Physics 207 – Lecture 4

Lecture 4

- Goals for Chapter 3 & 4
  - Perform vector algebra
    - (addition & subtraction) graphically or by xyz components
    - Interconvert between Cartesian and Polar coordinates
  - Work with 2D motion
    - Distinguish position-time graphs from particle trajectory plots
    - Trajectories
      - Obtain velocities
      - Acceleration: Deduce components parallel and perpendicular to the trajectory path
  - Solve classic problems with acceleration(s) in 2D
    (including linear, projectile and circular motion)
  - Discern different reference frames and understand how they relate to motion in stationary and moving frames

Assignment: Read thru Chapter 5.4
MP Problem Set 2 due this Wednesday

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Example of a 1D motion problem

- A cart is initially traveling East at a constant speed of 20 m/s. When it is halfway (in distance) to its destination its speed suddenly increases and thereafter remains constant. All told the cart spends a total of 10 s in transit with an average speed of 25 m/s.
- What is the speed of the cart during the 2nd half of the trip?
- Dynamical relationships (only if constant acceleration):

\[
\begin{align*}
\Delta x &= x_0 + v_{x_0} \Delta t + \frac{1}{2} a_x \Delta t^2 \\
v_x &= v_{x_0} + a_x \Delta t \\
a_x &= \text{const} \\
\end{align*}
\]

\[
\begin{align*}
v_{x}^2 - v_{x_0}^2 &= 2a_x (x - x_0) \\
v_{(\text{avg})} &= \frac{1}{2} (v_{x_0} + v_x) \\
\end{align*}
\]

And

\[
\vec{v} \text{(average velocity)} = \frac{\Delta x \text{(displacement)}}{\Delta t \text{(total time)}}
\]
The picture

- Plus the average velocity 
  \[ \bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_0}{t_2 - t_0} \]
  
- Knowns:
  - \( x_0 = 0 \text{ m} \)
  - \( t_0 = 0 \text{ s} \)
  - \( v_0 = 20 \text{ m/s} \)
  - \( t_2 = 10 \text{ s} \)
  - \( v_{\text{avg}} = 25 \text{ m/s} \)
  - relationship between \( x_1 \) and \( x_2 \)
  
- Four unknowns \( x_1 \), \( v_1 \), \( t_1 \), and \( x_2 \) and must find \( v_1 \) in terms of knowns

Using \[ x = x_0 + v_x \Delta t \]

- Four unknowns
- Four relationships

\[ v = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_0}{t_2 - t_0} \]
\[ x_1 = \frac{1}{2} (x_2 - x_0) \]
Using $x_0 = 0 \quad t_0 = 0$

1. $x_1 = v_0 t_1$
2. $x_2 = x_1 + v_1 (t_2 - t_1)$

- Eliminate unknowns
  - First $x_1$
    3. $x_1 = \frac{1}{2} x_2$
  - Next $t_1$
    1. $\frac{1}{2} x_2 = v_1 (t_2 - \frac{t_1}{v_1})$
  - Then $x_2$
    4. $\frac{1}{2} v t_2 = v_1 (t_2 - \frac{v_1}{v})$

  Mult. by $2/ t_2$

\[ \bar{v} = v_1 \left(2 - \frac{v_1}{v}\right) \]

Fini

- Plus the average velocity
- Given:
  - $v_0 = 20 \text{ m/s}$
  - $t_2 = 10 \text{ s}$
  - $v_{avg} = 25 \text{ m/s}$
Vectors and 2D vector addition

• The sum of two vectors is another vector.

\[ \mathbf{A} = \mathbf{B} + \mathbf{C} \]

\[ \mathbf{D} = \mathbf{B} + 2\mathbf{C} \]

2D Vector subtraction

• Vector subtraction can be defined in terms of addition.

\[ \mathbf{B} - \mathbf{C} = \mathbf{B} + (-1)\mathbf{C} \]
**Reference vectors: Unit Vectors**

- A **Unit Vector** is a vector having length 1 and no units.
- It is used to specify a direction.
- Unit vector $\mathbf{u}$ points in the direction of $\mathbf{U}$
  - Often denoted with a “hat”: $\mathbf{u} = \hat{\mathbf{u}}$

- Useful examples are the cartesian unit vectors [$\hat{i}, \hat{j}, \hat{k}$]
  - Point in the direction of the $x$, $y$ and $z$ axes.
  - $\mathbf{R} = r_x \hat{i} + r_y \hat{j} + r_z \hat{k}$

**Vector addition using components:**

- Consider, in 2D, $\mathbf{C} = \mathbf{A} + \mathbf{B}$.
  - (a) $\mathbf{C} = (A_x \hat{i} + A_y \hat{j}) + (B_x \hat{i} + B_y \hat{j}) = (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j}$
  - (b) $\mathbf{C} = (C_x \hat{i} + C_y \hat{j})$

- Comparing components of (a) and (b):
  - $C_x = A_x + B_x$
  - $C_y = A_y + B_y$
  - $|\mathbf{C}| = [ (C_x)^2 + (C_y)^2 ]^{1/2}$
**Example**

**Vector Addition**

- Vector \( \mathbf{A} = \{0,2,1\} \)
- Vector \( \mathbf{B} = \{3,0,2\} \)
- Vector \( \mathbf{C} = \{1,-4,2\} \)

What is the resultant vector, \( \mathbf{D} \), from adding \( \mathbf{A} + \mathbf{B} + \mathbf{C} \)?

A. \( \{3,-4,2\} \)
B. \( \{4,-2,5\} \)
C. \( \{5,-2,4\} \)
D. None of the above
Converting Coordinate Systems

- In **polar** coordinates the vector \( \mathbf{R} = (r, \theta) \)
- In Cartesian the vector \( \mathbf{R} = (r_x, r_y) = (x, y) \)
- We can convert between the two as follows:

\[
\begin{align*}
  r_x &= x = r \cos \theta \\
  r_y &= y = r \cos \theta \\
  \mathbf{R} &= x \mathbf{i} + y \mathbf{j} \\
  r &= \sqrt{x^2 + y^2} \\
  \theta &= \tan^{-1}(y / x)
\end{align*}
\]

- In 3D cylindrical coordinates \((r, \theta, z)\), \( r \) is the same as the magnitude of the vector in the \(x\)-\(y\) plane \(\sqrt{x^2 + y^2}\)

Resolving vectors into components

A mass on a frictionless inclined plane

- A block of mass \(m\) slides down a frictionless ramp that makes angle \(\theta\) with respect to horizontal. What is its acceleration \(a\) ?
Resolving vectors, little g & the inclined plane

- \( \mathbf{g} \) (bold face, vector) can be resolved into its \( x,y \) or \( x',y' \) components
  - \( \mathbf{g} = -g \mathbf{j} \)
  - \( \mathbf{g} = -g \cos \theta \mathbf{j}' + g \sin \theta \mathbf{i}' \)
  - The bigger the tilt the faster the acceleration..... along the incline

Dynamics II: Motion along a line but with a twist (2D dimensional motion, magnitude and directions)

- Particle motions involve a path or trajectory
- Recall instantaneous velocity and acceleration

\[
\mathbf{v} = \lim_{\Delta t \to 0} \frac{\Delta \mathbf{r}}{\Delta t} = \frac{d\mathbf{r}}{dt} \quad \text{and} \quad \mathbf{a} = \lim_{\Delta t \to 0} \frac{\Delta \mathbf{v}}{\Delta t} = \frac{d\mathbf{v}}{dt}
\]

- These are vector expressions reflecting \( x, y \) & \( z \) motion

\[
\mathbf{r} = \mathbf{r}(t) \quad \mathbf{v} = \frac{d\mathbf{r}}{dt} \quad \mathbf{a} = \frac{d^2\mathbf{r}}{dt^2}
\]
**Instantaneous Velocity**

- But how we think about requires knowledge of the path.
- The direction of the **instantaneous velocity** is along a line that is **tangent** to the path of the particle’s direction of motion.

\[
v = \lim_{{\Delta t \to 0}} \frac{\Delta r}{\Delta t} = \frac{dr}{dt}
\]

- The magnitude of the instantaneous velocity vector is the speed, \( s \).
  (Knight uses \( v \))
  \[s = (v_x^2 + v_y^2 + v_z^2)^{1/2}\]

**Average Acceleration**

- The average acceleration of particle motion reflects changes in the instantaneous velocity vector (divided by the time interval during which that change occurs).

\[
\overline{a} = \frac{v_f - v_i}{t_f - t_i} = \frac{\Delta v}{\Delta t}
\]

- The average acceleration is a vector quantity directed along \( \Delta v \)
  (a **vector**!)

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**Instantaneous Acceleration**

- The instantaneous acceleration is the limit of the average acceleration as $\Delta v/\Delta t$ approaches zero

$$a \equiv \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

- The instantaneous acceleration is a vector with components parallel (tangential) and/or perpendicular (radial) to the tangent of the path

- Changes in a particle’s path may produce an acceleration
  - The **magnitude** of the velocity vector may change
  - The **direction** of the velocity vector may change
    (Even if the magnitude remains constant)
  - Both may change simultaneously (depends: path vs time)

**Generalized motion with non-zero acceleration:**

- $\vec{a}_t \equiv \vec{a}_{||}$
- $\vec{a}_r \equiv \vec{a}_\perp$

$$\vec{a} \neq 0 \text{ with } |\vec{a}| = \sqrt{a_{||}^2 + a_{\perp}^2}$$

Two possible options:
- Change in the magnitude of $\vec{v}$
  - $\vec{a}_{||} \neq 0$
- Change in the direction of $\vec{v}$
  - $\vec{a}_{\perp} \neq 0$
Kinematics

- The position, velocity, and acceleration of a particle in 3-dimensions can be expressed as:

\[ \mathbf{r} = x \mathbf{i} + y \mathbf{j} + z \mathbf{k} \]

\[ \mathbf{v} = v_x \mathbf{i} + v_y \mathbf{j} + v_z \mathbf{k} \quad (i, j, k \text{ unit vectors}) \]

\[ \mathbf{a} = a_x \mathbf{i} + a_y \mathbf{j} + a_z \mathbf{k} \]

\[ x = x(\Delta t) \quad y = y(\Delta t) \quad z = z(\Delta t) \]

\[ v_x = \frac{dx}{dt} \quad v_y = \frac{dy}{dt} \quad v_z = \frac{dz}{dt} \]

\[ a_x = \frac{d^2x}{dt^2} \quad a_y = \frac{d^2y}{dt^2} \quad a_z = \frac{d^2z}{dt^2} \]

with, if constant accel., e.g. \( x(\Delta t) = x_0 + v_x \Delta t + \frac{1}{2} a_x \Delta t \)

- All this complexity is hidden away in

\[ \mathbf{r} = \mathbf{r}(\Delta t) \quad \mathbf{v} = \frac{d\mathbf{r}}{dt} \quad \mathbf{a} = \frac{d^2\mathbf{r}}{dt^2} \]

Special Case

Throwing an object with \( x \) along the horizontal and \( y \) along the vertical.

\( x \) and \( y \) motion both coexist and \( t \) is common to both

Let \( g \) act in the \(-y \) direction, \( v_{0x} = v_0 \) and \( v_{0y} = 0 \)
**Another trajectory**

Can you identify the dynamics in this picture?
How many distinct regimes are there?
Are $v_x$ or $v_y = 0$? Is $v_x >, <$ or $= v_y$?

![Graph](image)

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**Another trajectory**

Can you identify the dynamics in this picture?
How many distinct regimes are there?

0 < $t$ < 3  
3 < $t$ < 7  
7 < $t$ < 10

- I. $v_x =$ constant $= v_0$; $v_y = 0$
- II. $v_x = v_y = v_0$
- III. $v_x = 0$; $v_y =$ constant $< v_0$

What can you say about the acceleration?
**Exercise 1 & 2**

**Trajectories with acceleration**

- A rocket is drifting sideways (from left to right) in deep space, with its engine off, from A to B. It is not near any stars or planets or other outside forces.
- Its “constant thrust” engine (i.e., acceleration is constant) is fired at point B and left on for 2 seconds in which time the rocket travels from point B to some point C
  - Sketch the shape of the path from B to C.
- At point C the engine is turned off.
  - Sketch the shape of the path after point C

**Exercise 1**

**Trajectories with acceleration**

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<th>B</th>
<th>C</th>
<th>D</th>
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</tr>
</tbody>
</table>

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Exercise 3
Trajectories with acceleration

After C?

A. A
B. B
C. C
D. D
E. None of these

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Lecture 4
Assignment: Read through Chapter 5.4

MP Problem Set 2 due Wednesday