Today’s Topics

### Magnetic Forces
- Review: magnetic force
- Motion of charge in uniform B field:
  - Applications: cyclotron, velocity selector, Hall effect

### Magnetic Fields and Forces: Recap

#### Magnetic Force:
- Experienced by moving charges

\[
\vec{F} = q\vec{v} \times \vec{B}
\]

(point charges)

\[\vec{F} = \int I dl \times \vec{B}\]

(currents)

#### Magnetic Field B:
- Sourced by moving charges
- Direction: as indicated by north pole of compass

Units: 1 Tesla (T) = 1 N/(A m)

Field lines: closed loops!
- Outside magnet: N to S
- Inside magnet: S to N

### Trajectory in Constant B Field

- Suppose charge \( q \) enters a uniform B-field with velocity \( v \). What will be the path that \( q \) follows?

\[
F = qvB
\]

Force perpendicular to velocity: uniform circular motion

Note: magnetic force does no work on the charge!

Kinetic energy constant

### Trajectory in Uniform B Field

- Force:

\[
F = qvB
\]

- Centripetal acc:

\[
a = \frac{v^2}{R}
\]

- Newton’s 2nd Law:

\[
F = ma \implies qvB = m\frac{v^2}{R}
\]

\[
R = \frac{mv}{qB}
\]

(important result, with useful experimental consequences!)

“Cyclotron” frequency:

\[
\omega = \frac{v}{R} = \frac{qB}{m} \quad T = \frac{2\pi}{\omega} = \frac{2\pi m}{qB}
\]
**Application: Cyclotron**

First Modern Particle Accelerator

First Cyclotron (1934) Lawrence & Livingston

Text example: 27.66

**Trajectory in Uniform B Field: 3D case**

General 3D case:

- In the plane perpendicular to B:
  \[ R = \frac{mv}{qB}, \quad T = \frac{2\pi m}{qB} \]

- Parallel to B: spacing b/w turns of helix
  \[ d = v_T = v_B = \frac{\nu B}{qB} \]

**Application: Velocity, Mass Selectors**

Velocity and mass selector:

- Speed selected:
  \[ v = \frac{E}{B} \]

- Mass selected:
  \[ \frac{m}{q} = \frac{rB_0}{v} = \frac{rB_0}{(E/B)} \]

**Magnetic Force On A Current Carrying Wire**

Top View
Forces on a Current Loop

For current loops in a uniform magnetic field as shown, what is the direction of the force on each side?

Case 1

recall: $\sum F_B = 0$

Case 2

Torque on a Current Loop

The Galvanometer

Potential difference on current-carrying conductor in B field:

The Hall Effect

positive charges moving counterclockwise: upward force, upper plate at higher potential

negative charges moving clockwise: upward force, upper plate at lower potential

Equilibrium between electrostatic & magnetic forces:

$F_{up} = qvI_B$

$F_{down} = qE_{md} = q\frac{V_H}{W}$

$V_H = v_eB_w = "Hall\ Voltage"

(first evidence that electrons are charge carriers in most metals)