Group Problems Week 3 Feb 5

Problem 1
(a) Compute the kinetic energy of a proton whose de Broglie wavelength is 0.25 nm.
(b) If a beam of such protons is reflected from a calcite crystal with crystal plane spacing of 0.304 nm at what angle will the first-order Bragg maximum occur?
(c) What is the wavelength of electrons with velocity \( v = 0.998 \) c.

Remember that: \( m_e = 9.11 \times 10^{-31} \) kg = 0.511 MeV, \( m_p = 1.67 \times 10^{-27} = 0.938 \times 10^9 \) GeV; \( hc = 1240 \) nm eV and \( h = 6.626 \times 10^{-34} \) J s; \( c = 3 \times 10^8 \) m/s.

(a)
\[
\lambda = \frac{\hbar}{p} = \frac{\hbar}{\sqrt{2m_pE_k}} = 0.25 \text{nm}
\]

Squaring and rearranging,
\[
E_k = \frac{\hbar^2}{2m_p\lambda^2} = \frac{hc^2}{2(m_pc^2)\lambda^2} = \frac{(1240 eV \cdot nm)^2}{2(938 \times 10^6 eV)(0.25 nm)^2}
\]
\[
E_k = 0.013 \text{eV}
\]

(b) \( n\lambda = D\sin\phi \rightarrow \sin\phi = n\lambda/D = (1)(0.25 \text{nm})/0.304 \text{nm} \)
\[
\sin\phi = 0.822 \rightarrow \phi = 55^\circ
\]
v = 0.998c electrons are relativistic.

\[
\lambda = \frac{h}{p} = \frac{h}{\gamma mv} = \frac{h}{mv} \sqrt{1 - \frac{v^2}{c^2}}
\]

\[
\lambda = 1.53 \times 10^{-13} \text{ m}.
\]

**Problem 2**

In order to locate a particle, e.g. an electron, to within \(5 \times 10^{-12}\) m using electromagnetic waves ("light"), the wavelength must be at least this small. Calculate the momentum and energy of a photon with \(\lambda = 5 \times 10^{-12}\) m. If the particle is an electron with \(\lambda = 5 \times 10^{-12}\) m what is the corresponding uncertainty in its momentum?
Problem 3

A particle moving in 1 dimension between rigid walls separated by a distance L has the wave function \( \psi(x) = A \sin \left( \frac{\pi x}{L} \right) \). Since the particle must remain between the walls, what must be the value of A?

Because the particle must be in the box:
\[
\int_{0}^{L} \psi^* \psi \, dx = A^2 \int_{0}^{L} \sin^2 \left( \frac{\pi x}{L} \right) \, dx = 1
\]

Let \( u = \frac{\pi x}{L} \); when \( x = 0 \Rightarrow u = 0 \) and when \( x = L \Rightarrow u = \pi \) and \( dx = \frac{L}{\pi} \, du \), so we have:
\[
\int_{0}^{\pi} A^2 \left( \frac{L}{\pi} \right) \sin^2 u \, du = A^2 \frac{L}{\pi} \left[ \frac{u}{2} - \frac{\sin 2u}{4} \right]_0^{\pi} = A^2 \frac{L}{\pi} \left( \frac{\pi}{2} - 0 \right) = 1 \Rightarrow A = \sqrt{\frac{2}{L}}
\]