Physics 202, Lecture 8

Today’s Topics

- Middle Term 1 Review

About Exam 1

- When and where
  - Monday Feb. 16th 5:30-7:00 pm
  - 2650, 3650 Humanities (room allocation to be announced)

- Format
  - Closed book
  - One 8x11 formula sheet allowed, must be self prepared, no photo copy of solutions, lecture slides, etc.
  - Four full problems (~20 questions)
  - Bring a calculator (but no computer). Only basic calculation functionality can be used.

- Special requests:
  - Need to be approved by now.
  - McBurney students, please inform your proctor.
  - All specially arranged tests (e.g. those at alternative time) are held in our 202 labs. (for approved requests only)

Chapters Covered

- Chapter 23: Electric Fields
- Chapter 24: Gauss’s Law
- Chapter 25: Electric Potential
- Chapter 26: Capacitance

Exam Topics (1)

- Key concepts
  ("key": those in summary box at the end of each chapter)

- Basic Quantities:
  - Electric Charge
  - Electric Force
  - Electric Field, Field Lines
  - Electric Flux
  - Electric Potential
  - Electric Potential Energy
  - Capacitance
  - Energy carried by electric field

Disclaimer

- This review is a supplement to your own preparation.
- Hints and exercises presented in this review are not meant to be complete.
Exam Topics(2)

- Electric charge
  - Two types
  - Total charge is conserved.
- Electric force
  - Can be attractive/repulsive
  - Coulomb’s Law
- Electric field
  - Electric field is a form of matter, it carries energy.
  - Electric field is independent of test charge.
  - Electric field is a vector quantity.
  - Three ways to calculate electric field:
    - Direct vector sum
    - Gauss’s Law
    - Derivative of \( V \)
  - \( F=qE \) (note: \( E \) does not include the one created by \( q \))

Exam Topics(3)

- Electric potential energy.
  - Electric force is a conservative force
  - Electric potential energy depends on both the source and test charge.
  - Like all potential energies, the electric potential energy is relative to a certain reference state. (Usually, an “infinity” state is taken as \( U=0 \).)
  - Energy conservation, work-kinetic energy theorem, etc. are applicable to electric potential energy too.
- Electric potential
  - Electric potential depends only on the source.
  - Electric Potential and Electric Field are closely related. \( (E \leftrightarrow V) \)
  - Electric potential \( (V) \) and electric potential energy \( (U) \) are different quantities.
    - \( \rightarrow \) higher \( V \) does not necessarily mean higher \( U \).
  - Electric potential and Electric potential energy are related: \( U=qV \)

Exam Topics(4)

- Conductors and Electrostatic Equilibrium
  - Regardless of shape:
    - The electric field is zero inside the conductor.
    - All net charges reside on the surface of conductor.
    - E field on the surface of conductor is always normal to the surface, and has a magnitude of \( \sigma/\varepsilon_0 \).
    - The electric field is also zero inside an empty cavity within the conductor.
    - Potential is the same throughout the whole conductor (Equipoential)
- Capacitance
  - \( C=Q/V \)
  - Connection in parallel and in series
  - Energy stored in capacitor
  - Dielectrics enhance the capacitance by a factor of \( \kappa \).

Reminder:

Three Ways to Calculate Electrostatic Field

- Superposition with Coulomb’s Law (first principle):
  \[
  E = k \sum \frac{q}{r^2} \Rightarrow E = k \int \frac{dq}{r^2}
  \]
- Apply Gauss’s Law:
  (Practical only for cases with high symmetry)
  \[
  \Phi = \oint E \cdot dA = \sum \frac{q_e}{\varepsilon_0}
  \]
- From a known potential:
  \[
  E_x = -\frac{\partial V}{\partial x}, \quad E_y = -\frac{\partial V}{\partial y}, \quad E_z = -\frac{\partial V}{\partial z}
  \]
**Exercise 1: Seven Point Charges**

- Six point charges are fixed at corners of a hexagon as shown. A seventh point charge \( q_7 = 2Q \) is placed at the center.

1. What is the force on \( q_7 \)?
2. What is the minimum energy required to bring \( q_7 \) from infinity to its current position at the center?

**Solutions:** (See board)

- First thoughts:
  - Point charges: Coulomb's Law
  - Be aware of symmetry
  - Energy: \( \Delta U = q \Delta V \)

**Exercise 2: Three shells of charge**

- As shown below three thin sphere shells have radius \( R, 2R, 4R \), and charges \( Q, -Q, 2Q \), respectively.

  - Use Gauss's law, find the electric field distribution.
  - The setting is highly symmetrical
  - Gauss' surface will be concentric sphere of radius \( r \).

  \[
  \oint \vec{E} \cdot d\vec{A} = 4\pi \varepsilon_0 \cdot \int \frac{1}{q_{\text{enclosed}}} E = \frac{1}{\varepsilon_0} q_{\text{enclosed}}
  \]

  - \( 0 < r < R: \quad q_{\text{enclosed}} = 0 \quad \Rightarrow \quad E = 0 \)
  - \( R < r < 2R: \quad q_{\text{enclosed}} = Q \quad \Rightarrow \quad E = Q/(4\pi \varepsilon_0 r^2) \)
  - \( 2R < r < 4R: \quad q_{\text{enclosed}} = Q - Q = 0 \quad \Rightarrow \quad E = 0 \)
  - \( 4R < r: \quad q_{\text{enclosed}} = Q - Q + 2Q = 2Q \quad \Rightarrow \quad E = 2Q/(4\pi \varepsilon_0 r^2) \)

**Exercise 3: Potential and Field**

- The electric potential of a field is described by

  \[ V = 3x^2y + y^2 + yz. \]

  - Find the force on a test charge \( q = 1C \) at \((x,y,z) = (1,1,1)\)

  **Solution:**

  - First thoughts:
    - \( E_x = -dV/dx = -6x = -6 @ (1,1,1) \)
    - \( E_y = -dV/dy = -(3x^2 + 2y + z) = -6 @ (1,1,1) \)
    - \( E_z = -dV/dz = -y = -1 @ (1,1,1) \)

  - \( F_x = qE_x, \quad F_y = qE_y, \quad F_z = qE_z \)

  \[ (F_x, F_y, F_z) = qE = (-6, -6, -1) \text{ N} \]

**Reminder: Conductors in Electrostatic Equilibrium**

- Oops! What is it? I forgot myself!

  - Please review Lecture 5 and corresponding text!
Quick Exercise: Charge Distribution On Conductors

The total charge on this conductor shell is $+5q$. A point charge of $+q$ is placed at the center ($r=0$). How is the charge distributed?

(shell radius: $r_{inner}=R$, $r_{outer}=2R$)

- $Q_{inner\_surface} = -q$, $Q_{outer\_surface} = 6q$, $Q_{body} = 0$
- $Q_{inner\_surface} = q$, $Q_{outer\_surface} = 4q$, $Q_{body} = 0$
- $Q_{inner\_surface} = 0$, $Q_{outer\_surface} = 5q$, $Q_{body} = 0$

Challenge to you

Are you able to calculate $E$ and $V$ at $r=0.5R$, $1.5R$, $2.5R$?

(discuss with your TAs if in puzzle)

Reminder: A Picture to Remember

- Field lines always point towards lower electric potential
- Field lines and equal-potential lines are always at a normal angle.
- In an electric field:
  - $+q$ is always subject to a force in the same direction of field line. (i.e. towards lower $V$)
  - $-q$ is always subject to a force in the opposite direction of field line. (i.e. towards higher $V$)

Exercise 4: Potential, Field, and Energy

The equal potential lines surrounding two conductors, $+10V$ and $+15V$, are shown below.

- Draw on the figure the direction of electric field at point C.
- If a charge of $Q=+0.5C$ is to be moved from point B to C, how much work is required?

\[
W_{B\rightarrow C} = U_C - U_B = q(V_C - V_B) = 0.5 \times (12 - 9) = 1.5J
\]

Exercise 5: Electric Potential Energy

A charged non-conducting ring of radius $R$ has charge $-Q$ distributed on the left side and $2Q$ distributed on the right side.

- How much energy is required to bring a point charge $q=0.5Q$ from infinity to the center of the ring?
- What if $q=-0.5Q$ instead?

Solution: See board

First Thought:
Energy required = external work required to bring in $q$.
Follow up: How to calculate work?
- integral $dw=FdS$ (won't work, too complicated)
- use energy conservation & the idea of electric potential

\[
F = qE, \quad \Delta U = q\Delta V, \quad W = \Delta U = \Delta V = -qE dS
\]
Reminder: Capacitors

- Capacitance: \( C = \frac{Q}{V} \)  \( (Q = CV) \)
- Two capacitors in series:
  \[ Q = Q_1 = Q_2, \quad V = V_1 + V_2 \Rightarrow \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} \]
- Two capacitors in parallel:
  \[ V = V_1 = V_2, \quad Q = Q_1 + Q_2 \Rightarrow \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} \]
- Energy storage: \( U = \frac{1}{2} CV^2 \)
- Dielectrics: \( C = \kappa C_0 \)

Exercise 6: Two Capacitors

- Two capacitors, \( C_1 = 1 \mu F \) and \( C_2 = 2 \mu F \), are connected in series as shown. A voltage of 30V is applied across the two capacitors.
  - What is the combined capacitance?
  - What is the charge on \( C_1 \)?
  - What is the voltage on \( C_1 \)?
  - What is the energy stored on \( C_1 \)?

Solution:

- \[ \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} \Rightarrow C = \frac{C_1 C_2}{C_1 + C_2} = 0.67 \mu F \]
- \[ Q = Q_1 = Q_2 = CV = 0.67 \mu F \times 30V = 2 \times 10^{-5} \text{ Coulomb} \]
- \[ V_1 = Q_1 / C_1 = 2 \times 10^{-5} / 1 \times 10^{-6} = 20 \text{ V} \]
- \[ U_1 = \frac{1}{2} C_1 V_1^2 = 2 \times 10^{-4} \text{ J} \]

Final Reminder

Write your solutions in clear steps!

- You’ll have better chance to get your answers right
- You may get partial credits