Today’s Topics

- Wave Nature of Waves: Interference
  - Breakdown of ray approximation
  - Huygen’s principle
  - Light as Waves
  - Double-Slit Interference
  - Multi-Slit Interference

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**Ray Approximation**

- When the wavelength of the light is much smaller than the size of the optical objects it encounters, it can be treated as (colored) rays.

  ![Ray approximation](image)

  Ray approximation is valid when \( \lambda \ll d \)

  Ray approximation is not valid near the gap when \( \lambda \sim d \). OK elsewhere

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**The Huygens’ Principle**

- Every point on a wave front can be considered as a secondary source of waves that spread out in the forward direction. The new wave is the result of the superposition of these secondary waves.
Reminder: Light Waves

- Nature of Lights:
  Rays (classical), EM waves, Photons.
- Review: Electromagnetic plane waves
  \( E = E_{\text{max}} \sin(\omega t-kx+\phi) \), \( B = B_{\text{max}} \sin(\omega t-kx+\phi) \), \( E/B = c \)
  - As the E component and B component of an EM wave are 100% correlated, we can use just one of them to represent an EM wave.

Interference of Light Waves

- When two light waves meet at certain location, the resulting effect is determined by the superposition (i.e. sum) of the two individual waves
  - e.g. Two light waves with same color and amplitude.
    \( E_1 = E_0 \sin(\omega t-kx+\phi_1) = E_0 \sin(\omega t+\phi_1) \)
    \( E_2 = E_0 \sin(\omega t-kx+\phi_2) = E_0 \sin(\omega t+\phi_2) \)
  \( \rightarrow E = E_1 + E_2 = 2E_0 \cos(\Delta \phi/2) \sin(\omega t+\phi/2) \)
  \( \rightarrow \text{Resulting amplitude: } E_{\text{max}} = 2E_0 \cos(\Delta \phi/2) \)
    - Constructive interference: \( \Delta \phi = 0, 2\pi, 4\pi, \ldots \), \( E_{\text{max}} = 2E_0 \)
    - Destructive interference: \( \Delta \phi = \pi, 3\pi, 5\pi, \ldots \), \( E_{\text{max}} = 0 \)

Quiz: If the intensity of each incoming light is \( I \), what is the resulting intensity when (1): constructive, (2): destructive?

Test of the Wave Nature of Light: Double-Slit Experiment

- Rays or Waves:
  - If lights behave as rays
  - If lights behave as waves

Young's Famous Double-Slit Experiment

Thomas Young (1803)

- See demo
Double-Slit Experiment Explained

- The experiment can be easily explained by interference.

Constructive, $\Delta \phi = 0\pi, 2\pi, 4\pi,\ldots$  
Destructive, $\Delta \phi = \pi, 3\pi, 5\pi,\ldots$

Quantitatively

Path length difference $\delta = d\sin \theta - d\theta \sim d\gamma / l$

$\Delta \phi = k(r_2 - r_1) = kd \sin \theta = \frac{2\pi d}{\lambda} \sin \theta$

$I = I_o \cos^2 \left( \frac{\pi d \sin \theta}{\lambda} \right)$

Double-Slit Experiment Explained

- The experiment can be easily explained by interference.

Constructive, $\Delta \phi = 0\pi, 2\pi, 4\pi,\ldots$, or $2m\pi$, $m=0,1,2\ldots$

$\frac{2\pi d}{\lambda} \sin \theta = 2m\pi \quad \rightarrow \quad d \sin \theta = m\lambda$

Bright spots

Destructive, $\Delta \phi = \pi, 3\pi, 5\pi,\ldots$, or $(2m+1)\pi$, $m=0,1,2\ldots$

$\frac{2\pi d}{\lambda} \sin \theta = 2(m+1)\pi \quad \rightarrow \quad d \sin \theta = \left( m + \frac{1}{2} \right)\lambda$

Dark spots
Multi-Slit Interference

- # secondary maxima = N - 2
- Higher N → more suppression on secondary minima

(Grating: N > 1000, highly sensitive to λ, good for measuring λ.)