Maxwell's Equations:

\[
\frac{\partial \mathbf{E}}{\partial t} = \nabla \times \mathbf{H} = \sigma \mathbf{J} + \nabla \phi
\]

\[
\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \mathbf{E} = -\frac{\partial \mathbf{D}}{\partial t} - \mu_0 \mathbf{J}
\]

Differential form (using vector calc. results for flux & circulation):

\[
\oint \mathbf{E} \cdot d\mathbf{A} = \frac{\partial}{\partial t} \int \mathbf{D} \cdot d\mathbf{V}
\]

\[
\oint \mathbf{B} \cdot d\mathbf{A} = \mu_0 I_{\text{enclosed}} + \mu_0 \epsilon_0 \oint \mathbf{E} \cdot d\mathbf{A}
\]

Wave solutions:

\[
\begin{align*}
\mathbf{E} & = E_x (x, t) \mathbf{\hat{x}} \\
\mathbf{B} & = B_y (y, t) \mathbf{\hat{y}}
\end{align*}
\]

From Maxwell: time-dependent \( \mathbf{E} \) fields source \( \mathbf{B} \) fields.

What about \( \nabla \cdot \mathbf{E} + \partial \mathbf{B} / \partial t \)? → insist that \( E_z + B_z \) are both 0

(no component in direction of propagation)
Our 1-d case:
\[
\frac{\partial^2 E_x}{\partial t^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E_x}{\partial t^2}, \quad \frac{\partial^2 B_y}{\partial t^2} = \mu_0 \varepsilon_0 \frac{\partial^2 B_y}{\partial t^2}
\]

Wave equation with speed \( c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \).

This is in vacuum. Note: no source needed for continued propagation of wave!

How about in a material? \( \varepsilon_0 \to \varepsilon = K \varepsilon_0 \) in dielectric

\( \mu_0 \to \mu = K \mu_0 \) in magnetic material

(We didn't talk about magnetic materials, and from now on let's restrict only to the case with \( K = 1 \)).

Then \( n = \frac{c_0}{c} \) is the new speed of the wave.

\( n = \frac{c}{\sqrt{K}} \)

\( n = \frac{c}{\sqrt{K}} \to n = \text{index of refraction} \)

\( K \) is material dependent (not same value as for static fields, in general much smaller)

Back to wave eqn. Solution here

\[
E_x = E_0 \cos (kz - wt + \phi) \quad \text{and} \quad E_0 = c B_0
\]

\( \vec{E} \) in phase!

Nothing sacred about axes, just that \( \vec{E} \cdot \vec{B} = 0 \) and \( \vec{E} \times \vec{B} = \vec{0} \)

direction of \( \vec{E} \) = polarization (here linear) (Next week's lab)
EM Waves carry energy + momentum —

Energy → associated with the fields themselves.

Energy density: \( U_E = \frac{1}{2} \varepsilon_0 E^2 \)

Magnetic energy density: \( U_M = \frac{1}{2 \mu_0} B^2 \)

Since \( E_0 = cB_0 \), we see that energy is shared equally \( \frac{1}{2} \mu_0 E^2 + \frac{1}{2 \varepsilon_0} B^2 \):

\[
U_M = \frac{B^2}{2 \mu_0} = \frac{E^2}{2 \varepsilon_0} = \frac{1}{2} \varepsilon_0 E^2 = U_E
\]

Total energy density is the sum:

\[
U = U_E + U_M = \frac{1}{2} \varepsilon_0 E^2 + \frac{1}{2 \mu_0} B^2 = \frac{EB}{\mu_0} \frac{1}{\varepsilon_0 c}
\]

Transport — consider energy transport:

\[
\text{(Energy transfer/time/area)} = \text{Power/area}
\]

Consider EM \( U \) contained in specific volume element at time \( dt \) in area \( A \):

\[
dU = u (A \text{cdt})\]

Rate of energy transport: \( dU/dt = cuA \)

\[
\text{Rate of energy transport/area} = \text{Power/area} = cu
\]

Call the quantity \( \frac{1}{A} \frac{dU}{dt} = cu = \frac{c \varepsilon_0 E^2 + cB^2}{\mu_0} \)

\[
\text{units:} \quad \frac{W}{m^2}
\]
Define vector quantity \( \vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B} \)

Powering Vector - magnitude is energy flow per unit area per unit time

direction = direction of wave propagation.

\[
Power = \oint \vec{S} \cdot d\vec{A}
\]

Sinusoidal waves: useful to consider averaged values.

\[
|\vec{S}|_{\text{avg}} = \text{Intensity} = I
\]

\[
I = |\vec{S}|_{\text{avg}} = \frac{1}{2} \frac{E_0 B_0}{\mu_0} - \frac{1}{2} \frac{E_0^2}{\mu_0} = \frac{c B_0^2}{2 \mu_0}
\]

For such waves, also useful to consider the \textit{rms} values:

\[
\text{rms} = \text{root-mean-square}
\]

\[
E_{\text{rms}} = E_0 / \sqrt{2}
\]

\[
B_{\text{rms}} = B_0 / \sqrt{2}
\]

\[
\text{then } I = \frac{E_{\text{rms}} B_{\text{rms}}}{\mu_0}
\]

EM waves also carry momentum, with momentum density given by

\[
\frac{d\vec{p}}{dV} = \frac{\vec{E} \cdot \vec{B}}{\mu_0 c^2} = \frac{1}{c^2} \frac{\vec{S}}{c} \quad \text{by momentum flow in direction of } \vec{S}.
\]

Momentum flow rate per unit area: \(dV = A \, dt\)

\[
\frac{d\vec{p}}{A} \, dt = \frac{|\vec{S}|}{c}
\]

Average rate of momentum transfer per unit area:

\[
\frac{1}{A} \frac{d\vec{p}}{dt} = \frac{I}{c} = \frac{1}{c} \frac{S_{\text{avg}}}{c} \quad \Rightarrow \text{this is a force/area}
\]

\[
\text{"radiation pressure"}
\]
Radiation pressure - describes force/area on surface due to momentum transfer from EM wave

Wave completely absorbed: \( P_r = \frac{1}{2} \beta n c = \frac{T}{c} \) (momentum transfer \( 2\beta p \))

Wave totally reflected: twice that amount (momentum transfer \( 2\beta p \))

\[ P_r = \frac{1}{2} \beta n c = \frac{2T}{c} \]

More generally: at finite percent reflection, fraction \( f \):

\[ P_r = \frac{1}{2} \beta n c (1 + f) = \frac{T}{c} (1 + f) \]

Typically very small, but measurable
Sources of EM radiation: accelerating charges.

Prototype: dipole antenna (half-wave antenna)

At particular time:

2 metal rods with sinusoidal voltage source (oscillating end)

\[ E \times B \text{ fields for away from dipole: } 1/r \text{ dependence} \]

(not static fields!)

\[ \text{Intensity } \sim \frac{\sin^2 \theta}{r^2} \]

\[ \text{not } \sim r^{-1} \]

Intensity pattern

Detect radiation: can take water dipole antenna

in the direction of the \( E \) field

\[ \dot{E} \parallel A = E \]

or, can take a loop \( \perp \) to \( B \) field direction, \( r \)

Look for induced emf a la Faraday:

\[ \oint \vec{B} \cdot d\vec{A} = \frac{d\Phi_B}{dt} \]

End here with a comment →

accelerating charge radiate, radiation carries away energy.

This brings us back to Bohr + stability of classical orbitals

spiral + recall → led to quantization hypothesis

immediately: at heart of quantum mechanics!
Polarization - orientation of electric field vector in EM wave.

Polarized $\rightarrow$ $\vec{E}$ field in specific direction (one example: linear polarization: $\vec{E} = E_x \hat{x}$)

Unpolarized $\rightarrow$ mixture of waves with different $\vec{E}$ directions such that no direction of $\vec{E}$ is preferred.

Can polarize such a beam with a grid/polaroid.

\[ \vec{E} \rightarrow \text{charges inside conducting wires of grid can respond, absorb energy} \]
\[ \text{wave transmitted, reduced in magnitude} \]
\[ \text{wave direct} \rightarrow \text{grid transparent if oriented \perp to} \ \vec{E} \rightarrow \text{charges can't respond} \]
\[ \vec{E} \rightarrow \text{can't respond} \]
\[ \rightarrow \text{let wave transmit, no reduction of amplitude} \]

More quantities: $I = Io \cos^2 \theta$  $\theta =$ angle of polarizer + analyzer transmission axis.

Mieus' law

Can also polarize by reflection: we know $\theta_i = \theta_r$  $\tan \theta_p = n$

$\vec{E}$ field $\rightarrow$ electrons at boundary oscillate, radiate wave parallel to surface.