Physics 208, Lecture 22

Today's Topics

- Electromagnetic Waves (EM Waves)
- Review: Waves and Wave Equation
- Maxwell's Equations
- Propagation of E and B
- Energy Carried by EM Wave, Poynting Vector
- Momentum Carried by EM Wave
- Spectrum of EM wave.

Maxwell Equations

\[
\oint \mathbf{E} \cdot d\mathbf{A} = \frac{q}{\varepsilon_0},
\]
\[
\oint \mathbf{B} \cdot d\mathbf{A} = 0
\rightarrow \text{Gauss's Law/ Coulomb's Law}
\]
\[
\oint \mathbf{E} \cdot d\mathbf{l} = -\frac{d\Phi_B}{dt},
\]
\[
\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I + \varepsilon_0 \mu_0 \frac{d\Phi_E}{dt}
\rightarrow \text{Faraday's Law}
\]
\[
\oint \mathbf{E} \cdot d\mathbf{A} = 0
\rightarrow \text{Gauss's Law of Magnetism, no magnetic charge}
\]
\[
\oint \mathbf{B} \cdot d\mathbf{l} = 0
\rightarrow \text{Ampere Maxwell Law}
\]

Also, Lorentz force Law \[\mathbf{F} = q\mathbf{E} + q\mathbf{v} \times \mathbf{B}\]

These are the foundations of electromagnetism

EM Fields in Space

- Maxwell equations when there is no charge and current:

\[
\oint \mathbf{E} \cdot d\mathbf{A} = 0
\]
\[
\oint \mathbf{B} \cdot d\mathbf{A} = 0
\]

Differential forms:

- (single polarization)

\[
\frac{\partial E_y}{\partial x} = - \frac{\partial B_z}{\partial t},
\]
\[
\frac{\partial B_z}{\partial x} = -\mu_0 \varepsilon_0 \frac{\partial E_y}{\partial t}
\]

Linear Wave Equation

- Linear wave equation

\[
\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2}
\]

- Sinusoidal wave

\[
y = A \sin \left( \frac{2\pi}{\lambda} x - 2\pi ft + \phi \right)
\]

General wave: sum of sinusoidal waves

- Certain physical quantity

\[v = \lambda f, \quad \kappa = 2\pi/\lambda, \quad \omega = 2\pi f\]
Electromagnetic Waves

- EM wave equations:
  \[ \frac{\partial^2 E_y}{\partial x^2} = \mu_0 \epsilon_0 \frac{\partial^2 E_y}{\partial t^2} \quad \frac{\partial^2 B_z}{\partial x^2} = \mu_0 \epsilon_0 \frac{\partial^2 B_z}{\partial t^2} \]

- Plane wave solutions:
  \[ E = E_{\text{max}} \sin(kx - \omega t + \phi) \quad B = B_{\text{max}} \sin(kx - \omega t + \phi) \]

- Properties:
  - No medium is necessary.
  - E and B are normal to each other
  - E and B are in phase
  - Direction of wave is normal to both E and B (EM waves are transverse waves)
  - Speed of EM wave:
    \[ c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 2.9972 \times 10^8 \text{ m/s} \]
  - E/B = E_{\text{max}}/B_{\text{max}} = c
  - Transverse wave: two polarizations possible

Wavelength and Frequency

- Because of the wave equation the wavelength of and frequency of a EM wave in vacuum are related by:
  \[ \lambda f = c = 3 \cdot 10^8 \text{ m/s} \]

- Example: Determine the wavelength of an EM wave of frequency 50 MHz in free space
  \[ \lambda = \frac{c}{f} = \frac{3 \cdot 10^8 \text{ m/s}}{50 \text{ MHz}} = \frac{3 \cdot 10^8 \text{ m/s}}{5 \times 10^7 \text{ s}^{-1}} = 6 \text{ m} \]

Energy Carried By EM Waves

- Recall: energy densities \( u_E = \frac{1}{2} \epsilon_0 E^2 \), \( u_B = \frac{1}{2} B^2/\mu_0 \)
- For a EM wave, at any time/location,
  \[ u_E = \frac{1}{2} \epsilon_0 E^2 = \frac{1}{2} B^2/\mu_0 = u_B \quad \text{(using } E/B = c) \]
  \[ \Rightarrow \text{In an electromagnetic wave, the energies carried by electric field and magnetic field are always the same.} \]

- Total energy stored (per unit of volume):
  \[ u = u_E + u_B = \epsilon_0 E^2 = B^2/\mu_0 \]

- Power transmitted per unit of area is equal to uc in the direction of wave

- Averaging over time:
  \[ u_{av} = \frac{1}{2} \epsilon_0 E_{\text{max}}^2 = \frac{1}{2} B_{\text{max}}^2/\mu_0 \quad u_{av} c = I \text{ (intensity)} \]
The Poynting Vector

- The rate of flow of energy in an electromagnetic wave is described by a vector, \( \mathbf{S} \), called the Poynting vector.
- The Poynting vector is defined as:
  \[
  \mathbf{S} = \frac{1}{\mu_0} \mathbf{E} \times \mathbf{B}
  \]
- Its direction is the direction of propagation.
- This is time dependent:
  - Its magnitude varies in time.
  - Its magnitude reaches a maximum at the same instant as the electric field \( \mathbf{E} \) and magnetic field \( \mathbf{B} \).

Example: Solar Energy

- The average intensity of the EM radiation from the Sun on Earth is \( S \approx 10^3 \text{ W/m}^2 \).
  - What is the average radiation pressure for 100% absorption:
    \[
    P = \frac{S}{c} = \frac{10^3 \text{ W/m}^2}{3 \cdot 10^8 \text{ m/s}} = 3.3 \cdot 10^{-6} \text{ N/m}^2
    \]
  - What is the force exerted by EM radiation by the Sun on a surface of 1 m²:
    \[
    F = PA = 3.3 \cdot 10^{-6} \text{ N/m}^2 \cdot 1 \text{ m}^2 = 3.3 \cdot 10^{-6} \text{ N}
    \]

Momentum Carried By EM Waves

- EM waves: \( \text{momentum} = \frac{\text{energy}}{c} \)
  - Change of momentum in 100% absorption:
    \[
    \Delta p = \frac{\Delta U}{c} = \frac{uAc \Delta t}{c} = uA \Delta t
    \]
  - Change of momentum in 100% reflection:
    \[
    \Delta p = 2 \frac{\Delta U}{c} = 2 \frac{uAc \Delta t}{c} = 2uA \Delta t
    \]
- Radiation Pressure (\( P \)):
  \[
  P = \frac{F}{A} = \frac{\Delta p}{\Delta A} = \frac{u}{c} = \frac{S}{c}
  \]

Antennas

- Antennas are essentially arrangements of conductors for transmitting and receiving radio waves.
- Parameters: gain, impedance, frequency, orientation, polarization, etc.

Example diagrams of antennas, including:
- Loop
- Half-wave antenna
- \( \lambda/4 \) antennas
- Beverage
- Rhombic
- Yagi
- Helical
- Microstrip
- Log-periodic

Change of momentum in 100% absorption:
\[
\Delta p = \rho \rightarrow P = \frac{S}{c}
\]

Change of momentum in 100% reflection:
\[
\Delta p = 2\rho \rightarrow P = 2\frac{S}{c}
\]
An electromagnetic wave is propagating in the 
+ \( \hat{i} \) direction and its electric field is given by
\[ \vec{E} = E_0 \sin (kx - \omega t) \hat{j}. \]
The corresponding magnetic field wave is (where \( B_0 = E_0 / c \))

A. \( \vec{B} = B_0 \sin (kx - \omega t) \hat{i}. \)
B. \( \vec{B} = B_0 \sin (kx - \omega t) \hat{k}. \)
C. \( \vec{B} = B_0 \cos (kx - \omega t) \hat{j}. \)
D. \( \vec{B} = -B_0 \sin (kx - \omega t) \hat{k}. \)
E. \( \vec{B} = -B_0 \cos (kx - \omega t) \hat{k}. \)