Today’s Topics

- Inductance (Ch 28)
  - Self Inductance
  - Mutual Inductance
- Energy Stored in B Field
- RL Circuits

Self Inductance

- When the current in a conducting device changes, an induced emf is produced in the opposite direction of the source current  ⇒ self inductance

- The magnetic flux due to self inductance is proportional to I:
\[ \Phi_B = LI \]

- The induced emf is proportional to \( \frac{dI}{dt} \):
\[ \epsilon_L = -L \frac{dI}{dt} \]

\[ \mathbf{L}: \text{Inductance, unit: Henry (H)} \]

Exercise: Calculate Inductance of a Solenoid

- show that for an ideal solenoid:

\[ L = \frac{\mu_0 N^2 A}{l} \]

(see board)

Reminder: magnetic field inside the solenoid

\[ B = \mu_0 \frac{N}{l} I \]

Examples of Coupled Coils (Transformers)

\[ \frac{\epsilon_S}{\epsilon_P} = \frac{N_S}{N_P} \]
Energy Stored in a Magnetic Field

- When an inductor of inductance \( L \) is carrying a current changing at a rate \( \frac{dI}{dt} \), the power supplied is

\[
P = I\epsilon = LI\frac{dI}{dt}
\]

- The work needed to increase the current in an inductor from zero to some value \( I \)

\[
U = \int_0^t P \, dt = \int_0^I LI \, dI = \frac{1}{2} LI^2
\]

Energy in an Inductor

- Energy stored in an inductor is \( U = \frac{1}{2} LI^2 \)

- This energy is stored in the form of magnetic field:
  - energy density: \( u_B = \frac{1}{2} B^2/\mu_0 \) (recall: \( u_E = \frac{1}{2} \varepsilon_0 E^2 \))

Compare:
- Inductor: energy stored \( U = \frac{1}{2} LI^2 \)
- Capacitor: energy stored \( U = \frac{1}{2} C(\Delta V)^2 \)
- Resistor: no energy stored, (all energy converted to heat)

Basic Circuit Components

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<th>Component</th>
<th>Symbol</th>
<th>Behavior in circuit</th>
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<tr>
<td>Ideal battery, emf</td>
<td>( \text{--} )</td>
<td>( \Delta V = V_+ - V_- = \varepsilon )</td>
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<tr>
<td>Resistor</td>
<td>( \text{--} )</td>
<td>( \Delta V = -IR )</td>
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<tr>
<td>Realistic Battery</td>
<td>( \text{--} )</td>
<td>( \Delta V \rightarrow \varepsilon )</td>
</tr>
<tr>
<td>(Ideal) wire</td>
<td>( \text{--} )</td>
<td>( \Delta V = 0 ) (( R = 0, L = 0, C = 0 ))</td>
</tr>
<tr>
<td>Capacitor</td>
<td>( \text{--} )</td>
<td>( \Delta V = V_+ - V_- = -q/C, dq/dt = I )</td>
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<tr>
<td>Inductor</td>
<td>( \text{--} )</td>
<td>( \Delta V = -LdI/dt )</td>
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<tr>
<td>(Ideal) Switch</td>
<td>( \text{--} )</td>
<td>( L = 0, C = 0, R = 0 ) (on), ( R = \infty ) (off)</td>
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<td>Transformer</td>
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RL Circuit

- An inductor and are resistance constitute a RL circuit
  - Any inductor has a resistance, \( R \)
  - \( R \) could also include any other additional resistance
- When the current starts to flow a voltage drop will occur a the resistance and the inductance
- Once current stabilizes, reaches maximum of \( I = \frac{V_0}{R} \)
**Exercise: Turn on RL Circuit**

Apply Kirchhoff loop rule

\[ V_0 - IR - L \frac{dI}{dt} = 0 \]

\[ V_0 \, dt - IR \, dt - L \, dI = 0 \]

\[ \frac{dI}{V_0 - IR} = \frac{dt}{L} \]

\[ \int_{0}^{t} \frac{dI}{V_0 - IR} = \int_{0}^{t} \frac{dt}{L} \]

\[- \frac{1}{R} \ln \left( \frac{V_0 - IR}{V_0} \right) = \frac{t}{L} \]

\[ I = \frac{V_0}{R} \left( 1 - e^{-\frac{t}{L/R}} \right) \]

**Exercise: Turn on RL Circuit (cont)**

Note: the time constant is \( \tau = \frac{L}{R} \)

Quiz: What is the current when \( t = \infty \)?

**Exercise: Turn off RL Circuit**

Apply Kirchhoff loop rule

\[ IR + L \frac{dI}{dt} = 0 \]

\[ \frac{dI}{I} = - \frac{R}{L} \, dt \]

\[ \int_{I_0}^{t} \frac{dI}{I} = - \int_{0}^{t} \frac{R}{L} \, dt \]

\[ \ln \frac{I}{I_0} = - \frac{R}{L} \, t \]

\[ I = I_0 e^{-\frac{t}{L/R}} \]

**Exercise: Turn off RL Circuit (cont)**

Note: the time constant is \( \tau = \frac{L}{R} \)

Quiz: What is the current when \( t = \infty \)?