Chapter 30. Potential and Field

To understand the production of electricity by solar cells or batteries, we must first address the connection between electric potential and electric field.

Chapter Goal: To understand how the electric potential is connected to the electric field.
Potential and Field

Work and potential energy are associated with assembling a charge distribution.
Potential and Field

The work per unit charge done by an electric force = the change in potential energy of a unit charge and the change in the electrostatic potential $V$ measured in volts.

\[ F \cdot ds = dW \]
\[ dV = dW / q \]
\[ E \cdot ds = dV \]
Batteries

- A chemical reaction entails an adjustment of electron concentration on different atoms.
- A battery uses a chemical reaction (pair), intercepting an electron forcing it to pass through a circuit.
- The battery terminals are in effect at different electric potentials.
- The battery potential is typically a few volts – the scale of atomic energy potential differences.

http://www.av8n.com/physics/battery.htm
http://www.batteryuniversity.com/index.htm

Copyright © 2008 Pearson Education, Inc., publishing as Pearson Addison-Wesley.
What total potential difference is created by these three batteries?

A. 1.0 V
B. 2.0 V
C. 5.0 V
D. 6.0 V
E. 7.0 V
What total potential difference is created by these three batteries?

A. 1.0 V
B. 2.0 V
C. 5.0 V
D. 6.0 V
E. 7.0 V
Conductors in equilibrium

- Move charge from one conductor to another.
- The excess charge distributes on the surfaces in a unique way such that the two conductors are free of E field so volumes of constant potential.
- The unique equilibrium surface charges densities are not uniform and in general tricky to calculate.
**Electric power transmission**

- Imagine this picture is a cross section of two wires.
- Do work to transport additional + charge from – to + at one end. The charge distributes along the length reproducing the picture but the total charge, the strength of the electric field, the potential difference are increased.
- Get work at the other end by transporting + charge from + to -. 
- The other end might be 1000 miles away!
Capacitance

- In electrostatic equilibrium, the E field is perpendicular to the surface or each conductor, else the transverse component would cause current.
- Doubling the surface charge distribution maintains the equilibrium conditions (E perpendicular to surfaces) but doubles |E| everywhere and the potential difference V.
- In general $V=Q/C$ where C [Coulomb/Volt = Farad] is called the capacitance and depends only on the geometry.

$V = \frac{1}{C} Q$
Energy stored in a capacitor

To transfer a charge $dq$ from $-$ to $+$ given an existing potential difference $V(q) = q/C$ requires differential work
$$dW = Vdq = (q/C)dq$$
Summing the differential work as $q$ ranges from 0 to $Q$ gives the total work to establish the $+Q$ and $-Q$ charge separation and the total energy stored $U$.

Aside: One can show $U$ is a volume integral of an energy density $u \sim E^2$. 

$$W = \int_{0}^{Q} \frac{q}{C} dq = \frac{Q^2}{2C}$$

$$U = \frac{Q^2}{2C} = \frac{1}{2} Q \Delta V = \frac{1}{2} C (\Delta V)^2$$
Parallel plate capacitor

The electric field and surface charge densities are uniform. The potential difference is $V = Ed$

\[ V_+ - V_- = -\left(-qEd\right)/q \]

\[ \Delta V = Ed = \frac{\eta d}{\varepsilon_o} = \frac{Q}{\varepsilon_o A} \left( \frac{d}{\varepsilon_o A} \right) \]

\[ C = \frac{\varepsilon_o A}{d} \]

Note the capacity is proportional to area $A$ and inversely proportional to separation.
Spherical capacitor

Charge $Q$ moved from outer to inner sphere

Gauss’ law says $E = \frac{kQ}{r^2}$

Potential difference $\Delta V = \int_{a}^{b} E \cdot ds$

Along path shown

$$\Delta V = \int_{a}^{b} \frac{kQ}{r^2} = -kQ \frac{1}{r} \bigg|_{a}^{b} = kQ \left( \frac{1}{a} - \frac{1}{b} \right)$$

$$C = \frac{Q}{\Delta V} = 4\pi \varepsilon_0 \frac{ab}{a + b}$$
Example capacitor

Example 30.6 Charging a capacitor

The spacing between the plates of a 1.0 μF capacitor is 0.050 mm.

a. What is the surface area of the plates?
b. How much charge is on the plates if this capacitor is attached to a 1.5 V battery?

Solve

a. From the definition of capacitance,

\[ A = \frac{dC}{\epsilon_0} = 5.65 \text{ m}^2 \]

b. The charge is \( Q = C \Delta V_C = 1.5 \times 10^{-6} \text{ C} = 1.5 \mu\text{C} \).
Capacitor combinations

**FIGURE 30.24** Parallel and series capacitors.

The circuit symbol for a capacitor is two parallel lines.

Parallel capacitors are joined top to top and bottom to bottom.

Series capacitors are joined end to end in a row.
Combinations of Capacitors

If capacitors \( C_1, C_2, C_3, \ldots \) are in parallel, their equivalent capacitance is (note \( C \) is proportional to area)

\[
C_{eq} = C_1 + C_2 + C_3 + \cdots \quad \text{(parallel capacitors)}
\]

If capacitors \( C_1, C_2, C_3, \ldots \) are in series, their equivalent capacitance is

\[
C_{eq} = \left( \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots \right)^{-1} \quad \text{(series capacitors)}
\]
Equivalent Capacitors

- Two capacitors connected in parallel
  - Same voltage: $V_1 = V_2 = V_{eq}$
  - Add Areas: $C_{eq} = C_1 + C_2$
  - Add Charge: $Q_{eq} = Q_1 + Q_2$

\[ C = \frac{\varepsilon_0 A}{d} \]
The Energy Stored in a Capacitor

A defibrillator, which can restore a normal heartbeat, discharges a capacitor through the patient’s chest.
EXAMPLE 30.9 Storing energy in a capacitor

QUESTIONS:

EXAMPLE 30.9  Storing energy in a capacitor
How much energy is stored in a 2.0 $\mu$F capacitor that has been charged to 5000 V? What is the average power dissipation if this capacitor is discharged in 10 $\mu$s?
EXAMPLE 30.9 Storing energy in a capacitor

**SOLVE** The energy stored in the charged capacitor is

\[ U_c = \frac{1}{2} C(\Delta V_c)^2 = \frac{1}{2} (2.0 \times 10^{-6} \text{ F})(5000 \text{ V})^2 = 25 \text{ J} \]

If this energy is released in 10 \( \mu \text{s} \), the average power dissipation is

\[ P = \frac{\Delta E}{\Delta t} = \frac{25 \text{ J}}{1.0 \times 10^{-5} \text{ s}} = 2.5 \times 10^6 \text{ W} = 2.5 \text{ MW} \]
The Energy in the Electric Field

The energy density of an electric field, such as the one inside a capacitor, is

\[ u_E = \frac{\text{energy stored}}{\text{volume in which it is stored}} = \frac{U_C}{Ad} = \frac{\varepsilon_0}{2} E^2 \]

The energy density has units J/m\(^3\).
Effect of insulators in the gap

Capacitance $C_0$ in vacuum

Dielectric

Polarized molecule

Negative charge on surface

Positive charge on surface

(b)

Electrostatic field lines

(b)
Dielectric physics

Insulator is polarized by the field between the conducting plates forming in effect two narrow gap (large C) capacitors in series. Put another way, for fixed charge on the conductors the polarization produces an opposing E field reducing the voltage between the conductors by some constant $k$.

$$C = \frac{Q}{\Delta V_C} = \frac{Q_0}{(\Delta V_C)_0/k} = \kappa \frac{Q_0}{(\Delta V_C)_0} = \kappa C_0$$
Dielectrics

• The dielectric constant, like density or specific heat, is a property of a material.
• Easily polarized materials have larger dielectric constants than materials not easily polarized.
• Vacuum has $\kappa = 1$ exactly.
• **Filling a capacitor with a dielectric increases the capacitance by a factor equal to the dielectric constant.**

\[
C = \frac{Q}{\Delta V_C} = \frac{Q_0}{(\Delta V_C)_0/\kappa} = \kappa \frac{Q_0}{(\Delta V_C)_0} = \kappa C_0
\]
The polarizability can be significant!

But unlike vacuum, an insulator will breakdown (spark) beyond a critical field strength.

### Table 26.1

Approximate Dielectric Constants and Dielectric Strengths of Various Materials at Room Temperature

<table>
<thead>
<tr>
<th>Material</th>
<th>Dielectric Constant $\kappa$</th>
<th>Dielectric Strength* $(10^6 \text{ V/m})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air (dry)</td>
<td>1.00059</td>
<td>3</td>
</tr>
<tr>
<td>Bakelite</td>
<td>4.9</td>
<td>24</td>
</tr>
<tr>
<td>Fused quartz</td>
<td>3.78</td>
<td>8</td>
</tr>
<tr>
<td>Mylar</td>
<td>3.2</td>
<td>7</td>
</tr>
<tr>
<td>Neoprene rubber</td>
<td>6.7</td>
<td>12</td>
</tr>
<tr>
<td>Nylon</td>
<td>3.4</td>
<td>14</td>
</tr>
<tr>
<td>Paper</td>
<td>3.7</td>
<td>16</td>
</tr>
<tr>
<td>Paraffin-impregnated paper</td>
<td>3.5</td>
<td>11</td>
</tr>
<tr>
<td>Polystyrene</td>
<td>2.56</td>
<td>24</td>
</tr>
<tr>
<td>Polyvinyl chloride</td>
<td>3.4</td>
<td>40</td>
</tr>
<tr>
<td>Porcelain</td>
<td>6</td>
<td>12</td>
</tr>
<tr>
<td>Pyrex glass</td>
<td>5.6</td>
<td>14</td>
</tr>
<tr>
<td>Silicone oil</td>
<td>2.5</td>
<td>15</td>
</tr>
<tr>
<td>Strontium titanate</td>
<td>233</td>
<td>8</td>
</tr>
<tr>
<td>Teflon</td>
<td>2.1</td>
<td>60</td>
</tr>
<tr>
<td>Vacuum</td>
<td>1.00000</td>
<td>-</td>
</tr>
<tr>
<td>Water</td>
<td>80</td>
<td>-</td>
</tr>
</tbody>
</table>

* The dielectric strength equals the maximum electric field that can exist in a dielectric without electrical breakdown. Note that these values depend strongly on the presence of impurities and flaws in the materials.
Which potential-energy graph describes this electric field?
Which potential-energy graph describes this electric field?
Which set of equipotential surfaces matches this electric field?

(a) 0 V  50 V  0 V  50 V  0 V  50 V

(b) 0 V  50 V  0 V  50 V  0 V  50 V

(c) 0 V  50 V  0 V  50 V  0 V  50 V

(d) 50 V  0 V

(e) 50 V  0 V

Copyright © 2008 Pearson Education, Inc., publishing as Pearson Addison-Wesley.
Which set of equipotential surfaces matches this electric field?

- (a) 0 V 50 V 0 V 50 V
- (b) 0 V 50 V 0 V 50 V
- (c) 0 V 50 V
- (d) 50 V 0 V
- (e) 50 V 0 V
Three charged, metal spheres of different radii are connected by a thin metal wire. The potential and electric field at the surface of each sphere are $V$ and $E$. Which of the following is true?

A. $V_1 = V_2 = V_3$ and $E_1 > E_2 > E_3$
B. $V_1 > V_2 > V_3$ and $E_1 = E_2 = E_3$
C. $V_1 = V_2 = V_3$ and $E_1 = E_2 = E_3$
D. $V_1 > V_2 > V_3$ and $E_1 > E_2 > E_3$
E. $V_3 > V_2 > V_1$ and $E_1 = E_2 = E_3$
Three charged, metal spheres of different radii are connected by a thin metal wire. The potential and electric field at the surface of each sphere are $V$ and $E$. Which of the following is true?

A. $V_1 = V_2 = V_3$ and $E_1 > E_2 > E_3$
B. $V_1 > V_2 > V_3$ and $E_1 = E_2 = E_3$
C. $V_1 = V_2 = V_3$ and $E_1 = E_2 = E_3$
D. $V_1 > V_2 > V_3$ and $E_1 > E_2 > E_3$
E. $V_3 > V_2 > V_1$ and $E_1 = E_2 = E_3$
Rank in order, from largest to smallest, the equivalent capacitance \((C_{eq})_a\) to \((C_{eq})_d\) of circuits a to d.

A. \((C_{eq})_d > (C_{eq})_b > (C_{eq})_a > (C_{eq})_c\)
B. \((C_{eq})_d > (C_{eq})_b = (C_{eq})_c > (C_{eq})_a\)
C. \((C_{eq})_a > (C_{eq})_b = (C_{eq})_c > (C_{eq})_d\)
D. \((C_{eq})_b > (C_{eq})_a = (C_{eq})_d > (C_{eq})_c\)
E. \((C_{eq})_c > (C_{eq})_a = (C_{eq})_d > (C_{eq})_b\)
Rank in order, from largest to smallest, the equivalent capacitance \((C_{eq})_a\) to \((C_{eq})_d\) of circuits a to d.

A. \((C_{eq})_d > (C_{eq})_b > (C_{eq})_a > (C_{eq})_c\)
B. \((C_{eq})_d > (C_{eq})_b = (C_{eq})_c > (C_{eq})_a\)
C. \((C_{eq})_a > (C_{eq})_b = (C_{eq})_c > (C_{eq})_d\)
D. \((C_{eq})_b > (C_{eq})_a = (C_{eq})_d > (C_{eq})_c\)
E. \((C_{eq})_c > (C_{eq})_a = (C_{eq})_d > (C_{eq})_b\)