Group Problems Week 13 Apr 16

Problem 1:
(a) Determine the magnetic field (magnitude and direction) at point P in terms of \( I \) and \( d \) due to the current loop shown below.

\[
\vec{B}_1 = \frac{\mu_0 I}{4\pi} \int \frac{dy \, \hat{z}}{d + y} \\
\vec{B}_2 = -2 \frac{\mu_0 I}{4\pi} \int dx \frac{\hat{x}}{(d + x)^2} \\
\vec{B}_3 = -2 \frac{\mu_0 I}{4\pi} \int dy \frac{\hat{y}}{(d + y)^2} \\
\vec{B}_{\text{total}} = \vec{B}_1 + \vec{B}_2 + \vec{B}_3
\]

(b) What is the torque (magnitude and direction) on a small current loop of current \( I' \) and radius \( a \) (\( a \ll d \)) at point P? The plane of the loop is parallel to the yz plane and the current is in the clockwise direction when looking down at the loop from a large distance on the positive x axis. In what direction will the loop rotate?

\[
\vec{\tau} = \vec{\mu} \times \vec{B} = \frac{\mu_0 I' a^2}{2\pi} (\sqrt{2} - 1) \hat{z}
\]

Loop will rotate such that \( \vec{\mu} \parallel \vec{B} \), ccw around \( -\hat{y} \) direction (i.e., in xy plane)
Problem 2:
A long cylindrical conductor of radius $R$ and total current $I$ has current density $J(r) = ar^2$. Compute the magnetic field (magnitude and direction) everywhere, and sketch it as a function of radius. (Take the cylinder length to be infinite.)

Magnitude

$$I = \int F \cdot d\vec{A} = a \int_0^R r^2 r dr 2\pi = 2\pi a R^4$$

$r<R$:

$$\oint \vec{B} \cdot d\vec{L} = \mu_0 I \text{enclosed}$$

$$B = \mu_0 a r^2 r dr 2\pi = \frac{\mu_0 a r^2 2\pi}{4}$$

$$B = \frac{\mu_0 a r^2}{4} = \frac{\mu_0 I r}{2\pi R^3}$$

$r>R$:

$$\oint \vec{B} \cdot d\vec{L} = \mu_0 I \text{enclosed} = \mu_0 I$$

$$B = \frac{\mu_0 I r}{2\pi R}$$

\[
\begin{align*}
B & \quad \text{vs} \quad r \\
\text{Note:} & \quad \vec{B} \text{ continuous at } r=R, \text{ as it should be.}
\end{align*}
\]