Lecture 2
Goals: (Highlights of Chaps. 1 & 2.1-2.4)
- Units and scales, order of magnitude calculations, significant digits (on your own for the most part)
- Distinguish between Position & Displacement
- Define Velocity (Average and Instantaneous), Speed
- Define Acceleration
- Understand algebraically, through vectors, and graphically the relationships between position, velocity and acceleration
- Perform Dimensional Analysis

Reading Assignment:
- For next class: Finish reading Ch. 2, read Chapter 3 (Vectors)
## Length

<table>
<thead>
<tr>
<th>Distance</th>
<th>Length (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radius of Visible Universe</td>
<td>$1 \times 10^{26}$</td>
</tr>
<tr>
<td>To Andromeda Galaxy</td>
<td>$2 \times 10^{22}$</td>
</tr>
<tr>
<td>To nearest star</td>
<td>$4 \times 10^{16}$</td>
</tr>
<tr>
<td>Earth to Sun</td>
<td>$1.5 \times 10^{11}$</td>
</tr>
<tr>
<td>Radius of Earth</td>
<td>$6.4 \times 10^{6}$</td>
</tr>
<tr>
<td>Sears Tower</td>
<td>$4.5 \times 10^{2}$</td>
</tr>
<tr>
<td>Football Field</td>
<td>$1 \times 10^{2}$</td>
</tr>
<tr>
<td>Tall person</td>
<td>$2 \times 10^{0}$</td>
</tr>
<tr>
<td>Thickness of paper</td>
<td>$1 \times 10^{-4}$</td>
</tr>
<tr>
<td>Wavelength of blue light</td>
<td>$4 \times 10^{-7}$</td>
</tr>
<tr>
<td>Diameter of hydrogen atom</td>
<td>$1 \times 10^{-10}$</td>
</tr>
<tr>
<td>Diameter of proton</td>
<td>$1 \times 10^{-15}$</td>
</tr>
</tbody>
</table>

See [http://micro.magnet.fsu.edu/primer/java/scienceopticsu](http://micro.magnet.fsu.edu/primer/java/scienceopticsu)

## Time

<table>
<thead>
<tr>
<th>Interval</th>
<th>Time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age of Universe</td>
<td>$5 \times 10^{17}$</td>
</tr>
<tr>
<td>Age of Grand Canyon</td>
<td>$3 \times 10^{14}$</td>
</tr>
<tr>
<td>Avg age of college student</td>
<td>$6.3 \times 10^{8}$</td>
</tr>
<tr>
<td>One year</td>
<td>$3.2 \times 10^{7}$</td>
</tr>
<tr>
<td>One hour</td>
<td>$3.6 \times 10^{3}$</td>
</tr>
<tr>
<td>Light travel from Earth to Moon</td>
<td>$1.3 \times 10^{0}$</td>
</tr>
<tr>
<td>One cycle of guitar A string</td>
<td>$2 \times 10^{-3}$</td>
</tr>
<tr>
<td>One cycle of FM radio wave</td>
<td>$6 \times 10^{-8}$</td>
</tr>
<tr>
<td>One cycle of visible light</td>
<td>$1 \times 10^{-15}$</td>
</tr>
<tr>
<td>Time for light to cross a proton</td>
<td>$1 \times 10^{-24}$</td>
</tr>
</tbody>
</table>
Order of Magnitude Calculations / Estimates

Question: If you were to eat one french fry per second, estimate how many years would it take you to eat a linear chain of trans-fat free french fries, placed end to end, that reach from the Earth to the moon?

- Need to know something from your experience:
  - Average length of french fry: 3 inches or 8 cm, 0.08 m
  - Earth to moon distance: 250,000 miles
  - In meters: $1.6 \times 2.5 \times 10^5$ km = $4 \times 10^8$ m
  - $1 \text{ yr} \times 365 \text{ d/yr} \times 24 \text{ hr/d} \times 60 \text{ min/hr} \times 60 \text{ s/min} = 3 \times 10^7 \text{ sec}$

\[
\text{ff} \approx \frac{4 \times 10^8 \text{ m}}{8 \times 10^{-2}} \approx 0.5 \times 10^{10} \text{ (to moon)}
\]

\[
0.5 \times 10^{10} \text{ sec.} = \frac{5 \times 10^9 \text{ s}}{3 \times 10^7 \text{ s/yr}} = 200 \text{ years}
\]
**Dimensional Analysis (reality check)**

- This is a very **important** tool to check your work
  - Provides a reality check (if dimensional analysis fails then no sense in putting in the numbers)

- Example
  When working a problem you get an expression for distance
  \[ d = v t^2 \text{ (velocity \cdot time^2)} \]

  Quantity on left side \( d \rightarrow L \rightarrow \text{length} \)
  (also \( T \rightarrow \text{time} \) and \( v \rightarrow \text{m/s} \rightarrow L / T \))

  Quantity on right side = \( L / T \times T^2 = L \times T \)

- Left units and right units don’t match, so answer is nonsense

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**Exercise 1**

**Dimensional Analysis**

- The force (\( F \)) to keep an object moving in a circle can be described in terms of:
  - velocity (\( v \), dimension \( L / T \)) of the object
  - mass (\( m \), dimension \( M \))
  - radius of the circle (\( R \), dimension \( L \))

Which of the following formulas for \( F \) **could** be correct ?

<table>
<thead>
<tr>
<th>Note: Force has dimensions of ( ML/T^2 ) or ( kg \cdot m / s^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) ( F = mvR )</td>
</tr>
</tbody>
</table>
Moving between pictorial and graphical representations

- Example: Initially I walk at a constant speed along a line from left to right, next smoothly slow down somewhat, then smoothly speed up, and, finally walk at the same constant speed.
  1. Draw a pictorial representation of my motion by using a particle model showing my position at equal time increments.
  2. Draw a graphical “xy” representation of my motion with time on the x-axis and position along the y-axis.

Need to develop quantitative method(s) for algebraically describing:

1. Position
2. Rate of change in position (vs. time)
3. Rate of change in the change of position (vs. time)

Tracking changes in position: VECTORS

- Position

- Displacement (change in position)

- Velocity (change in position with time)

- Acceleration
Motion in One-Dimension (Kinematics)

Position / Displacement

- Position is usually measured and referenced to an origin:

  - At time $= 0$ seconds Joe is 10 meters to the right of the lamp
  - origin = lamp
  - positive direction = to the right of the lamp
  - position vector:

    \[ \text{position vector: } \begin{pmatrix} 10 \text{ meters} \\ 0 \end{pmatrix} \]

- One second later Joe is 15 meters to the right of the lamp

Displacement is just change in position.

\[ \Delta x = x_f - x_i \]

\[ x_f = x_i + \Delta x \]

\[ \Delta x = x_f - x_i = 5 \text{ meters} \]

\[ \Delta t = t_f - t_i = 1 \text{ second} \]
Speed & Velocity
Changes in position vs Changes in time

- Average velocity = net distance covered per total time,
  \[ \bar{v} (\text{average velocity}) = \frac{\Delta x (\text{net displacement})}{\Delta t (\text{total time})} \]
- Speed, \( s \), is usually just the magnitude of velocity.
  - The “how fast” without the direction.
- Average speed references the total distance travelled
  \[ \bar{s} (\text{average speed}) = \frac{\text{distance taken along path}}{\Delta t (\text{total time})} \]

Active Figure 1  http://www.phy.ntnu.edu.tw/ntnujava/main.php?l=282

Representative examples of speed

<table>
<thead>
<tr>
<th>Speed</th>
<th>(m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Speed of light</td>
<td>(3 \times 10^8)</td>
</tr>
<tr>
<td>Electrons in a TV tube</td>
<td>(10^7)</td>
</tr>
<tr>
<td>Comets</td>
<td>(10^6)</td>
</tr>
<tr>
<td>Planets orbital speeds</td>
<td>(10^5)</td>
</tr>
<tr>
<td>Satellite orbital speeds</td>
<td>(10^4)</td>
</tr>
<tr>
<td>Mach 3</td>
<td>(10^3)</td>
</tr>
<tr>
<td>Car</td>
<td>(10^1)</td>
</tr>
<tr>
<td>Walking</td>
<td>1</td>
</tr>
<tr>
<td>Centipede</td>
<td>(10^{-2})</td>
</tr>
<tr>
<td>Motor proteins</td>
<td>(10^{-6})</td>
</tr>
<tr>
<td>Molecular diffusion in liquids</td>
<td>(10^{-7})</td>
</tr>
</tbody>
</table>
**Instantaneous velocity**  
**Changes in position vs Changes in time**

- Instantaneous velocity, velocity at a given instant

\[
v(\text{velocity}) = \lim_{\Delta t \to 0} \frac{\Delta x(\text{displacement})}{\Delta t(\text{time})} = \frac{dx}{dt}
\]

---

**Exercise 2 Average Velocity**

<table>
<thead>
<tr>
<th>x (meters)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>t (seconds)</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

What is the **average** velocity over the first 4 seconds?

- (A) -1 m/s  
- (B) 4 m/s  
- (C) 1 m/s  
- (D) not enough information to decide.
Average Velocity Exercise 3

What is the average velocity in the last second (t = 3 to 4)?

- A. 2 m/s
- B. 4 m/s
- C. 1 m/s
- D. 0 m/s

Exercise 4

Instantaneous Velocity

What is the instantaneous velocity at the fourth second?

- (A) 4 m/s
- (B) 0 m/s
- (C) 1 m/s
- (D) not enough information to decide.
Average Speed Exercise 5

What is the average speed over the first 4 seconds?
Here we want the ratio: total distance travelled / time
(Could have asked “what was the speed in the first 4 seconds?”)

Key point:
• If the position $x$ is known as a function of time, then we can find both velocity $v$

$$x = x(t)$$
$$v_x = \frac{dx}{dt}$$
$$x = \int dt \ v(t)$$

• “Area” under the $v(t)$ curve yields the change in position
• Algebraically, a special case, if the velocity is a constant then

$$x(\Delta t) = v \ \Delta t + x_0$$
Motion in Two-Dimensions (Kinematics)
Position / Displacement

- Amy has a different plan (top view):

  - At time = 0 seconds Amy is 10 meters to the right of the lamp (East)
  - origin = lamp
  - positive x-direction = east of the lamp
  - position y-direction = north of the lamp

\[ \Delta \vec{r} \equiv \text{Displacement vector} = \vec{r}_f - \vec{r}_i \]

\[ \vec{v}_{avg} \equiv \Delta \vec{r} / \Delta t = \text{Average velocity} \]
**Average Acceleration**

- The average acceleration of a particle as it moves is defined as the change in the instantaneous velocity vector divided by the time interval during which that change occurs.

- **Note:** bold fonts are vectors

\[
\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i} = \frac{\Delta \vec{v}}{\Delta t}
\]

- The average acceleration is a vector quantity directed along \(\Delta \vec{v}\)

**Instantaneous Acceleration**

- The instantaneous acceleration is the limit of the average acceleration as \(\Delta v/\Delta t\) approaches zero

\[
a \equiv \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{d\vec{v}}{dt}
\]
Instantaneous Acceleration

- The instantaneous acceleration is the limit of the average acceleration as $\Delta v/\Delta t$ approaches zero

\[
a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}
\]

- Quick Comment: Instantaneous acceleration is a vector with components parallel (tangential) and/or perpendicular (radial) to the tangent of the path (more in Chapter 6)

Assignment Recap

- Reading for Tuesday’s class
  » Finish Chapter 2 & all of 3 (vectors)
  » And first assignment is due this Wednesday