Physics 202, Lecture 22

Today’s Topics

- Electromagnetic Waves (EM Waves)
- Review: Waves and Wave Equation
- Maxwell’s equation
- Propagation of $E$ and $B$
- Energy Carried by EM Wave, Poynting Vector
- Momentum Carried by EM Wave
- Spectrum of EM wave.

Maxwell Equations

\[
\oint E \cdot d\mathbf{A} = \frac{q}{\varepsilon_0} \quad \Rightarrow \text{Gauss’s Law/ Coulomb’s Law}
\]

\[
\oint B \cdot d\mathbf{A} = 0 \quad \Rightarrow \text{Gauss’s Law of Magnetism, no magnetic charge}
\]

\[
\oint E \cdot d\mathbf{l} = -\frac{d\Phi_B}{dt} \quad \Rightarrow \text{Faraday’s Law}
\]

\[
\oint B \cdot d\mathbf{l} = \mu_0 I + \varepsilon_0 \mu_0 \frac{d\Phi_E}{dt} \quad \Rightarrow \text{Ampere Maxwell Law}
\]

Also, Lorentz force Law $\Rightarrow$

\[
\mathbf{F} = q\mathbf{E} + q\mathbf{v} \times \mathbf{B}
\]

These are the foundations of the electromagnetism

Electromagnetic Waves

- EM wave equations:
  \[
  \frac{\partial^2 E_y}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E_y}{\partial t^2}, \quad \frac{\partial^2 B_z}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 B_z}{\partial t^2}
  \]

- Plane wave solutions:
  \[
  E = E_{\text{max}} \sin(kx-\omega t+\phi), \quad B = B_{\text{max}} \sin(kx-\omega t+\phi)
  \]

- Properties:
  - No medium is necessary.
  - $E$ and $B$ are normal to each other.
  - $E$ and $B$ are in phase.
  - Direction of wave is normal to both $E$ and $B$.
  - (EM waves are transverse waves)
  - Speed of EM wave:
    \[
    c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 2.9972 \times 10^8 \text{ m/s}
    \]
  - $E/B = E_{\text{max}}/B_{\text{max}} = c$
  - Transverse wave: two polarizations possible

The EM Wave

Two polarizations possible
Wavelength and Frequency

- Because of the wave equation the wavelength of and frequency of a EM wave in vacuum are related by:
  \[ \lambda f = c = 3 \cdot 10^8 \text{ m/s} \]

- Example: Determine the wavelength of an EM wave of frequency 50 MHz in free space

\[ \lambda = \frac{c}{f} = \frac{3 \cdot 10^8 \text{ m/s}}{50 \text{ MHz}} = 6 \text{ m} \]

Energy Carried By EM Waves

- Recall: energy densities \( u_E = \frac{1}{2} \varepsilon_0 E^2 \), \( u_B = \frac{1}{2} B^2/\mu_0 \)

- For a EM wave, at any time/location,
  \( u_E = \frac{1}{2} \varepsilon_0 E^2 = \frac{1}{2} B^2/\mu_0 = u_B \) (using \( E/B = c \))
  
  In an electromagnetic wave, the energies carried by electric field and magnetic field are always the same.

- Total energy stored (per unit of volume):
  \( u = u_E + u_B = \varepsilon_0 E^2 = B^2/\mu_0 \)

- Power transmitted per unit of area is equal to \( uc \) in the direction of wave

- Averaging over time:
  \( u_{av} = \frac{1}{2} \varepsilon_0 E_{max}^2 = \frac{1}{2} B_{max}^2/\mu_0 \), \( u_{av}c = I \) (intensity)

The Poynting Vector

- The rate of flow of energy in an EM wave is described by a vector, \( \vec{S} \), called the Poynting vector

- The Poynting vector is defined as
  \[ \vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B} \]

- Its direction is the direction of propagation

- This is time dependent
  - Its magnitude varies in time
  - Its magnitude reaches a maximum at the same instant as \( E \) and \( B \)
Momentum Carried By EM Waves

- **EM waves**: momentum = energy/c

\[ \Delta p = \frac{\Delta U}{c} = \frac{uA\Delta t}{c} = \frac{uA\Delta t}{c} \]

**Change of momentum in 100% absorption**

\[ \Delta p = \frac{\Delta U}{c} = \frac{uA\Delta t}{c} = uA\Delta t \]

**Change of momentum in 100% reflection**

\[ \Delta p = 2 \frac{\Delta U}{c} = 2 \frac{uA\Delta t}{c} = 2uA\Delta t \]

**Radiation Pressure (P):**

\[ P = \frac{F}{A} = \frac{\Delta p}{\Delta t} = u = \frac{S}{c} \]

100% absorption

\[ \Delta p = p \rightarrow P = \frac{S}{c} \]

100% reflection

\[ \Delta p = 2p \rightarrow P = 2\frac{S}{c} \]

Example: Solar Energy

- The average intensity of the EM radiation from the Sun on Earth is \( S \sim 10^3 \text{ W/m}^2 \)

  - What is the average radiation pressure for 100% absorption:
    \[ P = \frac{S}{c} = \frac{10^3 \text{ W/m}^2}{3 \cdot 10^8 \text{ m/s}} = 3.3 \cdot 10^{-6} \text{ N/m}^2 \]

  - What is the force exerted by EM radiation by the Sun on a surface of 1 m²
    \[ F = PA = 3.3 \cdot 10^{-6} \text{ N/m}^2 \cdot 1\text{ m}^2 = 3.3 \cdot 10^{-6} \text{ N} \]

Antennas

- Antennas are essentially arrangement of conductors for transmitting and receiving radio waves.
- Parameters: gain, impedance, frequency, orientation, polarization, etc.

- half-wave antenna

- maximum strength

- \( \lambda/4 \)

- \( \lambda/4 \)

- Beverages
  - rhombic
  - v-antenna
  - Yagi
  - helical
  - loop
  - microstrip
  - log-periodic