Lasers

Light Amplification by the Stimulated Emission of Radiation

Beer's Law:

\[ I = c u \]

Want \( \frac{dI}{dx} \): Absorption \( \frac{dI}{dx} = - (\text{# of } \gamma\text{'s absorbed}) \cdot c \cdot \text{hw} \)

\[ \frac{dI}{dx} = - (B_0 u N_0) c \frac{\text{little} \text{ hw}}{X} \]

\[ = - \text{hw} \text{B} \int \text{I} \cdot N_0 = - N_0 \sigma \text{I} \]

Stimulated Emission: the intensity is increased

\[ \frac{dI}{dx} = + N_1 \sigma \text{I} \]

\[ \frac{dI}{dx} = (N_1 - N_0) \sigma \text{I} \]

can be positive or negative depending on sign of \( N_1 - N_0 \)

If \( N_1 > N_0 \), we have a "population inversion" and the light intensity goes up:

\[ I(x) = I(0) e^{(N_1 - N_0) \sigma x} \]
The hard part of making a laser is getting $N_1 > N_0$; your book discusses this in some detail in Section 13-4.

Assume we have $N_1 > N_0$, how does this make a laser?

$\begin{array}{c}
\quad \\
\quad 0 \quad 1 \quad 2 \quad 3 \\
\quad 4 \quad 5 \\
\end{array}$

Transmits a fraction $T$ of the light

reflects $R = 1 - T$

Follow the light around the cavity.

Start at $O$: assume the light has intensity $I_0$.

$I_1 = I_0$ (no atoms in between!)

$I_2 = I_0 \cdot (N_2 - N_1) \sigma l$

$I_3 = I_2$

$I_4 = R I_3 = I_0 R E$

$I_5 = I_4 \cdot (N_2 - N_1) \sigma l = 2 (N_2 - N_1) \sigma l$

Completing the loop, we must have $I_0 = I_5$

Then $I_0 = I_5 = I_0 R E$

$2(N_2 - N_1) \sigma l$


2 solutions:  
1) \( I_0 = 0 \) (no laser!)  
2) \( I = R e^{2(N_r-N_0)0.2l} \)

> Laser oscillation condition: for 1 trip around the cavity, the loss at the mirror \( R \) must be balanced by the gain due to the population inversion

\[
\ln \frac{1}{R} = 2(N_r-N_0)0.2l
\]

\[
\Rightarrow N_r-N_0 = \frac{1}{20l} \ln \frac{1}{R} \tag{1}
\]

When this condition is met, light will leave the cavity through the mirror: the laser intensity \( I_{\text{las}} = T I_3 \)

What determines \( I_3 \)? Suppose we increased the inversion beyond the minimum of Eq. \( \text{1} \). Then the laser intensity would keep growing forever! It turns out that \( N_r-N_1 \) is not fixed but decreases with increasing intensity. Therefore, the laser intensity \( I_L \) is found from

\[
(N_r-N_0)(I) = \frac{1}{20l} \ln \frac{1}{R}
\]
This is an example of feedback; if the population inversion changes, the laser intensity automatically changes to keep \[ N_f - N_0 = \frac{1}{2 \pi e} \frac{\text{ln} I}{I} \]