Group Problems Week 12 Apr 9

Problem 1:
Two particles, one with charge $q_1$ and mass $m_1$ and the other with charge $4q_1$ and mass $2m_1$, are accelerated from rest through a potential difference $\Delta V$ and fired at an angle $\theta$ with respect to the horizontal (the x-axis) in the xy plane into a region with a uniform magnetic field $\vec{B} = B_0 \hat{z}$, resulting in helical orbits.

(a) What is the ratio of the periods of their orbits?

\[
\frac{T_1}{T_2} = \frac{\frac{2\pi m_1}{q_1 B}}{\frac{2\pi m_2}{q_2 B}} = \left( \frac{m_1}{m_2} \right) \left( \frac{q_2}{q_1} \right) = \frac{4}{2} = 2
\]

(b) What is the ratio of the radii of the orbits? Which particle has a larger radius?

First, accelerate: \[ \frac{1}{2} m v^2 = q_1 \Delta V \] \[ v = \sqrt{\frac{2q_1 \Delta V}{m}} \]

\[ r_1 = \frac{m_1 q_1}{q_1 B} = \sqrt{\frac{m_1 q_1}{m_1 q_1}} = \sqrt{2} \]

\[ r_2 = \frac{m_2 q_2}{q_2 B} = \frac{m_1 q_2}{m_2 q_1} = \frac{4}{2} = 2 \]

This larger radius

(c) What is the ratio of the pitches (distance between spirals) of the helical orbits?

\[ d \text{ pitch} = v_B T \]

\[ v_B = v \cos \theta = \sqrt{\frac{2q_1 \Delta V}{m}} \cos \theta \]

\[ d = \sqrt{\frac{2q_1 \Delta V}{m} \cos \theta} \]

\[ d_1 / d_2 = \sqrt{\frac{m_1 q_1}{m_2 q_2}} = \sqrt{2} \]

(d) If the initial potential difference used to accelerate the particles is $\Delta V = 1$ kV and the angle at which they are fired is $\theta = 30^\circ$, what is the magnitude of the magnetic field needed to spiral the particle of mass $m_1$ in a helical orbit of radius 4.0 cm? Here $m_1$ is the proton mass.

\[ v = \sqrt{\frac{2q_1 \Delta V}{m}} \]

\[ B = \frac{m v_1}{q r} = \frac{m}{q r} \sqrt{\frac{2q_1 \Delta V}{m} \sin \theta} \]

\[ \therefore B = \frac{\sqrt{2 \Delta V \sin \theta}}{r} \frac{m}{\sqrt{q}} \]

\[ = \sqrt{2 \cdot 1 \times 10^{-3} V} \cdot \frac{1}{2} \cdot \frac{1.67 \times 10^{-27} kg}{4 \times 10^{-2} m} \cdot \frac{1.6 \times 10^{-19} C}{1.6 \times 10^{-19} C} = 0.06 T \]
Problem 2
A square current loop of side \( d = 0.1 \text{ m} \) that carries a current \( I = 2 \) \( \text{A} \) is surrounded by a region of uniform magnetic field \( B = 2.4 \) \( \text{T} \) pointing in the positive \( y \) direction, as shown.

(a) What is the magnetic dipole moment (magnitude and direction) of the loop?

\[
\vec{\mu} = I \hat{A} = (2 \text{A})(0.1 \text{ m})^2 \hat{\varepsilon} = 0.2 \text{ A} \cdot \text{m}^2 \hat{\varepsilon}
\]

(b) What is the magnetic force (magnitude and direction) on each side of the loop? What is the net force on the loop?

\[
\vec{F} = I d B (\hat{\varepsilon}) = -(2 \text{A})(0.1 \text{m})(2.4 \text{T}) \hat{\varepsilon} = -0.48 \text{ N} \hat{\varepsilon}
\]

\[
\vec{F} = I d B (\hat{\varepsilon}) = 0.48 \text{ N} \hat{\varepsilon}
\]

Net \( \vec{F} = 0 \)

(c) What is the torque acting on the loop? In which direction will the loop rotate?

\[
\vec{\tau} = \vec{\mu} \times \vec{B}
\]

\[
= (0.2 \text{ A} \cdot \text{m}^2) \hat{\varepsilon} \times 2.4 \text{T} \hat{y}
\]

\[
= -\hat{\varepsilon} \times (0.048 \text{ N} \cdot \text{m})
\]

Loop will rotate such that \( \vec{\mu} \) is aligned with \( \vec{B} \)

Direction of rotation