Chapter 27. The Electric Field

Electric fields are responsible for the electric currents that flow through your computer and the nerves in your body. Electric fields also line up polymer molecules to form the images in a liquid crystal display (LCD).

Chapter Goal: To learn how to calculate and use the electric field.
**Electric force of a point charge**

The electric field at a point in space \( \mathbf{r} \) of a point charge \( q \) at the origin, \( r = 0 \), is the force per unit charge placed at \( \mathbf{r} \):

\[
E = \frac{1}{4\pi \varepsilon_0} \frac{q}{r^2} \hat{r}
\]

(electric field of a point charge)
At the position of the dot, the electric field points

A. Up.
B. Down.
C. Left.
D. Right.
E. The electric field is zero.
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Motion of a Charged Particle in an Electric Field

The electric field exerts a force

\[ \vec{F}_{\text{on } q} = q \vec{E} \]

on a charged particle. If this is the only force acting on \( q \), it causes the charged particle to accelerate with

\[ \vec{a} = \frac{\vec{F}_{\text{on } q}}{m} = \frac{q}{m} \vec{E} \]

In a uniform field, the acceleration is:

\[ \frac{|q_e|}{m_e} = \frac{1.6 \times 10^{-19} \text{ C}}{9.1 \times 10^{-31} \text{ kg}} = 1.7 \times 10^{11} \frac{\text{C}}{\text{kg}} \]

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Electric Field Superposition

The net electric field due to a group of point charges is

\[ \vec{E}_{\text{net}} = \frac{F_{\text{on}q}}{q} = \frac{F_{1\text{on}q}}{q} + \frac{F_{2\text{on}q}}{q} + \cdots = \vec{E}_1 + \vec{E}_2 + \cdots = \sum \vec{E}_i \]

where \( E_i \) is the field from point charge \( i \).

**SOLVE** The mathematical representation is \( \vec{E}_{\text{net}} = \sum \vec{E}_i \).

- For each charge, determine its distance from \( P \) and the angle of \( \vec{E}_i \) from the axes.
- Calculate the field strength of each charge’s electric field.
- Write each vector \( \vec{E}_i \) in component form.
- Sum the vector components to determine \( \vec{E}_{\text{net}} \).
- If needed, determine the magnitude and direction of \( \vec{E}_{\text{net}} \).
Quick question

Which vector best represents the electric field at the red dot?
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The Electric Field of a Dipole

For two opposite charges $\pm q$ separated by the small distance $s$, the dipole moment is defined as the vector

$$\mathbf{p} = \sum q \mathbf{x}_q = q(\mathbf{x}_+ - \mathbf{x}_-) \equiv qs$$

An effective dipole-moment vector characterizes the distant electric field of any neutral charge distribution.
The Electric Field of a Dipole

On the $y$-axis

$E = E_y \hat{y}$

$E_y = E_+ + E_-$

$= kq \frac{1}{(y - s/2)^2} + (-kq) \frac{1}{(y + s/2)^2}$

$= kq \frac{2ys}{(y - s/2)^2 (y + s/2)^2}$

$\approx k(2qs) \frac{1}{y^3}$ for $y \gg s$

Since $\vec{p}$ points from - charge to + charge

$\vec{E} = k \frac{2\vec{p}}{r^3}$ on $y$-axis of dipole only
The Electric Field of a Dipole

The electric field at a point on the axis of a dipole is

\[ \vec{E}_{\text{dipole}} \approx \frac{1}{4\pi \epsilon_0} \frac{2\vec{p}}{r^3} \]  
(on the axis of an electric dipole)

where \( r \) is the distance measured from the center of the dipole. (A net charge atop the dipole would add a \( 1/r^2 \) term.) The electric field in the plane that bisects and is perpendicular to the dipole is

\[ \vec{E}_{\text{dipole}} \approx -\frac{1}{4\pi \epsilon_0} \frac{\vec{p}}{r^3} \]  
(perpendicular plane)

This field is opposite to the dipole direction, and it is only half the strength of the on-axis field at the same distance.

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Dipoles in an Electric Field

In a uniform electric field, the net force on a dipole vanishes but the net torque need not. The torque on a dipole in an electric field is

\[ \tau = lF = (s \sin \theta)(qE) = pE \sin \theta \]

where \( \theta \) is the angle the dipole makes with the electric field.

**FIGURE 27.30** The torque on a dipole.

The torque due to a couple is \( \tau = lF = pE \sin \theta \).

In terms of vectors, \( \tau = \vec{p} \times \vec{E} \).
Electric field lines

Electric field lines follow the flow and help visualize the field. (Thanks Faraday!)

Field lines are tangent to the electric field and never cross.
Notice the density of lines corresponds to field strength.
Electric field of two like charges

You can almost “see” the repulsion in the field lines.
EXAMPLE 27.2 The electric field of a water molecule

The water molecule $\text{H}_2\text{O}$ has a permanent dipole moment of magnitude $6.2 \times 10^{-30}$ C·m. What is the electric field strength 1.0 nm from a water molecule at a point on the dipole’s axis?

SOLVE The on-axis electric field strength at $r = 1.0$ nm is

$$E \approx \frac{1}{4\pi \epsilon_0} \frac{2p}{r^3} = \left(9.0 \times 10^9 \text{ Nm}^2/\text{C}^2\right) \frac{2(6.2 \times 10^{-30} \text{ Cm})}{(1.0 \times 10^{-9} \text{ m})^3}$$

$$= 1.1 \times 10^8 \text{ N/C}$$

Water is a polar molecule, neutral but with charged legs. Those charges enable it to rip apart (dissolve) other molecules.
Continuous Charge Distributions

We can describe a smoothly varying charge distribution in terms of a charge density function akin to mass density. For example, the linear charge density of an object of length $L$ and charge $Q$ uniformly distributed along its length, is defined as

$$\lambda = \frac{Q}{L}$$

Linear charge density, which has units of C/m, is the amount of charge per meter of length.
Fields of Charge Distributions

We can calculate the electric field of a general charge distribution by dividing it into small pieces and summing the fields of the pieces using methods of calculus as the number of pieces tend to infinity.

\[ E(x) = \int k dq \hat{r} / r^2 = \int k \rho(x') dv' \hat{r} / r^2; r = x - x' \]

\[ dq = \rho(x') dv' \]
An Infinite Line of Charge

A very long, thin rod or taut wire, with uniform linear charge density $\lambda$, has an electric field pointing radially away from the rod.

$$E_{\text{line}} = \lim_{L \to \infty} \frac{1}{4\pi \varepsilon_0} \frac{|Q|}{r \sqrt{r^2 + (L/2)^2}} = \frac{1}{4\pi \varepsilon_0} \frac{|Q|}{rL/2} = \frac{1}{4\pi \varepsilon_0} \frac{2|\lambda|}{r}$$

where $r$ is the radial distance (cylindrical coordinate) away from the rod. Except for the factor 2, this can be guessed by dimensional analysis.

$$\left[ \frac{Q}{r^2} \right] = \left[ \frac{\lambda}{r} \right]$$

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A Disk of Charge

The on-axis electric field of a charged disk of radius $R$, centered on the origin with axis parallel to $z$, and surface charge density $\eta = Q/\pi R^2$ is

$$ (E_{\text{disk}})_z = \frac{\eta}{2\varepsilon_0} \left[ 1 - \frac{z}{\sqrt{z^2 + R^2}} \right] $$

NOTE: The field for $z < 0$ has the same magnitude but points in the opposite direction.

$$ \lim_{z<<R} E_{\text{disk}} = \frac{\eta}{2\varepsilon_0} $$
A Plane of Charge

The electric field of an infinite plane with surface charge density $\eta$ is:

$$E_{\text{plane}} = \frac{\eta}{2\varepsilon_0} = \text{constant}$$

For a positively/negatively charged plane, the electric field points away from /towards the plane on both sides of the plane.
A Sphere of Charge

A sphere of charge $Q$ and radius $R$, be it a uniformly charged sphere or just a spherical shell, has an electric field *outside* the sphere that is exactly the same as that of a point charge $Q$ located at the *center* of the sphere:

$$\vec{E}_{\text{sphere}} = \frac{Q}{4\pi \varepsilon_0 r^2} \hat{r} \quad \text{for } r \geq R$$

But inside a spherical shell of charge, the field vanishes.
The Parallel-Plate Capacitor

• The figure shows two electrodes, one with charge \( +Q \) and the other with \( -Q \) placed face-to-face a distance \( d \) apart.
• This arrangement of two electrodes, charged equally but oppositely, is called a **parallel-plate capacitor**.
• Capacitors play important roles in many electric circuits.
The electric field inside a capacitor is

\[ \vec{E}_{\text{capacitor}} = \vec{E}_+ + \vec{E}_- = \left( \frac{\eta}{\varepsilon_0}, \text{from positive to negative} \right) \]

\[ = \left( \frac{Q}{\varepsilon_0 A}, \text{from positive to negative} \right) \]

where \( A \) is the surface area of each electrode. Outside the capacitor plates, where \( E_+ \) and \( E_- \) have equal magnitudes but opposite directions, the electric field is zero.
EXAMPLE 27.7 The electric field inside a capacitor

QUESTIONS:

EXAMPLE 27.7 The electric field inside a capacitor
Two 1.0 cm × 2.0 cm rectangular electrodes are 1.0 mm apart. What charge must be placed on each electrode to create a uniform electric field of strength 2.0 × 10^6 N/C? How many electrons must be moved from one electrode to the other to accomplish this?
EXAMPLE 27.7 The electric field inside a capacitor

**SOLVE** The electric field strength inside the capacitor is \( E = \frac{Q}{\varepsilon_0 A} \). Thus the charge to produce a field of strength \( E \) is

\[
Q = (8.85 \times 10^{-12} \text{ C}^2/\text{Nm}^2)(2.0 \times 10^{-4} \text{ m}^2)(2.0 \times 10^6 \text{ N/C})
\]

\[
= 3.5 \times 10^{-9} \text{ C} = 3.5 \text{ nC}
\]

The positive plate must be charged to +3.5 nC and the negative plate to −3.5 nC. In practice, the plates are charged by using a battery to move electrons from one plate to the other. The number of electrons in 3.5 nC is

\[
N = \frac{Q}{e} = \frac{3.5 \times 10^{-9} \text{ C}}{1.60 \times 10^{-19} \text{ C/electron}} = 2.2 \times 10^{10} \text{ electrons}
\]

Thus \( 2.2 \times 10^{10} \) electrons are moved from one electrode to the other. Note that the capacitor *as a whole* has no net charge.
Do work to charge the capacitor thereby storing energy. Discharge capacitor through light bulb filament converting stored energy to heating the filament and to radiant energy (light, heat).
Summary

The electric force field of a collection of charges may be calculated using superposition and Coulomb’s Law.

The electric field is cleverly visualized with field lines.

Electronics concerns the deployment and redeployment of a vast number of electrons on conducting surfaces as in a capacitor serving as information carriers and sources of force field.
A piece of plastic is uniformly charged with surface charge density $\eta_1$. The plastic is then broken into a large piece with surface charge density $\eta_2$ and a small piece with surface charge density $\eta_3$. Rank in order, from largest to smallest, the surface charge densities $\eta_1$ to $\eta_3$.

A. $\eta_2 = \eta_3 > \eta_1$
B. $\eta_1 > \eta_2 > \eta_3$
C. $\eta_1 > \eta_2 = \eta_3$
D. $\eta_3 > \eta_2 > \eta_1$
E. $\eta_1 = \eta_2 = \eta_3$
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A. $\eta_2 = \eta_3 > \eta_1$
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C. $\eta_1 > \eta_2 = \eta_3$
D. $\eta_3 > \eta_2 > \eta_1$
E. $\eta_1 = \eta_2 = \eta_3$
Which of the following actions will increase the electric field strength at the position of the dot?

A. Make the rod longer without changing the charge.
B. Make the rod fatter without changing the charge.
C. Make the rod shorter without changing the charge.
D. Remove charge from the rod.
E. Make the rod narrower without changing the charge.
Which of the following actions will increase the electric field strength at the position of the dot?

A. Make the rod longer without changing the charge.
B. Make the rod fatter without changing the charge.
C. Make the rod shorter without changing the charge. **Correct Answer**
D. Remove charge from the rod.
E. Make the rod narrower without changing the charge.
Rank in order, from largest to smallest, the forces \(F_a\) to \(F_e\) a proton would experience if placed at points a – e in this parallel-plate capacitor.

A. \(F_a = F_b = F_d = F_e > F_c\)
B. \(F_a = F_b > F_c > F_d = F_e\)
C. \(F_a = F_b = F_c = F_d = F_e\)
D. \(F_e = F_d > F_c > F_a = F_b\)
E. \(F_e > F_d > F_c > F_b > F_a\)
Rank in order, from largest to smallest, the forces $F_a$ to $F_e$ a proton would experience if placed at points a – e in this parallel-plate capacitor.

A. $F_a = F_b = F_d = F_e > F_c$
B. $F_a = F_b > F_c > F_d = F_e$
C. $F_a = F_b = F_c = F_d = F_e$
D. $F_e = F_d > F_c > F_a = F_b$
E. $F_e > F_d > F_c > F_b > F_a$

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