Lecture 28

Goals:

• Chapter 20
  ❖ Employ the wave model
  ❖ Visualize wave motion
  ❖ Analyze functions of two variables
  ❖ Know the properties of sinusoidal waves, including wavelength, wave number, phase, and frequency.
  ❖ Work with a few important characteristics of sound waves. (e.g., Doppler effect)

• Assignment
  ❖ HW11, Due Tuesday, May 5th
  ❖ HW12, Due Friday, May 8th
  ❖ For Tuesday, Read through all of Chapter 21

Waves

• A traveling wave is an organized disturbance propagating at a well-defined wave speed $v$.

• In transverse waves the particles of the medium move perpendicular to the direction of wave propagation.

• In longitudinal waves the particles of the medium move parallel to the direction of wave propagation.

• A wave transfers energy, but no material or substance is transferred outward from the source.
Energy is transported in wave but the motion of matter is local.

Types of Waves

- Mechanical waves travel through a material medium such as water or air.
- Electromagnetic waves require no material medium and can travel through vacuum.
- Matter waves describe the wave-like characteristics of atomic-level particles.
  - For mechanical waves, the speed of the wave is a property of the medium. Speed does not depend on the size or shape of the wave.
- Examples:
  - Sound waves (air moves locally back & forth)
  - Stadium waves (people move up & down)
  - Water waves (water moves up & down)
  - Light waves (an oscillating electromagnetic field)
Wave Graphs

- The displacement \( D \) of a wave is a function of both position (where) and time (when).

- A snapshot graph shows the wave’s displacement as a function of position at a single instant of time.

- A history graph shows the wave’s displacement as a function of time at a single point in space.

- The displacement, \( D \), is a function of two variables, \( x \) and \( t \), or \( D(x, t) \)

Wave Speed

- Speed of a transverse, mechanical wave on a string:
  \[
  v = \sqrt{\frac{T_s}{\mu}}
  \]
  where \( T_s \) is the string tension and \( \mu \) is linear string density

- Speed of sound (longitudinal mechanical wave) in air at 20°C: \( v = 343 \text{ m/s} \)

- Speed of light (transverse, EM wave) in vacuum: \( c = 3 \times 10^8 \text{ m/s} \)

- Speed of light (transverse, EM wave) in a medium: \( v = \frac{c}{n} \)
  where \( n \) = index of refraction of the medium (typically 1 to 4)
Wave Forms

- So far we have examined “continuous waves” that go on forever in each direction!

- We can also have “pulses” caused by a brief disturbance of the medium:

- And “pulse trains” which are somewhere in between.

Continuous Sinusoidal Wave

- Wavelength: The distance $\lambda$ between identical points on the wave.

- Amplitude: The maximum displacement $A$ of a point on the wave.
Wave Properties...

- Period: The time $T$ for a point on the wave to undergo one complete oscillation.

- Speed: The wave moves one wavelength $\lambda$ in one period $T$ so its speed is $v = \lambda / T$.

\[
v = \frac{\lambda}{T} = f \lambda
\]

Exercise Wave Motion

- The speed of sound in air is a bit over 300 m/s, and the speed of light in air is about 300,000,000 m/s.

- Suppose we make a sound wave and a light wave that both have a wavelength of 3 meters.

What is the ratio of the frequency of the light wave to that of the sound wave? (Recall $v = \frac{\lambda}{T} = \lambda f$)

(A) About 1,000,000
(B) About 0.000,001
(C) About 1000
Wave Properties

\[ D(x,t) = A \cos\left[\left( \frac{2\pi}{\lambda} x - \frac{t}{T} \right) + \phi_0 \right] \]

\[ D(x,t) = A \cos\left[ kx - \omega t + \phi_0 \right] \]

\( A = \text{amplitude} \quad k = \frac{2\pi}{\lambda} = \text{wave number} \]

\( \omega = 2\pi f = \text{angular frequency} \quad \phi_0 = \text{phase constant} \)

Look at the spatial part (Let \( t = 0 \)).

\[ D(x,0) = A \cos\left[\left( \frac{2\pi}{\lambda} x \right) \right] \]

- \( x = 0 \quad y = A \)
- \( x = \frac{\lambda}{4} \quad y = A \cos(\pi/2) = 0 \)
- \( x = \frac{\lambda}{2} \quad y = A \cos(\pi) = -A \)

Look at the temporal (time-dependent) part

\[ D(x,t) = A \cos\left[\left( \frac{2\pi}{\lambda} x \right) - \omega t \right] \]

- Let \( x = 0 \)

\[ D(0,t) = A \cos(-\omega t) = A \cos\left[-\left(\frac{2\pi}{T}\right) t\right] \]

- \( t = 0 \quad y = A \)
- \( t = T/4 \quad y = A \cos(-\pi/2) = 0 \)
- \( t = T/2 \quad y = A \cos(-\pi) = -A \)
Exercise Wave Motion

- A harmonic wave moving in the positive x direction can be described by the equation
  (The wave varies in space and time.)
- \( v = \frac{\lambda}{T} = \lambda f = \frac{\lambda}{2\pi} \cdot 2\pi f = \omega/\lambda \) and, by definition, \( \omega > 0 \)
- \( D(x,t) = A \cos \left( \frac{2\pi}{\lambda} x - \omega t \right) = A \cos (k x - \omega t) \)

- Which of the following equation describes a harmonic wave moving in the negative x direction?

  (A) \( D(x,t) = A \sin (k x - \omega t) \)
  (B) \( D(x,t) = A \cos (k x + \omega t) \)
  (C) \( D(x,t) = A \cos (-k x + \omega t) \)

Exercise Wave Motion

- A boat is moored in a fixed location, and waves make it move up and down. If the spacing between wave crests is 20 meters and the speed of the waves is 5 m/s, how long \( \Delta t \) does it take the boat to go from the top of a crest to the bottom of a trough? (Recall \( v = \lambda / T = \lambda f \))

  (A) 2 sec    (B) 4 sec    (C) 8 sec

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Exercise  Wave Motion

• A boat is moored in a fixed location, and waves make it move up and down. If the spacing between wave crests is 20 meters and the speed of the waves is 5 m/s, how long $\Delta t$ does it take the boat to go from the top of a crest to the bottom of a trough?

• $T = 4$ sec but crest to trough is half a wavelength

(A) 2 sec  (B) 4 sec  (C) 8 sec

Speed of  Waves, (again)

• The speed of sound waves in a medium depends on the compressibility and the density of the medium

• The compressibility can sometimes be expressed in terms of the elastic modulus of the material

• The speed of all mechanical waves follows a general form:

$$v = \sqrt{\frac{\text{elastic property}}{\text{inertial property}}}$$

Waves on a string →

$$v = \sqrt{\frac{T}{\mu}}$$
Waves on a string...

- So we find: \[ v = \sqrt{\frac{F}{\mu}} \]
  
  Making the tension bigger increases the speed.
  
  Making the string heavier decreases the speed.
  
  The speed depends only on the nature of the medium, not on amplitude, frequency etc of the wave.

Exercise  Wave Motion

- A heavy rope hangs from the ceiling, and a small amplitude transverse wave is started by jiggling the rope at the bottom.
  
  As the wave travels up the rope, its speed will:

    (a) increase
    (b) decrease
    (c) stay the same
Sound, A special kind of longitudinal wave

Consider a vibrating guitar string

String Vibrates

Piece of string undergoes harmonic motion

Air molecules alternatively compressed and rarefied

Sound

Consider the actual air molecules and their motion versus time,

<table>
<thead>
<tr>
<th>Time</th>
<th>Molecules</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>O O</td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

Individual molecules undergo harmonic motion with displacement in same direction as wave motion.
Speed of Sound in a Solid Rod

- The Young's modulus of the material is $Y$
- The density of the material is $\rho$
- The speed of sound in the rod is

\[
v = \sqrt{\frac{Y}{\rho}}
\]

Speed of Sound in Liquid or Gas

- The bulk modulus of the material is $B$
- The density of the material is $\rho$
- The speed of sound in that medium is

\[
v = \sqrt{\frac{B}{\rho}}
\]

<table>
<thead>
<tr>
<th>Medium</th>
<th>Speed (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air</td>
<td>343</td>
</tr>
<tr>
<td>Helium</td>
<td>972</td>
</tr>
<tr>
<td>Water</td>
<td>1500</td>
</tr>
<tr>
<td>Steel (solid)</td>
<td>5600</td>
</tr>
</tbody>
</table>

Speed of Sound in Air

- The speed of sound also depends on the temperature of the medium
- This is particularly important with gases
- For air, the relationship between the speed and temperature (if pressure is constant) is

\[v = (331 \text{ m/s}) \sqrt{1 + \frac{T_c}{273 \degree C}}\]

- (331 m/s is the speed at 0\degree C)
- $T_c$ is the air temperature in Centigrade
**Home Exercise**
Comparing Waves, He vs. Air

A sound wave having frequency $f_0$, speed $v_0$ and wavelength $\lambda_0$, is traveling through air when it encounters a large helium-filled balloon. Inside the balloon the frequency of the wave is $f_1$, its speed is $v_1$, and its wavelength is $\lambda_1$

Compare the speed of the sound wave inside and outside the balloon

(A) $v_1 < v_0$  (B) $v_1 = v_0$  (C) $v_1 > v_0$

Compare the frequency of the sound wave inside and outside the balloon

(A) $f_1 < f_0$  (B) $f_1 = f_0$  (C) $f_1 > f_0$

Compare the wavelength of the sound wave inside and outside the balloon

(A) $\lambda_1 < \lambda_0$  (B) $\lambda_1 = \lambda_0$  (C) $\lambda_1 > \lambda_0$

Waves, Wave fronts, and Rays

- Sound radiates away from a source in all directions.
- A small source of sound produces a spherical wave.
- Note any sound source is small if you are far enough away from it.
Waves, Wave fronts, and Rays

- Note that a small portion of a spherical wave front is well represented as a plane wave.

\[ I = \frac{P_{av}}{A} = \frac{P_{av}}{4\pi R^2} \]

Waves, Wave fronts, and Rays

- If the power output of a source is constant, the total power of any wave front is constant.

\[ I = \frac{P_{av}}{A} = \frac{P_{av}}{\text{const}} \]
**Exercise  Spherical Waves**

- You are standing 10 m away from a very loud, small speaker. The noise hurts your ears. In order to reduce the intensity to 1/4 its original value, how far away do you need to stand?

(A) 14 m  (B) 20 m  (C) 30 m  (D) 40 m

---

**Intensity of sounds**

- Intensity of a sound wave is
  \[ I = \frac{\Delta P^2}{2 \rho \nu} \]

  - Proportional to (amplitude)^2
  - This is a general result (not only for sound)
- Threshold of human hearing: \( I_0 = 10^{-12} \text{ W/m}^2 \)

- The range of intensities detectible by the human ear is very large
- It is convenient to use a logarithmic scale to determine the intensity level, \( \beta \)

  \[ \beta = 10 \log_{10} \left( \frac{I}{I_0} \right) \]
Intensity of sounds

- $I_0$ is called the reference intensity
  - It is taken to be the threshold of hearing
  - $I_0 = 1.00 \times 10^{-12}$ W/m$^2$
  - $I$ is the intensity of the sound whose level is to be determined $\beta$ is in decibels (dB)

- Threshold of pain: $I = 1.00$ W/m$^2$; $\beta = 120$ dB

- Threshold of hearing: $I_0 = 1.00 \times 10^{-12}$ W/m$^2$; $\beta = 0$ dB

Intensity of sounds

- Some examples (1 pascal $\equiv 10^{-5}$ atm):

<table>
<thead>
<tr>
<th>Sound Intensity</th>
<th>Pressure amplitude (Pa)</th>
<th>Intensity (W/m$^2$)</th>
<th>Level (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hearing threshold</td>
<td>$3 \times 10^{-5}$</td>
<td>$10^{-12}$</td>
<td>0</td>
</tr>
<tr>
<td>Classroom</td>
<td>0.01</td>
<td>$10^{-7}$</td>
<td>50</td>
</tr>
<tr>
<td>City street</td>
<td>0.3</td>
<td>$10^{-4}$</td>
<td>80</td>
</tr>
<tr>
<td>Car without muffler</td>
<td>3</td>
<td>$10^{-2}$</td>
<td>100</td>
</tr>
<tr>
<td>Indoor concert</td>
<td>30</td>
<td>1</td>
<td>120</td>
</tr>
<tr>
<td>Jet engine at 30 m</td>
<td>100</td>
<td>10</td>
<td>130</td>
</tr>
</tbody>
</table>
Sound Level, Example

- What is the sound level that corresponds to an intensity of $2.0 \times 10^{-7}$ W/m$^2$?
- $\beta = 10 \log_{10} \left( \frac{2.0 \times 10^{-7} \text{ W/m}^2}{1.0 \times 10^{-12} \text{ W/m}^2} \right)$
  $\quad = 10 \log_{10} 2.0 \times 10^5 = 53 \text{ dB}$

- Rule of thumb: An apparent “doubling” in the loudness is approximately equivalent to an increase of 10 dB.
- This factor is not linear with intensity

Loudness and Intensity

- Sound level in decibels relates to a *physical measurement* of the strength of a sound
- We can also describe a *psychological “measurement”* of the strength of a sound
- Our bodies “calibrate” a sound by comparing it to a reference sound
- This would be the threshold of hearing
- Actually, the threshold of hearing is this value for 1000 Hz
Physics 207 – Lecture 27

Doppler effect, moving sources/receivers

Lecture 28

• Assignment
  ✓ HW11, Due Tuesday, May 5\textsuperscript{th}
  ✓ HW12, Due Friday, May 8\textsuperscript{th}
  ✓ For Tuesday, Read through all of Chapter 21