Today’s Topics

- Electromagnetic Waves (EM Waves)
- The Hertz Experiment
- Review of the Laws of Electro-Magnetism
- Maxwell’s equation
- Propagation of $\mathbf{E}$ and $\mathbf{B}$
  - The Linear Wave Equation

Review: Gauss’s Law / Coulomb’s Law

- The relation between the electric flux through a closed surface and the net charge $q$ enclosed within that surface is given by the Gauss’s Law

$$ \oint \mathbf{E} \cdot d\mathbf{A} = \frac{q}{\epsilon_0} $$

Gauss’s Law for Magnetism

- The Gauss’s Law for the electric flux is a reflection of the existence of electric charge. In nature we have not found the equivalent, a magnetic charge, or monopole.
- We can express this result differently: if any closed surface as many lines enter the enclosed volume as they leave it

$$ \oint \mathbf{B} \cdot d\mathbf{A} = 0 $$

Demo: Hertz Experiment

In 1887, Heinrich Hertz first demonstrated that EM fields can transmit over space.
Review: Faraday’s Law

- The emf induced in a “circuit” is proportional to the time rate of change of magnetic flux through the “circuit” or closed path.
  \[ \mathcal{E} = -\frac{d\Phi_B}{dt} \]

- Since \( \mathcal{E} = \oint \vec{E} \cdot d\vec{l} \)

- Then \( \oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt} \)

Review: Ampere’s Law

- A magnetic field is produced by an electric current is given by the Ampere’s Law
  \[ \oint \vec{B} \cdot d\vec{l} = \mu_0 I \]

- A changing electric field will also produce a magnetic field

Finally;

\[ \oint \vec{B} \cdot d\vec{l} = \mu_0 I + \varepsilon_0 \mu_0 \frac{d\Phi_E}{dt} \]

Maxwell Equations

- Gauss’s Law/ Coulomb’s Law: \( \oint \vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_0} \)
- Gauss’s Law of Magnetism, no magnetic charge: \( \oint \vec{B} \cdot d\vec{A} = 0 \)
- Faraday’s Law: \( \oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt} \)
- Ampere Maxwell Law: \( \oint \vec{B} \cdot d\vec{l} = \mu_0 I + \varepsilon_0 \mu_0 \frac{d\Phi_E}{dt} \)

Also, Lotentz force Law \( \vec{F} = q\vec{E} + q\vec{v} \times \vec{B} \)

These are the foundations of the electromagnetism
Linear Wave Equation

- Linear wave equation
  - Certain physical quantity
  - Wave speed

\[
\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2}
\]

Sinusoidal wave

- f: frequency
- \(\phi\): Phase
- \(A\): Amplitude
- \(\lambda\): Wavelength

General wave: superposition of sinusoidal waves

Electromagnetic Waves

- EM wave equations:
  \[
  \frac{\partial^2 E_y}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E_y}{\partial t^2} \quad \frac{\partial^2 B_x}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 B_x}{\partial t^2}
  \]

- Plane wave solutions:
  \[
  E = E_{\text{max}} \cos(kx - \omega t + \phi) \quad B = B_{\text{max}} \cos(kx - \omega t + \phi)
  \]

- Properties:
  - No medium is necessary.
  - E and B are normal to each other
  - E and B are in phase
  - Direction of wave is normal to both E and B
    (EM waves are transverse waves)
  - Speed of EM wave:
    \[
    c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 2.9972 \times 10^8 \text{ m/s}
    \]
  - \(E/B = E_{\text{max}}/B_{\text{max}} = c\)
  - Transverse wave: two polarizations possible

The EM Wave

Two polarizations possible (showing one)