Physics 202, Lecture 21

Today’s Topics

- More Wave Review
  - Standing waves

- Electromagnetic Waves (EM Waves)
  - The Hertz Experiment
  - Review of the Laws of Electromagnetism
  - Maxwell’s equations
  - Propagation of $\mathbf{E}$ and $\mathbf{B}$
  - The Linear Wave Equation
Parameters For A Sinusoidal Wave

- Snapshot with fixed t: wavevector = k
  wavelength $\lambda=\frac{2\pi}{k}$
- Snapshot with fixed x:
  angular frequency = $\omega$
  frequency $f=\frac{\omega}{2\pi}$
  Period $T=\frac{1}{f}$
  Amplitude = A
- Wave Speed $v=\frac{\omega}{k}$
  $\rightarrow v=\lambda f$, or
  $\rightarrow v=\lambda/T$
- Phase angle difference between two positions
  $\Delta \phi = -k \Delta x$
Standing Waves

- When two waves of the same amplitude, same frequency but opposite direction meet standing waves occur.

\[ y_1 = A \sin(kx - \omega t) \quad y_2 = A \sin(kx + \omega t) \]

\[ y = y_1 + y_2 = 2A \sin(kx) \cos(\omega t) \]

- Points of destructive interference (nodes)

\[ kx = n\pi; \quad x = \frac{n\pi}{k} = \frac{n\lambda}{2}; \quad n = 1,2,3... \]

- Points of constructive interference (antinodes)

\[ kx = (2n - 1)\frac{\pi}{2}; \quad x = (2n - 1)\frac{\lambda}{4}; \quad n = 1,2,3... \]
Nodes and antinodes will occur at the same positions, giving impression that wave is standing.
Standing Waves (cont)

Standing waves with a string of given length $L$ are produced by waves of natural frequencies or resonant frequencies:

$$\lambda = \frac{2L}{n}; \quad n = 1,2,3...$$

$$f = \frac{\nu}{\lambda} = \frac{nv}{2L} = \sqrt{\frac{T}{\mu}} \frac{n}{2L}$$
Demo: Hertz Experiment

In 1887, Heinrich Hertz first demonstrated that EM fields can transmit over space.
Review: Gauss’s Law / Coulomb’s Law

The relation between the electric flux through a closed surface and the net charge $q$ enclosed within that surface is given by the Gauss’s Law

$$\oint \vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_0}$$
Gauss’s Law for Magnetism

- The Gauss’s Law for the electric flux is a reflection of the existence of electric charge. In nature we have not found the equivalent, a magnetic charge, or monopole.
- We can express this result differently: if any closed surface as many lines enter the enclosed volume as they leave it.

\[ \oint \mathbf{B} \cdot d\mathbf{A} = 0 \]
Review: Faraday’s Law

- The emf induced in a “circuit” is proportional to the time rate of change of magnetic flux through the “circuit” or closed path.

\[ \mathcal{E} = - \frac{d\Phi_B}{dt} \]

- Since \[ \mathcal{E} = \int \vec{E} \cdot d\vec{l} \]

- Then \[ \int \vec{E} \cdot d\vec{l} = - \frac{d\Phi_B}{dt} \]

\[ \Phi_B = \int \vec{B} \cdot d\vec{A} \]
Review: Ampere’s Law

- A magnetic field is produced by an electric current is given by the Ampere’s Law

\[ \oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I \]

But Ampere’s Law can be ambiguous: currents enclosed by surfaces 1 and 2 are different!

But note time-dependent electric flux though surface 2.
Maxwell’s modification of Ampere’s Law

- A time-dependent electric flux induces a magnetic field (in analogy to Faraday’s law)

\[ \oint \vec{B} \cdot d\vec{l} = \mu_0 \left( I + \varepsilon_0 \frac{d\Phi_E}{dt} \right) \]

\[ \Phi_E = \int \vec{E} \cdot d\vec{A} \]

Ampere’s law

“Maxwell’s displacement current”
Maxwell Equations

<table>
<thead>
<tr>
<th>Equation</th>
<th>Interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\int \vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_0}$</td>
<td>Gauss’s Law/ Coulomb’s Law</td>
</tr>
<tr>
<td>$\int \vec{B} \cdot d\vec{A} = 0$</td>
<td>Gauss’s Law of Magnetism, no magnetic charge</td>
</tr>
<tr>
<td>$\int \vec{E} \cdot d\vec{l} = - \frac{d\Phi_B}{dt}$</td>
<td>Faraday’s Law</td>
</tr>
<tr>
<td>$\int \vec{B} \cdot d\vec{l} = \mu_0 I + \varepsilon_0 \mu_0 \frac{d\Phi_E}{dt}$</td>
<td>Ampere Maxwell Law</td>
</tr>
</tbody>
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Also, Lorentz force Law $\rightarrow$ $\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$

These are the foundations of the **electromagnetism**
EM Fields in Space

- Maxwell equations when there is no charge and current:

\[ \oint \mathbf{E} \cdot d\mathbf{A} = 0 \]
\[ \oint \mathbf{B} \cdot d\mathbf{A} = 0 \]
\[ \oint \mathbf{E} \cdot d\mathbf{l} = -\frac{d\Phi_B}{dt} \]
\[ \oint \mathbf{B} \cdot d\mathbf{l} = \varepsilon_0 \mu_0 \frac{d\Phi_E}{dt} \]

Differential forms: (single polarization)

\[ \frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t} \]
\[ \frac{\partial B_z}{\partial x} = -\mu_0 \varepsilon_0 \frac{\partial E_y}{\partial t} \]

\[ \frac{\partial^2 E_y}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E_y}{\partial t^2} \]
\[ \frac{\partial^2 B_z}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 B_z}{\partial t^2} \]
Linear Wave Equation

- **Linear wave equation**
  \[ \frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2} \]
  - Certain physical quantity
  - Wave speed

- **Sinusoidal wave**
  \[ y = A \sin \left( \frac{2\pi}{\lambda} x - 2\pi ft + \phi \right) \]
  - \( f \): frequency
  - \( \phi \): Phase
  - \( A \): Amplitude
  - \( \lambda \): Wavelength
  - \( v = \lambda f \)
  - \( k = \frac{2\pi}{\lambda} \)
  - \( \omega = 2\pi f \)

General wave: superposition of sinusoidal waves
Electromagnetic Waves

- **EM wave equations:**
  \[ \frac{\partial^2 E_y}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E_y}{\partial t^2} \quad \frac{\partial^2 B_z}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 B_z}{\partial t^2} \]

- **Plane wave solutions:**
  \[ E = E_{\text{max}} \cos(kx - \omega t + \phi) \quad B = B_{\text{max}} \cos(kx - \omega t + \phi) \]

- **Properties:**
  - No medium is necessary.
  - E and B are normal to each other
  - E and B are in phase
  - Direction of wave is normal to both E and B (EM waves are transverse waves)
  - Speed of EM wave:
    \[ c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} = 2.9972 \times 10^8 \text{ m/s} \]
  - \( E/B = E_{\text{max}}/B_{\text{max}} = c \)
  - Transverse wave: two polarizations possible
The EM Wave

Two polarizations possible (showing one)