Physics 202, Lecture 26

Today's Topics
- Lenses
  - Refraction and Lenses
    - Ray Diagrams
    - Lens Equation
  - Cameras
  - The Human Eye, Lenses and Magnifiers
- Combination of Lenses
  - Microscopes
  - Telescopes

converging lens: 3 easy rays

parallel rays

horizontal ray goes through focal point behind lens
ray through center of lens is undeflected
ray that goes through focal point in front of lens is deflected to be horizontal behind lens

image of an object in a converging lens

demo: white board
image of an object in a converging lens

converging lens

1 \over o + 1 \over i = 1 \over f

lens formula

Derivation of lens equation (1)

h_o = height of object

h_i = height of image

\frac{h_o}{h_i} = \frac{o - f}{f}
Derivation of lens equation (2)

\[ h_o = \text{height of object} \]
\[ h_i = \text{height of image} \]

From similar triangles,

\[ \frac{i}{h_i} = \frac{o}{h_o} \Rightarrow \frac{h_o}{h_i} = \frac{o}{i} \]

Derivation of lens equation (3)

\[ \frac{h_o}{h_i} = \frac{o - f}{f} \quad \text{and} \quad \frac{o}{i} = \frac{h_o}{h_i} \]

So,

\[ \frac{o}{i} = \frac{o - f}{f} \]
\[ \frac{0}{i} = \frac{0}{-f} = 1 \]
\[ \frac{1}{i} = \frac{1}{f} \]
\[ \frac{1}{o} = \frac{1}{f} \]
\[ \frac{i}{o} = -\frac{1}{f} \]

Lens equation

\[ \frac{h_i}{h_o} = \frac{i}{o} = |M| \]

magnification

\[ h_o \quad o \quad i \quad h_i \]

magnification M

\[ h_o \quad o \quad f \quad i \quad f \quad h_i \]

\[ h_i = \frac{i}{o} \cdot h_o \]
The equation for magnification is given as $M = -\frac{i}{o}$, where $i$ is the image distance and $o$ is the object distance.

- For $M < 1$, the image is virtual and the object is closer than the focal length ($i < o$).
- For $M > 1$, the image is real and smaller than the object ($i > o$).
- For $M = 0$, the object is at infinity ($o = \infty$) and the image is at the focal length ($i = f$).

The magnifying lens equation is $\frac{1}{\infty} + \frac{1}{i} = \frac{1}{f}$, implying $i = f$ and $M = 0$. The image is virtual when $i < 0$ (negative), and $|M| = \frac{i}{o} > 1$. For a converging lens with an object distance less than the focal distance, the magnifying lens equation is $\frac{1}{o} + \frac{1}{i} = \frac{1}{f}$.
summary: converging lens

converging lens, $o = \infty$

real image in focal point
$M = -i/\infty = 0$

converging lens, $o > f$

real inverted image
$M = -i/o < 0$

converging lens, $o < f$

virtual upright image
$M = -i/o > 1$

$M < 0$

BUT: $\{|M| > 1\}$ possible

$\{|M| < 1\}$ possible

example problem: projector

You set up a projector so that it is in focus, but the image is too small, and you move the projector farther from the screen.

1. Is the lens in the projector a converging lens or a diverging lens?

A converging lens. Any lens that makes a real image that can be seen on a screen is a converging lens.

example problem: projector

You set up a slide projector so that it is in focus, but the image is too small, and you move the projector farther from the screen.

1. Is the lens in the projector a converging lens or a diverging lens?

2. After you move the projector back, to focus the image again do you move the slide closer to or farther from the focal point of the lens?
example problem: projector

You set up a slide projector so that it is in focus, but the image is too small, and you move the projector farther from the screen.

1. Is the lens in the projector a converging lens or a diverging lens?
   A converging lens. Any lens that makes a real image that can be seen on a screen is a converging lens.
2. After you move the projector back, to focus the image again do you move the slide closer to or farther from the focal point of the lens?
   Closer to the focal point

example problem: converging lens

\[ f = 10 \text{ cm} \]
\[ i = ? \]

1. \( o = 20 \text{ cm} \)

2. Object farther away
   \( o = 50 \text{ cm} \)
   \( o \rightarrow \infty \)

3. Object closer than \( f \)
   \( o = 4 \text{ cm} \)

A nice applet for understanding converging lenses:


\[ \frac{1}{o} + \frac{1}{i} = \frac{1}{f} \]

1. \( o = 20 \text{ cm} \) \( i = 20 \text{ cm} \) \( M = 1 \)

2. Object farther away
   \( o = 50 \text{ cm} \) \( i = 12.5 \text{ cm} \) \( M = 0.25 \)
   \( o \rightarrow \infty \) \( i \rightarrow f \) \( M \rightarrow 0 \)

3. Object closer than \( f \)
   \( o = 4 \text{ cm} \) \( i = -6.7 \text{ cm} \) \( M = 1.7 \)
Images Formed by Converging Lens

- Object (O) is in front of $F_1$: real, inverted, enlarged or reduced

- Object (O) in between $F_1$ and lens: virtual, upright, enlarged.

For any $o$:

1. If $f > 0$:
   
   \[
   M = \frac{h_i}{h_o} = -\frac{i}{o}
   \]

2. If $f < 0$:
   
   \[
   \frac{1}{i} = \frac{1}{f} - \frac{1}{o}
   \]

Virtual image for any $o$.

**diverging lens**: 3 easy rays

- Horizontal ray is deflected so that it extrapolates back to focal point in front of lens.
- Ray from object to focal point in back of lens is deflected so that it is horizontal.
- Ray that goes through center of lens — this ray is undeflected.

**bb**: 3 easy rays

**bb2**: 3 easy rays
**Diverging Lens**

1. **Virtual Upright Image**
   - Object at infinity
   - Image in focal point

2. **Diverging Lens: Object at Infinity**
   - Diverging lens
   - Image at infinity

3. **Diverging Lens: $o < f$**
   - Diverging lens
   - Image at focal point
Diverging lens: o < f

Virtual upright image
Images Formed by Diverging Lenses

Images are always virtual, upright, and reduced.

Summary of sign conventions

<table>
<thead>
<tr>
<th></th>
<th>&gt;0</th>
<th>&lt;0</th>
</tr>
</thead>
<tbody>
<tr>
<td>f</td>
<td>concave mirrors</td>
<td>convex mirrors</td>
</tr>
<tr>
<td></td>
<td>converging lens</td>
<td>diverging lens</td>
</tr>
<tr>
<td>o</td>
<td>object side</td>
<td>the other side</td>
</tr>
<tr>
<td>i</td>
<td>real</td>
<td>virtual</td>
</tr>
<tr>
<td>M</td>
<td>upright</td>
<td>inverted</td>
</tr>
</tbody>
</table>

Converging and Diverging Lenses

Converging and diverging lenses have different sign conventions:

- For converging lenses ($f > 0$):
  - Real mirrors: front
  - Virtual mirrors: behind
  - Real lenses: behind
  - Virtual lenses: font

- For diverging lenses ($f < 0$):
  - Real mirrors: behind
  - Virtual mirrors: font
  - Real lenses: font
  - Virtual lenses: behind

M = \frac{h_i}{h_o} = -\frac{i}{o}
Combination of Lenses

\[ M = M_1 M_2 \]

Cameras

- A camera is essentially a converging lens with a short focal length. (Operating condition: \( o \gg f \rightarrow i \sim f \))

Eyes

- The eye is essentially an auto-focus camera

Simple Magnifier

- A simple magnifier is essentially also a converging lens with a short focal length.
  - Operating condition: \( o < f \) and \( i \sim 25 \text{ cm} \)
  - Simple magnifiers magnify the opening angle an object subtends at the eye (i.e. psychological size)

Angular magnification:

\[ m = \frac{\theta}{\theta_0} = \frac{25 \text{ cm}}{f} \text{ for near point} \]
Compound Microscopes (cont)

Robert Hooke’s Microscope (1665)

Telescopes (cont)

Galileo’s Telescope

Compound Microscope

- Compound microscope also does angular magnification.
- Configuration: \( L \gg f_e+f_o \)

Telescopes

- Telescope is another type of angular magnification device with configuration \( L \sim f_e+f_o \)

Note:
For telescope application, object distance can not be adjusted.