Physics 202, Lecture 5

Today’s Topics

- Electric Potential Energy, Electric Potential
  - Motion of charges in E fields
  - Methods for obtaining E, V
    - Point charges (review)
    - Continuous distributions
Field lines always point towards lower electric potential!

In an electric field:

- positive charges are always subject to a force in the direction of field lines, towards lower \( V \)
- negative charge is always subject a force in the opposite direction of field lines, towards higher \( V \)
Electric Potential - Uniform Field

\[ \Delta V = -\mathbf{E} \cdot d\mathbf{l} \Rightarrow V_B - V_A = \int_A^B -\mathbf{E} \cdot d\mathbf{l} \]

\[ = -\int_A^B \mathbf{E} \cdot d\mathbf{l} = -E \int_A^B d\mathbf{l} = -Ex \]

\[ \mathbf{E} \parallel d\mathbf{l} \quad E \text{ constant} \]

Recall: \[ \Delta U = q\Delta V \]

A positively charged particle moving from A to B gains kinetic energy \( K = \text{potential energy} \Delta U \) lost by the charge-field system

\[ \Delta K + \Delta U = 0 \quad \text{(energy conservation)} \]
Example: Uniform Electric Field

In the uniform electric field shown:

1. What is the potential at B, C, D, G?

2. If a charge +q is placed at B, what is the potential energy $U_B$?

3. If now a -q is at B, what is $U_B$?

4. If a -q is initially at rest at G, will it move to A or B? What is its final kinetic energy?

Particles will move to minimize their final potential energy.
Cathod Ray tube

- Cathode
- Anode
- Horizontal deflection plates
- Vertical deflection plates
- Bright spot on screen where electrons hit
- Fluorescent screen
- Path of electrons
Summary: Methods for Obtaining $E$, $V$

**Coulomb’s Law:**  
(lecture 2)

$$
\vec{E} = \sum k \frac{q}{r^2} \hat{r}
$$

$$
\vec{E} = k \int \frac{dq}{r^2} \hat{r}
$$

$$
V = -\int \vec{E} \cdot d\vec{l}
$$

**Gauss’s Law:**  
(lecture 3,4)

$$
\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{encl}}}{\varepsilon_0}
$$

**Potential:**  
(lecture 4, today)

$$
V = \sum k \frac{q}{r}
$$

$$
V = k \int \frac{dq}{r}
$$

$$
\vec{E} = -\nabla V
$$

$$
E_x = -\frac{\partial V}{\partial x}, E_y = -\frac{\partial V}{\partial y}, E_z = -\frac{\partial V}{\partial z}
$$

Know all methods and when to apply which
Electric Potential and Point Charges

For point charge $q$ shown below, what is $V_B - V_A$?

$$V_B - V_A = -\int_A^B E(r)dr = -kq\int_A^B \frac{dr}{r^2} = kq\left(\frac{1}{r_B} - \frac{1}{r_A}\right)$$

independent of path b/w A and B!

Potential of point charge:

$$V(r) = \frac{kq}{r}$$

Many point charges: superposition

$$V(r) = k\sum_i \frac{q_i}{r_i}$$
Continuous Charge Distributions: “Brute Force”

Finite charge distributions: usually set $V=0$ at infinity (can also set it zero at ground)

If charge distribution is known:

$$dV = k \frac{dq}{r} \quad V = \int dV$$

$r = \text{distance b/w source and obs. point}$

Note: scalar integral

Examples:
finite line, ring, disk…
Imagine a homogenously charged ring

\[ V = \int dV = k \int \frac{dq}{r} \]

\[ V = k \int \frac{dq}{\sqrt{x^2 + R^2}} = k \frac{1}{\sqrt{x^2 + R^2}} \int dq = k \frac{Q}{\sqrt{x^2 + R^2}} \]
Continuous Charge Distributions: Gauss’s Law

Obtain E by Gauss’s Law.
Integrate to get V.

$$\Delta V = - \int_{A}^{B} \vec{E} \cdot d\vec{l} = V_B - V_A$$

V = constant inside E = 0 region

Note: like spherical conductor

Example 1: spherical shell

$$E = 0 \quad r < R$$

$$E_r = \frac{kQ}{r^2} \quad r > R$$
Continuous Charge Distributions: Gauss’s Law

Uniformly charged sphere:

Use
\[ \Delta V = -\int_A^B \vec{E} \cdot d\vec{l} = V_B - V_A \]

since from Gauss’s law:

\[ E = \frac{kQr}{R^3} \quad r < R \]
\[ E = \frac{kQ}{r^2} \quad r > R \]
Imagine an electric potential of the following form

\[ V(x, y, z) = 2x^2 + 8y^2z + 2z^2 \]

Units of V

\[
E_x = -\frac{\partial V}{\partial x} = -4x \quad E_y = -\frac{\partial V}{\partial y} = -16zy \quad E_z = -\frac{\partial V}{\partial z} = -8y^2 - 4z
\]

Units of V/m