Lecture 14

Goals:
- Chapter 10
  - Understand spring potential energies & use energy diagrams
- Chapter 11
  - Understand the relationship between force, displacement and work
  - Recognize transformations between kinetic, potential, and thermal energies
  - Define work and use the work-kinetic energy theorem
  - Use the concept of power (i.e., energy per time)

Assignment:
- HW6 due Wednesday, Mar. 11
- For Tuesday: Read Chapter 12, Sections 1-3, 5 & 6
  do not concern yourself with the integration process in regards to “center of mass" or “moment of inertia"

Energy for a Hooke’s Law spring

\[ F_x = m \frac{dv_x}{dt} = m \frac{dv_x}{dx} \frac{dx}{dt} = m \frac{dv_x}{dx} v_x = -k(x - x_{eq}) \]

\[ F_x dx = mv_x dv_x = -k(x - x_{eq})dx \]

\[ \int_{x_i}^{x_f} F_x \, dx = \int_{v_i}^{v_f} mv_x \, dv_x \]

\[ \frac{1}{2} kx_i^2 + \frac{1}{2} mv_i^2 = \frac{1}{2} kx_f^2 + \frac{1}{2} mv_f^2 \]

- Associate \( \frac{1}{2} kx^2 \) with the “potential energy” of the spring
Energy for a Hooke’s Law spring

\[ \frac{1}{2} k x_i^2 + \frac{1}{2} m v_i^2 = \frac{1}{2} k x_f^2 + \frac{1}{2} m v_f^2 \]

\[ U_{si} + K_i = U_{sf} + K_f = \text{constant} \]

- Ideal Hooke’s Law springs are conservative so the mechanical energy is constant

Energy diagrams

- In general:

**Ball falling**

\[ E_{\text{mech}} - U \]

**Spring/Mass system**

\[ E_{\text{mech}} - U \]
Equilibrium

- Example
  - Spring: $F_x = 0 \Rightarrow \frac{dU}{dx} = 0$ for $x=x_{\text{eq}}$
  - The spring is in equilibrium position

- In general: $\frac{dU}{dx} = 0 \rightarrow$ for ANY function establishes equilibrium

Comment on Energy Conservation

- We have seen that the total kinetic energy of a system undergoing an inelastic collision is not conserved.
  - Mechanical energy is lost:
    - Heat (friction)
    - Bending of metal and deformation

- Kinetic energy is not conserved by these non-conservative forces occurring during the collision!

- Momentum along a specific direction is conserved when there are no external forces acting in this direction.
  - In general, easier to satisfy conservation of momentum than energy conservation.
Mechanical Energy

- Potential Energy (U)
- Kinetic Energy (K)
- If “conservative” forces (e.g., gravity, spring) then

$$E_{\text{mech}} = \text{constant} = K + U$$

During \( \Rightarrow U_{\text{spring}} + K_1 + K_2 = \text{constant} = E_{\text{mech}} \)

- Mechanical Energy conserved

Energy (with spring & gravity)

- \( E_{\text{mech}} = \text{constant} \) (only conservative forces)
- At 1: \( y_1 = h ; \ v_{y1} = 0 \)  
- At 2: \( y_2 = 0 ; \ v_{y2} = ? \)  
- At 3: \( y_3 = -x ; \ v_3 = 0 \)

- Given \( m, g, h \) & \( k \), how much does the spring compress?

\[ E_{m1} = E_{m3} = mgx + \frac{1}{2} kx^2 \rightarrow \text{Solve} \ \frac{1}{2} kx^2 - mgx + mgh = 0 \]
Energy (with spring & gravity)

- When is the child’s speed greatest?
  (A) At $y_1$ (top of jump)
  (B) Between $y_1$ & $y_2$
  (C) At $y_2$ (child first contacts spring)
  (D) Between $y_2$ & $y_3$
  (E) At $y_3$ (maximum spring compression)

A: Calculus soln. Find $v$ vs. spring displacement then maximize (i.e., take derivative and then set to zero)

B: Physics: As long as $F_{gravity} > F_{spring}$ then speed is increasing

Find where $F_{gravity} - F_{spring} = 0 \rightarrow -mg = kx_{V_{max}}$ or $x_{V_{max}} = -mg / k$

So $mgh = U_{g23} + U_{s23} + K_{23} = mg (-mg/k) + \frac{1}{2} k(-mg/k)^2 + \frac{1}{2} mv^2$

$\rightarrow 2gh = 2(-mg^2/k) + mg^2/k + v^2 \rightarrow 2gh + \frac{mg^2}{k} = v_{max}^2$
Inelastic Processes

- If non-conservative" forces (e.g, deformation, friction) then
  \[ E_{\text{mech}} \text{ is NOT constant} \]

- After → \( K_{1+2} < E_{\text{mech}} \) (before)

- Accounting for this loss we introduce

  - Thermal Energy (\( E_{\text{th}} \), new)

where \( E_{\text{sys}} = E_{\text{mech}} + E_{\text{th}} = K + U + E_{\text{th}} \)

Energy & Work

- Impulse (Force vs time) gives us momentum transfer

- Work (Force vs distance) tracks energy transfer

- Any process which changes the potential or kinetic energy of a system is said to have done work \( W \) on that system

\[ \Delta E_{\text{sys}} = W \]

\( W \) can be positive or negative depending on the direction of energy transfer

- Net work reflects changes in the kinetic energy

\[ W_{\text{net}} = \Delta K \]

This is called the “Net” Work-Kinetic Energy Theorem
Circular Motion

- I swing a sling shot over my head. The tension in the rope keeps the shot moving at constant speed in a circle.
- How much work is done after the ball makes one full revolution?

(A) $W > 0$

(B) $W = 0$

(C) $W < 0$

(D) need more info

Examples of “Net” Work ($W_{\text{net}}$)

$\Delta K = W_{\text{net}}$

- Pushing a box on a smooth floor with a constant force; there is an increase in the kinetic energy

Examples of No “Net” Work

$\Delta K = W_{\text{net}}$

- Pushing a box on a rough floor at constant speed
- Driving at constant speed in a horizontal circle
- Holding a book at constant height
This last statement reflects what we call the “system”
( Dropping a book is more complicated because it involves changes in $U$ and $K$, $U$ is transferred to $K$ )
Changes in K with a constant F

\[ \int_{x_i}^{x_f} F_x \, dx = \int_{v_i}^{v_f} mv_x \, dv_x \]

- If \( F \) is constant

\[ F_x \int_{x_i}^{x_f} dx = \int_{v_i}^{v_f} mv_x \, dv_x \]

\[ F_x (x_f - x_i) = F_x \Delta x = \frac{1}{2} mv_{xf}^2 - \frac{1}{2} mv_{xi}^2 = \Delta K \]

Net Work: 1-D Example
(constant force)

- A force \( F = 10 \, N \) pushes a box across a frictionless floor for a distance \( \Delta x = 5 \, m \).

\[ \theta = 0^\circ \]

\[ \Delta x \]

- Net Work is \( F \Delta x = 10 \times 5 \, N \, m = 50 \, J \)
- 1 Nm \( \equiv \) 1 Joule and this is a unit of energy
- Work reflects energy transfer
Units:

Force x Distance = Work

\[ \text{Newton} \times \text{Meter} = \text{Joule} \]

<table>
<thead>
<tr>
<th>mks</th>
<th>cgs</th>
<th>Other</th>
</tr>
</thead>
<tbody>
<tr>
<td>N-m (Joule)</td>
<td>Dyne-cm (erg)</td>
<td>BTU = 1054 J</td>
</tr>
<tr>
<td></td>
<td>(= 10^{-7} \text{ J} )</td>
<td>calorie = 4.184 J</td>
</tr>
<tr>
<td></td>
<td></td>
<td>foot-lb = 1.356 J</td>
</tr>
<tr>
<td></td>
<td></td>
<td>eV = 1.6x10^{-19} J</td>
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Net Work: 1-D 2\textsuperscript{nd} Example (constant force)

- A force \( F = 10 \text{ N} \) is opposite the motion of a box across a frictionless floor for a distance \( \Delta x = 5 \text{ m} \).

\[ \text{Start} \quad F \quad \text{Finish} \quad \theta = 180^\circ \]

\[ \Delta x \]

- Net Work is \( F \Delta x = -10 \times 5 \text{ N m} = -50 \text{ J} \)
- Work reflects energy transfer
Work in 3D….

- \( x, y \) and \( z \) with constant \( F \):

\[
\begin{align*}
F_x (x_f - x_i) &= F_x \Delta x = \frac{1}{2} mv_{xf}^2 - \frac{1}{2} mv_{xi}^2 \\
F_y (y_f - y_i) &= F_y \Delta y = \frac{1}{2} mv_{yf}^2 - \frac{1}{2} mv_{yi}^2 \\
F_z (z_f - z_i) &= F_z \Delta z = \frac{1}{2} mv_{zf}^2 - \frac{1}{2} mv_{zi}^2
\end{align*}
\]

\[
F_x \Delta x + F_y \Delta y + F_z \Delta z = \frac{1}{2} mv_f^2 - \frac{1}{2} mv_i^2 = \Delta K
\]

with \( v^2 = v_x^2 + v_y^2 + v_z^2 \)

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**Work: “2-D” Example**

(constant force)

- A force \( F = 10 \, \text{N} \) pushes a box across a frictionless floor for a distance \( \Delta x = 5 \, \text{m} \) and \( \Delta y = 0 \, \text{m} \)

![Diagram showing a box being pushed by a force \( F \) at an angle \( \theta = -45^\circ \) over a distance \( \Delta x \).](image)

- (Net) Work is \( F_x \Delta x = F \cos(-45^\circ) \Delta x = 50 \times 0.71 \, \text{Nm} = 35 \, \text{J} \)

- Work reflects energy transfer
**Scalar Product (or Dot Product)**

\[
\mathbf{A} \cdot \mathbf{B} \equiv |A| |B| \cos(\theta)
\]

- Useful for performing projections.
  \[
  \mathbf{A} \cdot \mathbf{i} = A_x \\
  \mathbf{i} \cdot \mathbf{i} = 1 \\
  \mathbf{i} \cdot \mathbf{j} = 0
  \]
- Calculation can be made in terms of components.
  \[
  \mathbf{A} \cdot \mathbf{B} = (A_x)(B_x) + (A_y)(B_y) + (A_z)(B_z)
  \]

Calculation also in terms of magnitudes and relative angles.
\[
\mathbf{A} \cdot \mathbf{B} \equiv |A| |B| \cos \theta
\]

*You choose the way that works best for you!*

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**Scalar Product (or Dot Product)**

Compare:

\[
\mathbf{A} \cdot \mathbf{B} = (A_x)(B_x) + (A_y)(B_y) + (A_z)(B_z)
\]

with \(\mathbf{A}\) as force \(\mathbf{F}\), \(\mathbf{B}\) as displacement \(\Delta \mathbf{r}\)

and apply the **Work-Kinetic Energy** theorem

Notice:

\[
\mathbf{F} \cdot \Delta \mathbf{r} = (F_x)(\Delta x) + (F_y)(\Delta z) + (F_z)(\Delta z)
\]

\[
F_x \Delta x + F_y \Delta y + F_z \Delta z = \Delta K
\]

So here

\[
\mathbf{F} \cdot \Delta \mathbf{r} = \Delta K = W_{net}
\]

More generally a Force acting over a Distance does Work

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Definition of Work, The basics

**Ingredients:** Force ($F$), displacement ($\Delta r$)

Work, $W$, of a constant force $F$ acts through a displacement $\Delta r$:

$$W = F \cdot \Delta r$$  \hspace{1cm} (Work is a scalar)

“Scalar or Dot Product” $\vec{F} \cdot d\vec{r}$

If we know the angle the force makes with the path, the dot product gives us $F \cos \theta$ and $\Delta r$

If the path is curved $dW = \vec{F} \cdot d\vec{r}$ at each point and

$$W = \int_{\vec{r}_i}^{\vec{r}_f} \vec{F} \cdot d\vec{r}$$

Remember that a real trajectory implies forces acting on an object

Two possible options:

- Change in the magnitude of $\vec{v}$ $\vec{a}_\parallel \neq 0$
- Change in the direction of $\vec{v}$ $\vec{a}_\perp \neq 0$

- Only tangential forces yield work!
- The distance over which $F_{\text{Tang}}$ is applied: Work
Definition of Work, The basics

**Ingredients:** Force (\( F \)), displacement (\( \Delta r \))

Work, \( W \), of a constant force \( F \) acts through a displacement \( \Delta r \):

\[
W = F \cdot \Delta r \quad \text{(Work is a scalar)}
\]

Work tells you something about what happened on the path!

Did something do work on you?

Did you do work on something?

If only one force acting: Did your **speed** change?

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**Exercise**

**Work in the presence of friction and non-contact forces**

- A box is pulled up a rough (\( \mu > 0 \)) incline by a rope-pulley-weight arrangement as shown below.
  - How many forces (including non-contact ones) are doing work on the box?
  - Of these which are positive and which are negative?
  - Use a Free Body Diagram
  - Compare force and path

A. 2
B. 3
C. 4
D. 5
Home Exercise
Work Done by Gravity

- An frictionless track is at an angle of 30° with respect to the horizontal. A cart (mass 1 kg) is released from rest. It slides 1 meter downwards along the track bounces and then slides upwards to its original position.

- How much total work is done by gravity on the cart when it reaches its original position? \((g = 10 \text{ m/s}^2)\)

\[
h = 1 \text{ m} \sin 30° = 0.5 \text{ m}
\]

(A) 5 J  (B) 10 J  (C) 20 J  (D) 0 J

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