\[ Z = \sqrt{R^2 + (X_L - X_C)^2} \]

\[ \phi = \tan^{-1} \left( \frac{X_L - X_C}{R} \right) \]

\[ \Delta V = \Delta V_{\text{max}} \sin(\omega t + \phi) \]

Quiz: Can the voltage amplitudes across each component, \( (\Delta V_R)_{\text{max}} \), \( (\Delta V_L)_{\text{max}} \), \( (\Delta V_C)_{\text{max}} \) be larger than the overall voltage amplitude \( \Delta V_{\text{max}} \)?

Impedance

- For general circuit configuration:
  \[ \Delta V = \Delta V_{\text{max}} \sin(\omega t + \phi) \]

  \[ Z = \frac{\Delta V}{\Delta I} \]

- \( Z \) is called Impedance.

  e.g. RLC circuit:
  \[ Z = \sqrt{R^2 + (X_L - X_C)^2} \]

- In general impedance is a complex number, \( Z = Z_e + Z_i \).

  It can be shown that impedance in series and parallel circuits follows the same rule as resistors.

  \[ Z = Z_1 + Z_2 + Z_3 + \ldots \] (in series)

  \[ \frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} + \ldots \] (in parallel)

  (All impedances here are complex numbers)

Resonances In Series RLC Circuit

- The impedance of an AC circuit is a function of \( \omega \).
  - e.g. Series RLC:
    \[ Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \omega^2 L^2 - \frac{1}{\omega^2 C^2}} \]

  - when \( \omega = \omega_0 \):
    \[ Z = \frac{1}{\sqrt{LC}} \] (i.e. \( X_L = X_C \))

  - lowest impedance \( \rightarrow \) largest current \( \rightarrow \) resonance

- For a general AC circuit, at resonance:
  - Impedance is at lowest
  - Phase angle is zero. (In phase)
  - \( I_{\text{max}} \) is at highest
  - Power consumption is at highest
## Summary of Impedances and Phases

The table below summarizes the impedances and phase angles for various circuit element combinations:

<table>
<thead>
<tr>
<th>Circuit Elements</th>
<th>Impedance</th>
<th>Phase Angle $\phi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R$</td>
<td>$R$</td>
<td>$0^\circ$</td>
</tr>
<tr>
<td>$\frac{X}{R}$</td>
<td>$-\frac{X}{R}$</td>
<td>$-90^\circ$</td>
</tr>
<tr>
<td>$\frac{X}{L}$</td>
<td>$\frac{X}{L}$</td>
<td>$+90^\circ$</td>
</tr>
<tr>
<td>$\frac{1}{jR}$</td>
<td>$\frac{1}{R}$</td>
<td>Negative, between $-90^\circ$ and $0^\circ$</td>
</tr>
<tr>
<td>$\frac{1}{jL}$</td>
<td>$\frac{1}{L}$</td>
<td>Positive, between $0^\circ$ and $90^\circ$</td>
</tr>
<tr>
<td>$\frac{1}{jC}$</td>
<td>$\frac{1}{C}$</td>
<td>Negative if $X_C &gt; X_L$</td>
</tr>
<tr>
<td>$\frac{1}{jC}$</td>
<td>$\frac{1}{C}$</td>
<td>Positive if $X_C &lt; X_L$</td>
</tr>
</tbody>
</table>

*In each case, an AC voltage (not shown) is applied across the elements.

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