Chapter 34. Electromagnetic Induction

Electromagnetic induction is the scientific principle that underlies many modern technologies, from the generation of electricity to communications and data storage.

**Chapter Goal:** To understand and apply electromagnetic induction.
If a magnet is moved towards a circuit, or a circuit moved towards a magnet, a current is *induced* in the circuit. The direction of the current is such that its field opposes the inducing field (Lenz’s rule).
Magnetic Induction

In the case of a stationary circuit, the induced current is driven by an electric field associated with the changing magnetic field. The “electromotive force” can be observed with a voltmeter. Whenever a magnetic field is changing, a new sort of circulating electric field exists.
Magnetic Induction Application

Magnetic data storage encodes information in a pattern of alternating magnetic fields. When these fields move past a small *pick-up coil*, the changing magnetic field implies an electric field which induces current in the coil. The current pulses represent the 0s and 1s of digital data.
Magnetic induction can be used to sense the positions of ferromagnetic steel strings in a guitar.

In this application, the motion of magnetized strings implies time dependent magnetic and hence electric fields.
Motional magnetic induction

In the case in which the circuit moves, the electromotive force is the magnetic force on the electrons forced to move moving with the circuit.

These effects are related – they are the same effect viewed in different frames of reference.

FIGURE 34.3 Two different ways to generate an emf.

(a) Magnetic forces separate the charges and cause a potential difference between the ends. This is a motional emf.
Motional magnetic example

The magnetic motional EMF in this sliding bar circuit is the integral of the magnetic force per unit charge $F/q = vB$ along the length $l$ of the bar.

\[ |\mathcal{E}| = B\ell v \]
EXAMPLE 34.1 Measuring the earth’s magnetic field

It is known that the earth’s magnetic field over northern Canada points straight down. The crew of a Boeing 747 aircraft flying at 260 m/s over northern Canada finds a 0.95 V potential difference between the wing tips. The wing span of a Boeing 747 is 65 m. What is the magnetic field strength there?
EXAMPLE 34.1 Measuring the earth’s magnetic field

**MODEL** The wing is a conductor moving through a magnetic field, so there is a motional emf.

**SOLVE** The magnetic field is perpendicular to the velocity, so we can use Equation 34.3 to find

\[
B = \frac{\mathcal{E}}{vL} = \frac{0.95 \text{ V}}{(260 \text{ m/s})(65 \text{ m})} = 5.6 \times 10^{-5} \text{ T}
\]
Electric generator

The loop rotation (driven by human or water or steam power) moves wires in a fixed B field generating an EMF by motional induction. Mechanical energy is converted to electrical energy.
Motional induction produces currents and braking forces when a conductor moves in a magnetic field.
Faraday’s Law

The EMF in any closed loop is the negative of the time rate of change of magnetic flux through any closed surface spanning the loop:

\[ \int \mathbf{E} \cdot d\mathbf{l} = -\frac{d}{dt} \int \mathbf{B} \cdot d\mathbf{s} \]

\[ \text{EMF} = -\frac{d}{dt} \Phi_B \]

Direction of flux related to direction of EMF by r.h. rule.

The 2\textsuperscript{nd} form applies to motional EMF also.
Alternating voltage/current

\[
\int \mathbf{B} \cdot ds = NBA \cos \theta = NBA \cos(\omega t)
\]

\[
EMF = -\frac{d}{dt} \Phi_B = \omega NBA \sin(\omega t)
\]
The generator may be run in reverse. If an alternating current is caused to flow in the loop, the magnetic force on the wires causes the loop to rotate. Electrical energy is converted to mechanical energy.
Maxwell’s generalization of Faraday’s Law

**FIGURE 34.35** Maxwell hypothesized the existence of induced magnetic fields.

A changing magnetic field creates an induced electric field.

Region of increasing $\vec{B}$

Induced electric field $\vec{E}$

A changing electric field creates an induced magnetic field.

Region of increasing $\vec{E}$

Induced magnetic field $\vec{B}$

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Maxwell’s Equations

\[ \oint E \cdot ds = 0 \quad \text{becomes} \quad \oint E \cdot ds = 0 - \frac{d\Phi_B}{dt} \]

\[ \oint B \cdot ds = \mu_0 I \quad \text{becomes} \quad \oint B \cdot ds = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} \]

- Not only charges produce E-field
  - a changing B-field also produces an E-field
- Not only currents produce B-field
  - a changing E-field produces a B-field

\[ \implies \mathbf{E} \neq -\nabla V \]
Maxwell’s Equations

\[ \int_S \mathbf{E} \cdot d\mathbf{a} = q/\varepsilon_0 \]  \hspace{1cm} \text{Gauss’s Law}

\[ \int_S \mathbf{B} \cdot d\mathbf{a} = 0 \]  \hspace{1cm} \text{No magnetic monopoles}

\[ \int \mathbf{E} \cdot d\mathbf{l} = -\frac{d}{dt} \int \mathbf{B} \cdot d\mathbf{s} \]  \hspace{1cm} \text{Faraday’s Law}

\[ \int \mathbf{B} \cdot d\mathbf{l} = \mu_0 I + \frac{1}{c^2} \frac{d}{dt} \int \mathbf{E} \cdot d\mathbf{s} \]  \hspace{1cm} \text{Ampere’s Law}

\[ c = 1/\sqrt{\mu_0 \varepsilon_0} = 3 \times 10^8 \, \text{m/s} \]
Maxwell’s Equations

\[ \int_S \mathbf{E} \cdot d\mathbf{a} = \frac{q}{\varepsilon_0} \]

The part of \( \mathbf{E} \) that diverges comes from charge

\[ \int_S \mathbf{B} \cdot d\mathbf{a} = 0 \]

No part of \( \mathbf{B} \) diverges

\[ \int \mathbf{E} \cdot d\mathbf{l} = -\frac{d}{dt} \int \mathbf{B} \cdot d\mathbf{s} \]

A part of \( \mathbf{E} \) circulates associated with time dependent \( \mathbf{B} \)

\[ \int \mathbf{B} \cdot d\mathbf{l} = \mu_0 I + \frac{1}{c^2} \frac{d}{dt} \int \mathbf{E} \cdot d\mathbf{s} \]

The circulation of \( \mathbf{B} \) comes from \( I \) and also is associated with time dependent \( \mathbf{E} \).

\[ c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3 \times 10^8 \text{ m/s} \]
Electromagnetic waves

Maxwell discovered there are self-sustaining wave solutions with the absence of charge and current that move at light speed.

\[
\int_S \mathbf{E} \cdot d\mathbf{a} = 0
\]

\[
\int_S \mathbf{B} \cdot d\mathbf{a} = 0
\]

\[
\int E \cdot dl = -\frac{d}{dt} \int B \cdot ds
\]

\[
\int B \cdot dl = \frac{1}{c^2} \frac{d}{dt} \int E \cdot ds
\]

FIGURE 34.36 A self-sustaining electromagnetic wave.

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Magnetic inductance in circuits

A current in one circuit generates a magnetic field and magnetic flux in another circuit.

A change in the current implies a change in magnetic field and linked flux and associated electric field, EMF, and induced current in the other circuit.

The flux in one circuit due to another is a geometric quantity called the mutual inductance.

The flux in a single circuit due to its own magnetic field is called its self inductance.
Magnetic inductance defined

\[ \Phi_B(2 \text{ due to } 1) = L_{21} I_1 \]

\[ EMF_2 = -\frac{d}{dt} \Phi_B(2 \text{ due to } 1) = L_{21} \frac{d}{dt} I_1 \]
Self Inductance

In changing the current in a circuit, the induced EMF opposes the increase in current. If the rate of change is fast enough, these induced EMFs can be as important as any static voltage source!

Prototype: The self inductance of a solenoid having \( N \) turns, length \( l \) and cross-section area \( A \) is the flux for \( N \) turns NBA per unit current:

\[
L_{\text{solenoid}} = \frac{\Phi_m}{I} = \frac{\mu_0 N^2 A}{l}
\]
EXAMPLE 34.12 The length of an inductor

An inductor is made by tightly wrapping 0.30-mm-diameter wire around a 4.0-mm-diameter cylinder. What length cylinder has an inductance of 10 $\mu$H?
**SOLVE** The cross-section area of the solenoid is \( A = \pi r^2 \). If the wire diameter is \( d \), the number of turns of wire on a cylinder of length \( l \) is \( N = l/d \). Thus the inductance is

\[
L = \frac{\mu_0 N^2 A}{l} = \frac{\mu_0 (l/d)^2 \pi r^2}{l} = \frac{\mu_0 \pi r^2 l}{d^2}
\]

The length needed to give inductance \( L = 1.0 \times 10^{-5} \text{ H} \) is

\[
l = \frac{d^2 L}{\mu_0 \pi r^2} = \frac{(0.00030 \text{ m})^2 (1.0 \times 10^{-5} \text{ H})}{(4\pi \times 10^{-7} \text{ T m/A}) \pi (0.0020 \text{ m})^2}
\]

\[
= 0.057 \text{ m} = 5.7 \text{ cm}
\]
Self Inductance circuit equivalent

An ideal inductor is has no resistance but nonnegligible self inductance and passively opposes changes in current.

**Figure 34.42** The potential difference across a resistor and an inductor.

- **Resistor**
  - $\Delta V_R = -IR$
  - The potential always decreases.

- **Inductor**
  - $\Delta V_L = -L \frac{dI}{dt}$
  - The potential decreases if the current is increasing.
  - The potential increases if the current is decreasing.
EXAMPLE 34.13 Large voltage across an inductor

QUESTION:

EXAMPLE 34.13 Large voltage across an inductor
A 1.0 A current passes through a 10 mH inductor coil. What potential difference is induced across the coil if the current drops to zero in 5.0 $\mu$s?
EXAMPLE 34.13 Large voltage across an inductor

**MODEL** Assume this is an ideal inductor, with \( R = 0 \ \Omega \), and that the current decrease is linear with time.

**SOLVE** The rate of current decrease is

\[
\frac{dI}{dt} \approx \frac{\Delta I}{\Delta t} = \frac{-1.0 \text{ A}}{5.0 \times 10^{-6} \text{ s}} = -2.0 \times 10^5 \text{ A/s}
\]

The induced voltage is

\[
\Delta V_L = -L \frac{dI}{dt} \approx -(0.010 \text{ H})(-2.0 \times 10^5 \text{ A/s}) = 2000 \text{ V}
\]
The current has decreased to 37% of its initial value at $t = \tau$.
The current has decreased to 13% of its initial value at $t = 2\tau$. 

$I(t) = I_o e^{-t/(L/R)}$

$\tau = R/L$
Energy in a magnetic field

To move charge dq against the back EMF while charging an inductor with current requires work

\[ dW = V \ dq = V \ Idt \]

Write this in terms of the increment of current and integrate

\[ dW = V \ Idt = L \frac{dI}{dt} \ Idt = LI \ dI \]

\[ W = \int dW = \int_0^I LI \ dI = \frac{1}{2} LI^2 \]

This stored energy is also the volume integral of a magnetic energy density

\[ u_B = \frac{1}{2\mu_0} B^2 \]
- Induced EMF extremely high
- Breaks down air gap at switch
- Air gap acts as resistor
LC Circuit

Combine:

\[ Q = CV \quad V = -L \frac{dI}{dt} \]

\[ \frac{dI}{dt} = \frac{d^2Q}{dt^2} = -\frac{V}{L} = -\frac{Q}{LC} \]

Find equation of an oscillator:

\[ \ddot{Q} = -\omega^2 Q \quad \omega = \frac{1}{\sqrt{LC}} \]

The solutions are harmonic functions. The energy oscillates between electric (capacitor charged, \( CV^2/2 \)) and magnetic (inductor, \( LI^2/2 \)).
The current in an LC circuit

The current in an \textit{LC} circuit where the initial charge on the capacitor is $Q_0$ is

$$I = -\frac{dQ}{dt} = \omega Q_0 \sin \omega t = I_{\text{max}} \sin \omega t$$

The oscillation frequency is given by

$$\omega = \sqrt{\frac{1}{LC}}$$
EXAMPLE 34.15 An AM radio oscillator

You have a 1.0 mH inductor. What capacitor should you choose to make an oscillator with a frequency of 920 kHz? (This frequency is near the center of the AM radio band.)

SOLVE The angular frequency is \( \omega = 2\pi f = 5.78 \times 10^6 \text{ rad/s} \). Using Equation 34.51 for \( \omega \) gives the required capacitor:

\[
C = \frac{1}{\omega^2 L} = \frac{1}{(5.78 \times 10^6 \text{ rad/s})^2(0.0010 \text{ H})}
\]

\[
= 3.0 \times 10^{-11} \text{ F} = 30 \text{ pF}
\]

A radio uses a tuned LC circuit to selectively respond to electric waves of a given frequency. Note the extreme frequency!
Chapter 34. Summary Slides
General Principles

Faraday’s Law

MODEL  Make simplifying assumptions.

VISUALIZE  Use Lenz’s law to determine the direction of the induced current.

SOLVE  The induced emf is

\[ \mathcal{E} = \left| \frac{d\Phi_m}{dt} \right| \]

Multiply by \( N \) for an \( N \)-turn coil.
The size of the induced current is \( I = \mathcal{E} / R \).

ASSESS  Is the result reasonable?
General Principles

**Lenz’s Law**

There is an induced current in a closed conducting loop if and only if the magnetic flux through the loop is changing. The direction of the induced current is such that the induced magnetic field opposes the *change* in the flux.
Magnetic flux

Magnetic flux measures the amount of magnetic field passing through a surface.

\[ \Phi_m = \vec{A} \cdot \vec{B} = AB \cos \theta \]
Important Concepts

Three ways to change the flux

1. A loop moves into or out of a magnetic field.

2. The loop changes area or rotates.

3. The magnetic field through the loop increases or decreases.
Important Concepts

Two ways to create an induced current

1. A **motional emf** due to magnetic forces on moving charge carriers.

2. An induced electric field due to a changing magnetic field.
## Applications

### Inductors

- Solenoid inductance: \( L_{\text{solenoid}} = \frac{\mu_0 N^2 A}{l} \)
- Potential difference: \( \Delta V_L = -L \frac{dI}{dt} \)
- Energy stored: \( U_L = \frac{1}{2} LI^2 \)
- Magnetic energy density: \( u_B = \frac{1}{2\mu_0} B^2 \)
Applications

**LC circuit**

Oscillates at \( \omega = \sqrt{\frac{1}{LC}} \)

**LR circuit**

Exponential change with \( \tau = \frac{L}{R} \)
A square conductor moves through a uniform magnetic field. Which of the figures shows the correct charge distribution on the conductor?
A square conductor moves through a uniform magnetic field. Which of the figures shows the correct charge distribution on the conductor?
Is there an induced current in this circuit? If so, what is its direction?

A. No
B. Yes, clockwise
C. Yes, counterclockwise
Is there an induced current in this circuit? If so, what is its direction?

A. No
B. Yes, clockwise
C. Yes, counterclockwise
A square loop of copper wire is pulled through a region of magnetic field. Rank in order, from strongest to weakest, the pulling forces $F_a$, $F_b$, $F_c$ and $F_d$ that must be applied to keep the loop moving at constant speed.

A. $F_b = F_d > F_a = F_c$
B. $F_c > F_b = F_d > F_a$
C. $F_c > F_d > F_b > F_a$
D. $F_d > F_b > F_a = F_c$
E. $F_d > F_c > F_b > F_a$
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C. $F_c > F_d > F_b > F_a$
D. $F_d > F_b > F_a = F_c$
E. $F_d > F_c > F_b > F_a$
A current-carrying wire is pulled away from a conducting loop in the direction shown. As the wire is moving, is there a cw current around the loop, a ccw current or no current?

A. There is no current around the loop.
B. There is a clockwise current around the loop.
C. There is a counterclockwise current around the loop.
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A. There is no current around the loop.

B. There is a clockwise current around the loop.
C. There is a counterclockwise current around the loop.
A conducting loop is halfway into a magnetic field. Suppose the magnetic field begins to increase rapidly in strength. What happens to the loop?

A. The loop is pulled to the left, into the magnetic field.
B. The loop is pushed to the right, out of the magnetic field.
C. The loop is pushed upward, toward the top of the page.
D. The loop is pushed downward, toward the bottom of the page.
E. The tension in the wires increases but the loop does not move.
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The potential at a is higher than the potential at b. Which of the following statements about the inductor current $I$ could be true?

A. $I$ is from b to a and is steady.
B. $I$ is from b to a and is increasing.
C. $I$ is from a to b and is steady.
D. $I$ is from a to b and is increasing.
E. $I$ is from a to b and is decreasing.
The potential at a is higher than the potential at b. Which of the following statements about the inductor current $I$ could be true?

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D. $I$ is from a to b and is increasing.
E. $I$ is from a to b and is decreasing.
Rank in order, from largest to smallest, the time constants $\tau_a$, $\tau_b$, and $\tau_c$ of these three circuits.

A. $\tau_a > \tau_b > \tau_c$
B. $\tau_b > \tau_a > \tau_c$
C. $\tau_b > \tau_c > \tau_a$
D. $\tau_c > \tau_a > \tau_b$
E. $\tau_c > \tau_b > \tau_a$
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D. $\tau_c > \tau_a > \tau_b$
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