Name: ___ Solutions ____________________________________________

Student ID: __________________________

Section #: _______

Physics 208 Final Exam December 15, 2008

Print your name and section clearly above. If you do not know your section number, write your TA’s name.

Your final answer must be placed in the box provided. You must show all your work to receive full credit. If you only provide your final answer (in the box), and do not show your work, you will receive very few points.

Problems will be graded on reasoning and intermediate steps as well as on the final answer. Be sure to include units, and also the direction of vectors.

You are allowed two double-sided 8½ x 11” sheets of handwritten notes and no other references. The exam lasts exactly 120 minutes.

Acceleration of gravity: \( g = 9.8 \) m/s\(^2\)
Coulomb constant: \( k_e = 9.0 \times 10^9 \) N \cdot m^2/C^2
Speed of light in vacuum: \( c = 3 \times 10^8 \) m/s
Permittivity of free space: \( \varepsilon_0 = 1/4\pi k_e = 8.85 \times 10^{-12} C^2/N \cdot m^2 \)
Permeability of free space: \( \mu_0 = 4\pi \times 10^{-7} T \cdot m/A \)
Planck’s constant: \( h = 6.626 \times 10^{-34} J \cdot s = 4.1357 \times 10^{-15} eV \cdot s \), \( \hbar = h/2\pi \)
Bohr radius: \( a_o = 0.053 \) nm
Atomic mass unit: 1 \( u = 1.66054 \times 10^{-27} \) kg
\[ = 931.494 \text{ MeV}/c^2 \]
Electron mass: \( m_e = 9.11 \times 10^{-31} \) kg
\[ = 0.00055u = 0.51 \text{ MeV}/c^2 \]
Proton mass: \( m_p = 1.67262 \times 10^{-27} \) kg
\[ = 1.00728u = 938.28 \text{ MeV}/c^2 \]
Neutron mass: \( m_n = 1.67493 \times 10^{-27} \) kg
\[ = 1.00866u = 939.57 \text{ MeV}/c^2 \]
Hydrogen atom mass: \( m_H = 1.67349 \times 10^{-27} \) kg
\[ = 1.007825u = 938.79 \text{ MeV}/c^2 \]
Fundamental charge: \( e = 1.60 \times 10^{-19} \) C
\( hc = 1240eV \cdot nm \)
\( 1eV = 1.602 \times 10^{-19} J \)

| Problem 1: _______ / 20 |
| Problem 2: _______ / 25 |
| Problem 3: _______ / 25 |
| Problem 4: _______ / 20 |
| Problem 5: _______ / 25 |
| Problem 6: _______ / 15 |
| TOTAL: _______ / 130 |

\[ 1 \text{ eV} = 1.602 \times 10^{-19} J \]
1) [20 pts, 4 pts each] **Multiple choice.** Show work/explanation for full credit.

   i) Plutonium $^{241}_{94}Pu$ decays directly into $^{241}_{95}Am$. What particle is emitted when $^{241}_{94}Pu$ decays?

   a) alpha particle  
   b) gamma particle  
   **c) electron**  
   d) positron  
   e) neutron  
   f) proton

   **Explanation/work:**
   Since $^{241}_{95}Am$ has one more proton than $^{241}_{94}Pu$ but the same total number of nucleons, a neutron must have changed into a proton in the nucleus. By charge conservation, an electron must have been emitted from the nucleus.

   ii) Calculate the current through the resistor $R_2$. The battery voltage is 20V.

   a) 0.1A  
   b) 0.2A  
   c) 0.25A  
   d) 0.3A  
   **e) 0.4A.**  
   f) 0.5A  
   g) 1.0A

   **Explanation/work:**
   Equivalent resistance is 25Ω. All the current goes through $R_1$, so the current through $R_1$ is 20V/25Ω=0.8A. The voltage drop across the resistor $R_1$ is then 16V, leaving 4V across the 10Ω resistor. The current through that is then 0.4A.
   Or can say $R_2$, $R_3$, $R_4$, and $R_5$ can be replaced by an equivalent resistance of 5Ω, which is in series with $R_1$. Since $R_1$ is four times as big as the equivalent resistance, the voltage is split in a 4:1 ratio, 16V across $R_1$ and 4V across $R_{eq}$. Same result.

   iii) The electron beam in one of your lab experiments was accelerated through a potential difference of 20V. Quantum mechanics says that these electrons have wave-like properties. What is the wavelength of each of the electrons?

   a) 0.27 nm  
   b) 1.23 nm  
   c) 62 nm  
   d) 125 nm  
   e) 620 nm

   **Explanation/Work**
   Each electron has an energy of 20eV, all of which is kinetic $\left( \frac{p^2}{2m} \right)$.

   DeBroglie says the wavelength is $\lambda = \frac{h}{p} = h / \sqrt{2mE_K} = \frac{hc}{\sqrt{2mc^2E_K}}$

   $\lambda = \frac{1240eV \cdot nm}{\sqrt{2(0.51 \times 10^8 eV)(20eV)}} = 0.2745nm$
iv) The $^{12}_6C$ atom has a mass of exactly 12u. The nucleons are bound together in the nucleus by the strong force. Which answer is closest to the energy required to separate the nucleus of the $^{12}_6C$ atom into isolated protons and neutrons?

- a. 92 MeV
- b. 120 MeV
- c. 920 MeV
- d. 940 MeV
- e. 11,200 MeV

**Explanation/Work:**

Need mass difference.

$$6m_H + 6m_N - 12u = 6(1.007825u) + 6(1.008665u) - 12u = 0.0989u$$

$$0.0989u \left( \frac{931.49 \text{ MeV}}{c^2} \right) = 92.12 \text{ MeV} / c^2$$

Then $E = mc^2 \Rightarrow \left(92.12 \text{ MeV} / c^2 \right)c^2 = 92.12 \text{ MeV}$

v) You use a lens of 1m focal length to form an image of the moon on a piece of paper. About what is the diameter of the image? The moon is $3.8 \times 10^8$ m away, and $3.5 \times 10^6$ m diameter.

- a. 0.1 cm
- b. 0.2 cm
- c. 0.5 cm
- d. 1.0 cm
- e. 1.5 cm

**Explanation/Work:**

Since the object distance is so large, the image is formed at the focal point. The magnification is $\text{image dist} / \text{obj dist} = 2.63 \times 10^{-9}$. The image size is then $$\left( 2.63 \times 10^{-9} \right) \left( 3.5 \times 10^6 \text{ m} \right) = 0.0092 \text{ m} = 0.01 \text{ m}$$
2) [25 points, 5 pts each] Short-answer questions

a) $^{241}\text{Am}$ (half-life is 432 years) is used in smoke detectors, with the alarm coming on when the alpha particles emitted from its nucleus are blocked by smoke and do not reach the detector. To make the detector operate, the $^{241}\text{Am}$ source must emit $10^7$ alpha particles / s.

How many micro-grams of $^{241}\text{Am}$ are required? (1 micro-gram $^{241}\text{Am} \sim 2.5 \times 10^{15}$ nuclei)

(There are 31,536,000 seconds in a year)

<table>
<thead>
<tr>
<th>micro-grams $^{241}\text{Am} =$</th>
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<tbody>
<tr>
<td>Value</td>
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b) Three infinitely long wires are arranged as shown. The lower two rest on the floor and carry a current of 100A each directed into the page. The top wire has a mass of 0.01 kg per meter of length. What current must the top wire carry in order for it to be levitated above the floor at a height of 1 cm?

![Diagram of three wires](image)

The force / unit length between two infinitely long current-carrying wires is $\frac{\mu_0 I_1 I_2}{2\pi d}$. This is directed as shown. The distance between them is 3 cm. The component along the $-z$ direction is $\cos(\theta) = \frac{1}{\sqrt{5}}$ of this, and there are two of them. So the force / length comes out to be

$$2 \cdot \cos \theta \frac{\mu_0 I_1 I_2}{2\pi d} \Rightarrow I_2 = \frac{g(m/L)2\pi d}{2\cos \theta \mu_0 I_1} = \frac{(9.8 m/s^2)(0.01 kg/m)2\pi(\sqrt{5}cm)(1 m/100 cm)}{2 \cdot \cos(\theta)(4\pi \times 10^{-7} T \cdot m/A)(100A)}$$

$$= 122.5 A.$$ Direction should be out of page so that force is repulsive.

$|I|=$

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<thead>
<tr>
<th>Magnitude</th>
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c) Carbon is the element with 6 electrons. Silicon is directly below carbon in the periodic table. How many electrons does silicon have? In this range of atomic number the subshells fill in order of increasing angular momentum.

Since C has 6 electrons, its configuration is 1s²2s²2p². Since silicon is directly below it, it must end in 3p². So it is 1s²2s²2p²3s²3p², so 14 electrons.

\[ \# \text{ electrons} = 14 \]

d) You have two separate, isolated capacitors, \( C_1 = 1 \mu F \), and \( C_2 = 2 \mu F \). \( C_1 \) has been charged to a potential difference of 6V, and \( C_2 \) has a potential difference of zero. Calculate the voltage across the capacitors after they are connected.

After they are connected, the effective capacitance is 3\( \mu \)F instead of 1\( \mu \)F, but the charge is the same. From \( V = \frac{Q}{C_{eff}} \), the voltage must be three times smaller, so it is 2 V.

\[ V_{after} = 2 \text{ V} \]

e) A light bulb is in a single-turn loop of wire as shown. The light bulb has a resistance of 0.02 \( \Omega \), and the loop resistance is negligible. The loop is near a long, straight wire that carries 10A of current. This 10 A current produces a magnetic flux of 0.05 Tesla·m² through the loop. Calculate the time \( \Delta t \) over which you must reduce the wire current to zero (at a constant rate) in order to dissipate 1 Watt of power in the light bulb.

The dissipated power is \( V^2/R = 1W \), so the required voltage is \( \sqrt{1W \times 0.02\Omega} = 0.141V \). The flux is 0.05 Tm²

\[ 0.141V = \left(0.05T \cdot m^2\right)/\Delta t \Rightarrow \Delta t = 0.355s = 355ms \]

\[ \Delta t = 355ms \]
3) [25 pts] A 100 turn rectangular coil of wire 20cm x 10cm is rotated in a 0.2T magnetic field as shown as constant angular velocity \( \omega_0 \). The plane of the loop is perpendicular to the field at \( \theta=0 \). The surface normal vector \( \hat{n} \) of the loop points upward at this angle, and rotates with the loop.

\[ \hat{n} \]

\[
\begin{array}{c}
100 \text{ turns} \\
\text{Rotate loop about this axis}
\end{array}
\]

\[
B=0.2T \\
\text{everywhere}
\]

a) [5pts] Make a plot of the magnetic flux through the loop as a function of angle \( \theta \). Flux is positive when it is in the direction of the surface normal.

\[
\begin{array}{c}
\theta=90^\circ \\
\theta=180^\circ \\
\theta=270^\circ \\
\theta=360^\circ
\end{array}
\]

b) [5 pts] At what angle(s) is the magnitude of the induced current in the loop a maximum? Explain.

The magnitude of the current is a maximum at \( \theta=90^\circ \) and \( \theta=270^\circ \). This is where the flux is changing the fastest. Can see this from the plot of flux vs angle. Constant rotation speed means \( \theta \) increases linearly with time. Slope is greatest at \( \theta=90^\circ \) and \( \theta=270^\circ \).

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<thead>
<tr>
<th>Angle(s)</th>
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</table>
c) [5 pts] Suppose you increase the rotation speed of the loop. Does the maximum current in the loop change? Explain.

d) [10 pts] The 20 cm X 10 cm rotating loop is used to charge a battery. It includes a circuit element that passes current only in the direction shown, and guarantees that all the charge flowing in that direction is delivered to the battery. Suppose that the voltage across the battery is negligible compared to the EMF around the loop.

The coil has N=100 turns, and a resistance of R=10Ω. How long will it take to charge the battery if you rotate the loop at 5 rotations/second? The battery requires 100 C of charge to be fully charged.

The instantaneous current in the loop is $EMF/R = -\frac{1}{R}d\Phi/dt$. When the loop is rotated from 0 to $\pi$, the flux goes from $(100\text{ turns})(0.2T)(0.1m \times 0.2m) = 0.4\text{ Wb}$ to $-0.4\text{ Wb}$. The total charge flowing in the loop during this time is $\Delta\Phi/R = 0.8\text{ Wb}/10\Omega = 0.08\text{ C}$. During the rest of the rotation the charge flows in the opposite direction and is blocked. The required number of rotations is $100\text{ C}/(0.08\text{ C}/\text{ rotation}) = 1250\text{ rotations}$. At 5 rotations/s this is 250 s.
4) [20 pts, 5 pts each] A solid conducting metal sphere with +10 µC charge and radius R₁ = 0.5 cm is at the center of a conducting metal shell. The conducting metal shell has a charge of -2 µC, an inner radius R₂ = 1 cm and outer radius R₃ = 2 cm.

a) What is the charge density on the inner surface of the conducting metal shell?

Since the electric field is zero inside a metal, by Gauss’ law there must be -10 µC on the inner surface to cancel the +10 µC on the inner sphere. The charge density is then

\[
\frac{-10 \mu C}{4\pi (1 cm)^2} = -0.796 \mu C/cm^2
\]

b) What is the charge density on the outer surface of the conducting metal shell?

Since the shell has a total charge of -2 µC, there must be +8 µC on the outer surface. The charge density is then

\[
\frac{+8 \mu C}{4\pi (2 cm)^2} = 0.159 \mu C/cm^2
\]
c) What is the work required to bring a charge of +1\(\mu\)C from infinitely far away from the sphere to a distance 5 cm from the center of the sphere?

The work is the electric potential times the charge. Since by Gauss' law the fields outside the sphere are identical to that of an 8\(\mu\)C point charge at the center. The electric potential is \(k(8 \times 10^{-6} \text{ C})/0.05 \text{ m}\), and so the required work is \((9 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(8 \times 10^{-6} \text{ C})(10^{-6} \text{ C})/0.05 \text{ m} = 1.44 \text{ J}\)

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d) Plot the electric potential \(V(r)\) as a function of distance from the center of the sphere.
5. [25 pts, 5 pts each] This problem is about a highway sodium lamp. It radiates 100 W of light power isotropically in all directions, and the emitted light has a wavelength of 590 nm.

a) What is the electric field amplitude 50 cm from the bulb?

\[
\text{At 50 cm, the intensity is } I = \frac{P}{4\pi r^2} = \frac{100\text{ W}}{4\pi(0.5\text{ m})^2} = 31.83\text{ W/m}^2.
\]

\[
\text{From } I = \frac{cE^2}{2}, E_{\text{max}} = \sqrt{\frac{2I}{c\varepsilon_0}} = \sqrt{\frac{2(31.83\text{ W/m}^2)}{\left(3 \times 10^8 \text{ m/s}\right)^2 \left(8.854 \times 10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2\right)}} = 154.8\text{ N/C}.
\]

\[
\text{E=}
\]

b) You look at the light using a spectrometer that has a diffraction grating with 600 lines/mm. At what angle \(\theta\) from the incident direction do you see the 590 nm light?

\[
\text{Use } d\sin\theta = m\lambda \text{ with } d = 10^6 \text{ nm} / 600 = 1667\text{ nm}.
\]

For \(m=1\), \(\sin\theta = \frac{590\text{ nm}}{1667\text{ nm}} \Rightarrow 20.73^\circ\)

For \(m=2\), \(\sin\theta = 2 \times \frac{590\text{ nm}}{1667\text{ nm}} \Rightarrow 45.1^\circ\)

\[
\theta=\]

c) Below are listed energies of the excited states of sodium. Use this information to describe what happens in the atom to produce the light described above.

The 590 nm photon has an energy of \(\frac{hc}{\lambda} = \frac{1240\text{ eV}\cdot\text{ nm}}{590\text{ nm}} = 2.1\text{ eV}\)

This matches only the energy difference \(-3.04\text{ eV} - (-5.14\text{ eV}) = 2.1\text{ eV}\)

So the outer electron made the transition \(3p\rightarrow3s\).

<table>
<thead>
<tr>
<th>Energy Level</th>
<th>Energy (eV)</th>
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<tbody>
<tr>
<td>(1s^22s^22p^63s^1)</td>
<td>-5.14 eV</td>
</tr>
<tr>
<td>(1s^22s^22p^63p^1)</td>
<td>-3.04 eV</td>
</tr>
<tr>
<td>(1s^22s^22p^64s^1)</td>
<td>-1.95 eV</td>
</tr>
<tr>
<td>(1s^22s^22p^63d^1)</td>
<td>-1.53 eV</td>
</tr>
<tr>
<td>(1s^22s^22p^64p^1)</td>
<td>-1.39 eV</td>
</tr>
</tbody>
</table>

d) The excited states of sodium are approximately described as the 3s electron excited into 3p, 4s, 3d, 4p, etc states. What is the magnitude of the orbital angular momentum of an electron in the 4p state?

\[\text{A } p\text{-state has } \ell = 1. \text{ So } |\ell| = \hbar\sqrt{\ell(\ell + 1)} = \hbar\sqrt{2}. \text{ Either } \sqrt{2} \text{ in units of } \hbar, \text{ or } 1.49 \times 10^{-34}\text{ J-s, or } 9.31 \times 10^{-16}\text{ eV-s}\]

Angular momentum=

<table>
<thead>
<tr>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
</table>

\[\text{Energy of each photon is } 2.1\text{ eV} = 3.364 \times 10^{-19}\text{ J.}\]

\[
(100\text{ J/s}) / (3.364 \times 10^{-19}\text{ J/photon}) = 2.97 \times 10^{20} \text{ photons/s}
\]

\[\text{# photons/sec=}\]
6) **[15 pts, 5 pts each]** For parts a) and b), an electron is in a one-dimensional box of length 1 nm. It has wavefunctions of \( \Psi(x) = \sqrt{\frac{2}{L}} \sin \left( \frac{n\pi x}{L} \right) \), and energies of \( E_n = \frac{n^2 h^2}{8m_e L^2} \)

a) What is the wavelength of the electron in the \( n=3 \) state?

> There are three half-wavelengths in the box in this state, so the wavelength is 0.66 nm.

\[ \lambda = \]

\[ \begin{array}{|c|c|} \hline \text{Value} & \text{Units} \\ \hline \end{array} \]

b) What is the probability \( P \) of finding the electron in a 0.01nm wide region about \( x=0.5\text{nm} \)?

At \( x=0.5\text{nm} \)

\[ \Psi^2(0.5\text{nm}) = \frac{2}{1nm} \sin^2 \left( \frac{3\pi}{2} \frac{0.5\text{nm}}{1\text{nm}} \right) = 2\sin^2 \left( \frac{3\pi}{2} \text{nm}^{-1} \right) = 2 nm^{-1} \]

The probability is then

\[ P = \Psi^2(x) \delta x = 0.02. \]

\[ \begin{array}{|c|c|} \hline \text{Value} & \text{Units} \\ \hline \end{array} \]

c) Now think about a 3-dimensional version of this box, each side still being 1 nm. Below is shown a constant probability surface for one of the quantum states. What are the quantum numbers of this state?

Quantum numbers = \( n_x=2, \; n_y=2, \; n_z=1 \)