Lecture 25

Today Review:

• Exam covers Chapters 14-17 plus angular momentum, rolling motion & torque

• Assignment
  ❖ HW11, Due Tuesday, May 6th
  ❖ For Thursday, read through all of Chapter 18

Angular Momentum  Exercise

• A mass $m=0.10$ kg is attached to a cord passing through a small hole in a frictionless, horizontal surface as in the Figure. The mass is initially orbiting with speed $\omega_i = 5$ rad / s in a circle of radius $r_i = 0.20$ m. The cord is then slowly pulled from below, and the radius decreases to $r = 0.10$ m.

• What is the final angular velocity ?
• Underlying concept: Conservation of Momentum
**Angular Momentum Exercise**

- A mass \( m = 0.10 \) kg is attached to a cord passing through a small hole in a frictionless, horizontal surface as in the Figure. The mass is initially orbiting with speed \( \omega_i = 5 \) rad / s in a circle of radius \( r_i = 0.20 \) m. The cord is then slowly pulled from below, and the radius decreases to \( r = 0.10 \) m.

- What is the final angular velocity ?

No external torque implies

\[
\Delta L = 0 \text{ or } L_i = L_f
\]

\[
I_i \omega_i = I_f \omega_f
\]

I for a point mass is \( mr^2 \) where \( r \) is the distance to the axis of rotation

\[
m r_i^2 \omega_i = m r_f^2 \omega_f
\]

\[
\omega_f = \frac{r_i^2 \omega_i}{r_f^2} = \left( \frac{0.20}{0.10} \right)^2 5 \text{ rad/s} = 20 \text{ rad/s}
\]

---

**Example: Throwing ball from stool**

- A student sits on a stool, initially at rest, but which is free to rotate. The moment of inertia of the student plus the stool is \( I \). They throw a heavy ball of mass \( M \) with speed \( v \) such that its velocity vector has a perpendicular distance \( d \) from the axis of rotation.

- What is the angular speed \( \omega_f \) of the student-stool system after they throw the ball ?

\[
Mv
\]

Top view: before after

\[
\omega_f
\]

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Example: Throwing ball from stool

- What is the angular speed $\omega_f$ of the student-stool system after they throw the ball?
- Process: (1) Define system (2) Identify Conditions

(1) System: student, stool and ball (No Ext. torque, $L$ is constant)
(2) Momentum is conserved (check $|L| = |r| |p| \sin \theta$ for sign)

$$L_{\text{init}} = 0 = L_{\text{final}} = -M v d + I \omega_f$$

Top view: before after

![Diagram](image)

Ideal Fluid

Bernoulli Equation $\Rightarrow P_1 + \frac{1}{2} \rho v_1^2 + \rho g y_1 = \text{constant}$
A 5 cm radius horizontal pipe carries water at 10 m/s into a 10 cm radius. ($\rho_{\text{water}} = 10^3 \text{ kg/m}^3$)

What is the pressure difference?

$$P_1 + \frac{1}{2} \rho v_1^2 = P_2 + \frac{1}{2} \rho v_2^2$$

$$\Delta P = \frac{1}{2} \rho v_2^2 - \frac{1}{2} \rho v_1^2$$

$$\Delta P = \frac{1}{2} \rho (v_2^2 - v_1^2)$$

and $A_1 v_1 = A_2 v_2$

$$\Delta P = \frac{1}{2} \rho v_2^2 (1 - (A_2/A_1)^2)$$

$$= 0.5 \times 1000 \text{ kg/m} \times 100 \text{ m}^2/\text{s}^2 (1 - (25/100)^2)$$

$$= 47000 \text{ Pa}$$
A water fountain

- A fountain, at sea level, consists of a 10 cm radius pipe with a 5 cm radius nozzle. The water sprays up to a height of 20 m.

- What is the velocity of the water as it leaves the nozzle?

- What volume of the water per second as it leaves the nozzle?

- What is the velocity of the water in the pipe?

- What is the pressure in the pipe?

- How many watts must the water pump supply?

Simple Picture: \( \frac{1}{2} m v^2 = m g h \rightarrow v = (2gh)^{\frac{1}{2}} = (2 \times 10 \times 20)^{\frac{1}{2}} = 20 \text{ m/s} \)

- What volume of the water per second as it leaves the nozzle?
  \[ Q = A_n v_n = 0.0025 \times 20 \times 3.14 = 0.155 \text{ m}^3/\text{s} \]

- What is the velocity of the water in the pipe?
  \[ A_n v_n = A_p v_p \rightarrow v_p = Q / A_p = 5 \text{ m/s} \]

- What is the pressure in the pipe?
  \[ 1 \text{ atm} + \frac{1}{2} \rho v_n^2 = 1 \text{ atm} + \Delta P + \frac{1}{2} \rho v_p^2 \rightarrow 1.9 \times 10^5 \text{ N/m}^2 \]

- How many watts must the water pump supply?

Power = \( Q \rho g h = 0.0155 \text{ m}^3/\text{s} \times 10^3 \text{ kg/m}^3 \times 9.8 \text{ m/s}^2 \times 20 \text{ m} = 3 \times 10^4 \text{ W} \) (Comment on syringe injection)
Fluids Buoyancy

- A metal cylinder, 0.5 m in radius and 4.0 m high is lowered, as shown, from a massless rope into a vat of oil and water. The tension, T, in the rope goes to zero when the cylinder is half in the oil and half in the water. The densities of the oil is 0.9 gm/cm\(^3\) and the water is 1.0 gm/cm\(^3\).

- What is the average density of the cylinder?

- What was the tension in the rope when the cylinder was submerged in the oil?

---

Fluids Buoyancy

- \(r = 0.5\) m, \(h = 4.0\) m
- \(\rho_{\text{oil}} = 0.9\) gm/cm\(^3\) \(\rho_{\text{water}} = 1.0\) gm/cm\(^3\)
- What is the average density of the cylinder?

When \(T = 0\)

\[
F_{\text{buoyancy}} = W_{\text{cylinder}}
\]

\[
F_{\text{buoyancy}} = \rho_{\text{oil}} \ g \ \frac{1}{2} \ V_{\text{cyl.}} + \rho_{\text{water}} \ g \ \frac{1}{2} \ V_{\text{cyl.}}
\]

\[
W_{\text{cylinder}} = \rho_{\text{cyl.}} \ g \ V_{\text{cyl.}}
\]

\[
\rho_{\text{cyl.}} \ g \ V_{\text{cyl.}} = \rho_{\text{oil}} \ g \ \frac{1}{2} \ V_{\text{cyl.}} + \rho_{\text{water}} \ g \ \frac{1}{2} \ V_{\text{cyl.}}
\]

\[
\rho_{\text{cyl.}} = \frac{1}{2} \rho_{\text{oil}} + \frac{1}{2} \rho_{\text{water}}
\]

What was the tension in the rope when the cylinder was submerged in the oil?

Use a Free Body Diagram!
**Fluids Buoyancy**

- \( r = 0.5 \text{ m}, \ h = 4.0 \text{ m} \) \( V_{\text{cyl.}} = \pi r^2 h \)
- \( \rho_{\text{oil}} = 0.9 \text{ gm/cm}^3 \) \( \rho_{\text{water}} = 1.0 \text{ gm/cm}^3 \)
- What is the average density of the cylinder?

When \( T = 0 \)

\[
F_{\text{buoyancy}} = W_{\text{cylinder}}
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\[
F_{\text{buoyancy}} = \rho_{\text{oil}} g \frac{1}{2} V_{\text{cyl.}} + \rho_{\text{water}} g \frac{1}{2} V_{\text{cyl.}}
\]

\[
W_{\text{cylinder}} = \rho_{\text{cyl}} g V_{\text{cyl.}}
\]

\[
\rho_{\text{cyl}} g V_{\text{cyl.}} = \rho_{\text{oil}} g \frac{1}{2} V_{\text{cyl.}} + \rho_{\text{water}} g \frac{1}{2} V_{\text{cyl.}}
\]

\[
\rho_{\text{cyl}} = \frac{1}{2} \rho_{\text{oil}} + \frac{1}{2} \rho_{\text{water}} = 0.95 \text{ gm/cm}^3
\]

What was the tension in the rope when the cylinder was submerged in the oil?

Use a Free Body Diagram!

\[
\sum F_z = 0 = T - W_{\text{cylinder}} + F_{\text{buoyancy}}
\]

\[
T = W_{\text{cyl}} - F_{\text{buoy}} = g (\ \rho_{\text{cyl}} - \rho_{\text{oil}}) V_{\text{cyl}}
\]

\[
T = 9.8 \times 0.05 \times 10^3 \times \pi \times 0.5^2 \times 4.0 = 1500 \text{ N}
\]

**A new trick**

- Two trapeze artists, of mass 100 kg and 50 kg respectively are testing a new trick and want to get the timing right. They both start at the same time using ropes of 10 meter in length and, at the turnaround point the smaller grabs hold of the larger artist and together they swing back to the starting platform. A model of the stunt is shown at right.

- How long will this stunt require if the angle is small?
A new trick

- How long will this stunt require?

Period of a pendulum is just

$$\omega = (g/L)^{\frac{1}{2}}$$

$$T = 2\pi (L/g)^{\frac{1}{2}}$$

Time before \(\frac{1}{2}\) period

Time after \(\frac{1}{2}\) period

So, \(t = T = 2\pi(L/g)^{\frac{1}{2}} = 2\pi\) sec

Key points: Period is one full swing and independent of mass

(this is SHM but very different than a spring. SHM requires only a linear restoring force.)

Example

- A Hooke’s Law spring, \(k=200\) N/m, is on a horizontal frictionless surface is stretched 2.0 m from its equilibrium position. A 1.0 kg mass is initially attached to the spring however, at a displacement of 1.0 m a 2.0 kg lump of clay is dropped onto the mass. The clay sticks.

What is the new amplitude?
Example

- A Hooke’s Law spring, \( k = 200 \text{ N/m} \), is on a horizontal frictionless surface is stretched 2.0 m from its equilibrium position. A 1.0 kg mass is initially attached to the spring however, at a displacement of 1.0 m a 2.0 kg lump of clay is dropped onto the mass.

What is the new amplitude?

Sequence: SHM, collision, SHM

\[
\frac{1}{2} k A_0^2 = \text{const.}
\]

\[
\frac{1}{2} k A_0^2 = \frac{1}{2} m v^2 + \frac{1}{2} k (A_0/2)^2
\]

\[
\frac{3}{4} k A_0^2 = m v^2 \quad \Rightarrow \quad v = \left( \frac{3}{4} k A_0^2 / m \right)^{\frac{1}{2}} = 24.5 \text{ m/s}
\]

Conservation of x-momentum:

\[
mv = (m+M) V \quad \Rightarrow \quad V = \frac{mv}{m+M}
\]

\[V = \frac{24.5}{3} \text{ m/s} = 8.2 \text{ m/s}
\]

Example

- A Hooke’s Law spring, \( k = 200 \text{ N/m} \), is on a horizontal frictionless surface is stretched 2.0 m from its equilibrium position. A 1.0 kg mass is initially attached to the spring however, at a displacement of 1.0 m a 2.0 kg lump of clay is dropped onto the mass. The clay sticks.

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\[V = \frac{24.5}{3} \text{ m/s} = 8.2 \text{ m/s}
\]

\[
\frac{1}{2} k A_f^2 = \text{const.}
\]

\[
\frac{1}{2} k A_f^2 = \frac{1}{2} (m+M)V^2 + \frac{1}{2} k (A_f)^2
\]

\[A_f^2 = \left[ (m+M)V^2 / k + (A_0)^2 \right]^{\frac{1}{2}}
\]

\[A_f^2 = [3 \times 8.2^2 / 200 + (1)^2]^{\frac{1}{2}}
\]

\[A_f^2 = [1 + 1]^{\frac{1}{2}} \rightarrow A_f^2 = 1.4 \text{ m}
\]

Key point: \( K+U \) is constant in SHM
Fluids Buoyancy & SHM

- A metal cylinder, 0.5 m in radius and 4.0 m high is lowered, as shown, from a rope into a vat of oil and water. The tension, T, in the rope goes to zero when the cylinder is half in the oil and half in the water. The densities of the oil is 0.9 gm/cm³ and the water is 1.0 gm/cm³
- Refer to earlier example
- Now the metal cylinder is lifted slightly from its equilibrium position. What is the relationship between the displacement and the rope’s tension?
- If the rope is cut and the drum undergoes SHM, what is the period of the oscillation if undamped?

\begin{align*}
0 &= T + F_{\text{buoyancy}} - W_{\text{cylinder}} \\
T &= -F_{\text{buoyancy}} + W_{\text{cylinder}} \\
T &= -[\rho_o g (h/2 + \Delta y) A_c + \rho_w g A_c (h/2 - \Delta y)] + W_{\text{cyl}} \\
T &= -[\rho_w g A_c (\rho_o - \rho_w) + \Delta y g A_c (\rho_o - \rho_w)] + W_{\text{cyl}} \\
T &= -[W_{\text{cyl}} + \Delta y g A_c (\rho_o - \rho_w)] + W_{\text{cyl}} \\
T &= [g A_c (\rho_w - \rho_o)] \Delta y \\
\text{If the rope is cut, net force is towards equilibrium position with a proportionality constant} \\
g A_c (\rho_w - \rho_o) & \quad [\text{with } g=10 \text{ m/s}^2] \\
\text{If } F = -k \Delta y \text{ then } k &= g A_c (\rho_o - \rho_w) = \pi/4 \times 10^3 \text{ N/m}
\end{align*}
Fluids Buoyancy & SHM

- A metal cylinder, 0.5 m in radius and 4.0 m high is lowered, as shown, from a rope into a vat of oil and water. The tension, $T$, in the rope goes to zero when the cylinder is half in the oil and half in the water. The densities of the oil is 0.9 gm/cm$^3$ and the water is 1.0 gm/cm$^3$.
- If the rope is cut and the drum undergoes SHM, what is the period of the oscillation if undamped?

\[ F = ma = -k \Delta y \] and with SHM \[ \omega = (k/m)^{\frac{1}{2}} \] where $k$ is a “spring” constant and $m$ is the inertial mass (resistance to motion), the cylinder

So \[ \omega = \left( \frac{1000\pi}{4 \rho_{\text{cyl}}} \right)^{\frac{1}{2}} \]
\[ = \left( \frac{1000\pi}{4 \rho_{\text{cyl}} V_{\text{cyl}}} \right)^{\frac{1}{2}} = \left( \frac{0.25}{0.95} \right)^{\frac{1}{2}} \]
\[ = 0.51 \text{ rad/sec} \]
\[ T = 3.2 \text{ sec} \]

Underdamped SHM

\[ x(t) = A \exp \left(-\frac{bt}{2m}\right) \cos(\omega t + \phi) \text{ if } \omega_0 > b/2m \]

If the period is 2.0 sec and, after four cycles, the amplitude drops by 75%, what is the time constant?

Four cycles implies 8 sec

So

\[ 0.25 A_0 = A_0 \exp(-4 b/m) \]
\[ \ln(1/4) = -4/\tau \]
\[ \tau = -4 / \ln(1/4) = 2.9 \text{ sec} \]
**Ch. 12**

**General Principles**

### Rotational Dynamics

Every point on a rigid body rotating about a fixed axis has the same angular velocity \( \omega \) and angular acceleration \( \alpha \).

**Newton’s second law** for rotational motion is:

\[
\alpha = \frac{\tau_{net}}{I}
\]

Use rotational kinematics to find angles and angular velocities.

### Conservation Laws

Energy is conserved for an isolated system.

- Pure rotation: \( E = K_{rot} + U = \frac{1}{2} I \omega^2 + M g \ell_{cm} \)
- Rolling: \( E = K_{rot} + K_{trans} + U = \frac{1}{2} I \omega^2 + \frac{1}{2} M v_{rms}^2 + M g \ell_{cm} \)

Angular momentum is conserved if \( \tau_{net} = 0 \).

- Particle: \( \vec{L}_{net} = \vec{m} \vec{v} \times \vec{r} \)
- Rigid body rotating about an axis of symmetry: \( \vec{L} = I \ell \omega \)

### Important Concepts

**Torque** is the rotational equivalent of force:

\[
\tau = r F \sin \theta = r F = d \vec{F}
\]

The vector description of torque is:

\[
\vec{F} = \vec{r} \times \vec{F}
\]

A system of particles on which there is no net force undergoes unconstrained rotation about the center of mass:

- \( x_{cm} = \frac{1}{M} \int x \, dm \)
- \( y_{cm} = \frac{1}{M} \int y \, dm \)

The gravitational torque on a body can be found by treating the body as a particle with all the mass \( M \) concentrated at the center of mass.

**Vector description of rotation**

Angular velocity \( \vec{\omega} \) points along the rotation axis in the direction of the right-hand rule.

For a rigid body rotating about an axis of symmetry, the angular momentum is \( \vec{L} = I \ell \omega \).

**The moment of inertia**

\[
I = \sum m_i r_i^2
\]

is the rotational equivalent of mass. The moment of inertia depends on how the mass is distributed around the axis. If \( I \) is known, the \( I \) about a parallel axis distance \( d \) away is given by the **parallel-axis theorem**:

\[
I = I_{cm} + M d^2
\]

**Hooke’s Law Springs and a Restoring Force**

**General Principles**

- **Dynamics**
  - SHM occurs when a linear restoring force acts to return a system to an equilibrium position.
  - **Horizontal spring**: \( F = -kx \)
  - **Vertical spring**: \( F = -ky \)
  - \( \omega = \sqrt{\frac{k}{m}} \)
  - \( T = 2\pi \sqrt{\frac{m}{k}} \)
  - **Pendulum**: \( F = -\frac{mg}{L} \sin \theta \)
  - \( \omega = \sqrt{\frac{g}{L}} \)
  - \( T = 2\pi \sqrt{\frac{L}{g}} \)

- **Energy**
  - If there is no friction or dissipation, kinetic and potential energy are alternately transformed into each other, but the total mechanical energy \( E = K + U \) is conserved:
    \[
    E = \frac{1}{2} m v^2 + \frac{1}{2} k x^2 = \frac{1}{2} M (v_{rms})^2 = \frac{1}{2} I \omega^2
    \]
  - In a damped system, the energy decays exponentially:
    \[
    E = E_0 e^{-\frac{t}{\tau}}
    \]
    where \( \tau \) is the time constant.

- **Key fact**: \( \omega = (k / m)^{1/2} \) is a general result where \( k \) reflects the constant of the linear restoring force and \( m \) is the inertial response (e.g., the “physical pendulum” where \( \omega = (\kappa / l)^{1/2} \))
Simple Harmonic Motion

Important Concepts

Simple harmonic motion (SHM) is a sinusoidal oscillation with period $T$ and amplitude $A$.

Frequency $f = \frac{1}{T}$

Angular frequency

$\omega = 2\pi f = \frac{2\pi}{T}$

Position $x(t) = A \cos(\omega t + \phi_0)$

$= A \cos\left(\frac{2\pi t}{T} + \phi_0\right)$

Velocity $v_x(t) = -v_{\text{max}} \sin(\omega t + \phi_0)$ with maximum speed $v_{\text{max}} = \omega A$

Acceleration $a_x = -\omega^2 x$

Resonance and damping

- Energy transfer is optimal when the driving force varies at the resonant frequency.

Applications

Resonance

When a system is driven by a periodic external force, it responds with a large-amplitude oscillation if $f = f_0$, where $f_0$ is the system’s natural oscillation frequency, or resonant frequency.

Damping

If there is a drag force $\vec{D} = -bv$, where $b$ is the damping constant, then (for lightly damped systems)

$x(t) = Ae^{-\alpha t} \cos(\omega t + \phi_0)$

The time constant for energy loss is $\tau = \frac{\alpha}{b}$.

- Types of motion
  - Undamped
  - Underdamped
  - Critically damped
  - Overdamped
Fluid Flow

General Principles

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<td>- Freely moving particles</td>
<td>- Incompressible</td>
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<tr>
<td>- Compressible</td>
<td>- Smooth, laminar flow</td>
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<tr>
<td>- Pressure primarily thermal</td>
<td>- Nonviscous</td>
</tr>
<tr>
<td>- Pressure is constant in a laboratory-size container</td>
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</table>

Fluid particles move along streamlines.

Equation of continuity
\[ \dot{V}_A = \dot{V}_B \]

Bernoulli's equation
\[ p_1 + \frac{1}{2} \rho v_1^2 + \rho g y_1 = p_2 + \frac{1}{2} \rho v_2^2 + \rho g y_2 \]

Bernoulli’s equation is a statement of energy conservation.

Density and pressure

Important Concepts

Density \( \rho = \frac{m}{V} \), where \( m \) is mass and \( V \) is volume.

Pressure \( p = \frac{F}{A} \), where \( F \) is the magnitude of the fluid force and \( A \) is the area on which the force acts.

- Pressure exists at all points in a fluid.
- Pressure pushes equally in all directions.
- Pressure is constant along a horizontal line.
- Gauge pressure is \( p_g = p - 1 \text{ atm} \).
Response to forces

Applications

Buoyancy is the upward force of a fluid on an object.

Archimedes' principle

The magnitude of the buoyant force equals the weight of the fluid displaced by the object.

Sink: \( \rho_\text{fl} > \rho_f \) \( F_B < m_g \)
Rise to surface: \( \rho_\text{fl} < \rho_f \) \( F_B > m_g \)
Neutrally buoyant: \( \rho_\text{fl} = \rho_f \) \( F_B = m_g \)

Elasticity describes the deformation of solids and liquids under stress.

Linear stretch and compression

\[ F = Y(\Delta L/L) \] Strain

Tensile stress: Young’s modulus

Volume compression

\[ \rho = -B(\Delta V/V) \] Bulk modulus: Volume strain

States of Matter and Phase Diagrams

General Principles

Three Phases of Matter

Liquid: Molecules loosely held together by molecular bonds, but able to move around. Nearly incompressible.
Gas: Molecules move freely through space. Compressible.

The different phases exist for different conditions of temperature \( T \) and pressure \( p \).
The boundaries separating the regions of a phase diagram are lines of phase equilibrium. Any amounts of the two phases can coexist in equilibrium. The triple point is the one value of temperature and pressure at which all three phases can coexist in equilibrium.
Ideal gas equation of state

**Important Concepts**

**Ideal-Gas Model**
- Atoms and molecules are small, hard spheres that travel freely through space except for occasional collisions with each other or the walls.
- The model is valid when the density is low and the temperature well above the condensation point.

**Ideal-Gas Law**

The state variables of an ideal gas are related by the ideal-gas law

\[ pV = nRT \quad \text{or} \quad pV = N k_B T \]

where \( R = 8.31 \text{ J/mol K} \) is the universal gas constant and \( k_B = 1.38 \times 10^{-23} \text{ J/K} \) is Boltzmann’s constant.

\( p, V \), and \( T \) must be in SI units of Pa, m³, and K. For a gas in a sealed container, with constant \( n \):

\[ \frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2} \]

Counting atoms and moles

A macroscopic sample of matter consists of \( N \) atoms (or molecules), each of mass \( m \) (the atomic or molecular mass):

\[ N = \frac{M}{m} \]

Alternatively, we can state that the sample consists of \( n \) moles:

\[ n = \frac{N}{N_A} \quad \text{or} \quad \frac{M(\text{in grams})}{M_{\text{mol}}} \]

\( N_A = 6.02 \times 10^{23} \text{ mol}^{-1} \) is Avogadro’s number.

The numerical value of the molar mass \( M_{\text{mol}} \), in g/mol, equals the numerical value of the atomic or molecular mass \( m \), in u. The atomic or molecular mass \( m \), in atomic mass units \( u \), is well approximated by the atomic mass number \( A \):

\[ 1 \text{ u} = 1.66 \times 10^{-27} \text{ kg} \]

The number density of the sample is \( \frac{N}{V} \).

**Applications**

**Temperature scales**

\[ T_F = \frac{9}{5} T_C + 32 \quad \text{and} \quad T_K = T_C + 273 \]

The Kelvin temperature scale is based on:
- Absolute zero at \( T_0 = 0 \text{ K} \)
- The triple point of water at \( T_T = 273.16 \text{ K} \)

**Three basic gas processes**

1. Isochoric, or constant volume
2. Isobaric, or constant pressure
3. Isothermal, or constant temperature

\[ pV = nRT \]

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Thermodynamics

General Principles

First Law of Thermodynamics
\[ \Delta E_s = W + Q \]

The first law is a general statement of energy conservation.

Work \( W \) and heat \( Q \) depend on the process by which the system is changed.

The change in the system depends only on the total energy exchanged \( W + Q \), not on the process.

Energy

Thermal energy \( E_a \). Microscopic energy of moving molecules and stretched molecular bonds. \( \Delta E_a \) depends on the initial/final states but is independent of the process.

Work \( W \). Energy transferred to the system by forces in a mechanical interaction.

Heat \( Q \). Energy transferred to the system via atomic-level collisions when there is a temperature difference. A thermal interaction.

Work, Pressure, Volume, Heat

Important Concepts

The work done on a gas is
\[ W = -\int_p V \, dp \]
\[ = -\text{(area under the } pV \text{ curve)} \]

An adiabatic process is one for which \( V \) holds constant, where \( \gamma = \frac{C_p}{C_v} \) is the specific heat ratio. An adiabatic process changes the temperature of the gas without heating it. \( T \) can change!

Calorimetry. When two or more systems interact thermally, they come to a common final temperature determined by
\[ Q_{in} = Q_{out} + Q_1 + \cdots + Q_n = 0 \]

The heat of transformation \( L \) is the energy needed to cause 1 kg of substance to undergo a phase change
\[ Q = \pm ML \]

The specific heat \( c \) of a substance is the energy needed to raise the temperature of 1 kg by 1 K:
\[ Q = mc \Delta T \]

The molar specific heat \( C \) is the energy needed to raise the temperature of 1 mol by 1 K:
\[ Q = nC \Delta T \]

The molar specific heat of gases depends on the process by which the temperature is changed:
\[ C_v = \text{molar specific heat at constant volume} \]
\[ C_p = \text{molar specific heat at constant pressure} \]

Heat is transferred by conduction, convection, radiation, and evaporation.

Conduction:
\[ Q/\Delta t = (\lambda A L) \Delta T \]

Radiation:
\[ Q/\Delta t = e r A T^4 \]

In steady-state \( T = \text{constant} \) and so heat in equals heat out.
Gas Processes

Summary of Basic Gas Processes

<table>
<thead>
<tr>
<th>Process</th>
<th>Definition</th>
<th>Stays constant</th>
<th>Work</th>
<th>Heat</th>
</tr>
</thead>
<tbody>
<tr>
<td>Isochoric</td>
<td>$\Delta V = 0$</td>
<td>$V$ and $pT$</td>
<td>$W = 0$</td>
<td>$Q = nC_v\Delta T$</td>
</tr>
<tr>
<td>Isobaric</td>
<td>$\Delta p = 0$</td>
<td>$p$ and $V/T$</td>
<td>$W = -p\Delta V$</td>
<td>$Q = nC_p\Delta T$</td>
</tr>
<tr>
<td>Isothermal</td>
<td>$\Delta T = 0$</td>
<td>$T$ and $pV$</td>
<td>$W = -nRT\ln(V/V_i)$</td>
<td>$\Delta E_h = 0$</td>
</tr>
<tr>
<td>Adiabatic</td>
<td>$Q = 0$</td>
<td>$pV^{\gamma}$</td>
<td>$W = \Delta E_h$</td>
<td>$Q = 0$</td>
</tr>
</tbody>
</table>

All gas processes

First law: $\Delta E_h = W + Q = nC_v\Delta T$

Ideal-gas law: $pV = nRT$

Lecture 25

- Exam covers Chapters 14-17 plus angular momentum, rolling motion & torque

- Assignment
  - HW11, Due Tuesday, May 6th
  - For Thursday, read through all of Chapter 18