Physics 202, Lecture 5

Today’s Topics

- Electric Potential Energy, Electric Potential
- Motion of charges in E fields
- Methods for obtaining E, V
  - Point charges (review)
  - Continuous distributions

Electric Potential and Electric Field Lines

Field lines always point towards lower electric potential!

In an electric field:
- positive charges are always subject to a force in the direction of field lines, towards lower V
- negative charge is always subject a force in the opposite direction of field lines, towards higher V

Electric Potential - Uniform Field

\[
\Delta V = -\vec{E} \cdot d\vec{l} = \int_{A}^{B} \vec{E} \cdot d\vec{l} = \int_{A}^{B} -E d\vec{l} = -Ex
\]

Recall: \( \Delta U = q\Delta V \)

A positively charged particle moving from A to B gains kinetic energy \( K = \) potential energy \( \Delta U \) lost by the charge-field system

\[ \Delta K + \Delta U = 0 \] (energy conservation)

Example: Uniform Electric Field

In the uniform electric field shown:
1. What is the potential at B,C,D,G?
2. If a charge +q is placed at B, what is the potential energy \( U_B \)?
3. If now a –q is at B, what is \( U_B \)?
4. If a -q is initially at rest at G, will it move to A or B? What is its final kinetic energy?

Particles will move to minimize their final potential energy
**Cathod Ray tube**

**Electric Potential and Point Charges**

For point charge q shown below, what is \( V_B - V_A \)?

\[
V_B - V_A = \int_A^B E(r)dr = -kq \int_A^B \frac{dr}{r^2} = kq \left( \frac{1}{r_B} - \frac{1}{r_A} \right)
\]

independent of path b/w A and B!

Potential of point charge:

\[
V(r) = \frac{kq}{r}
\]

Many point charges: superposition

\[
V(r) = k \sum \frac{q_i}{r_i}
\]

**Summary: Methods for Obtaining E, V**

- **Coulomb’s Law:**
  (lecture 2)
  \[
  \vec{E} = \sum k \frac{dq}{r^2} \vec{r}
  \]
  \[
  V = -\int \vec{E} \cdot d\vec{l}
  \]

- **Gauss’s Law:**
  (lecture 3,4)
  \[
  \oint \vec{E} \cdot d\vec{A} = \frac{q_{net}}{\varepsilon_0}
  \]

- **Potential:**
  (lecture 4, today)
  \[
  V = k \sum \frac{q}{r}
  \]
  \[
  \vec{E} = -\nabla V
  \]

Know all methods and when to apply which

**Continuous Charge Distributions: “Brute Force”**

- **Finite** charge distributions: usually set \( V=0 \) at infinity (can also set it zero at ground)

If charge distribution is known:

\[
dV = k \frac{dq}{r} \quad V = \int dV
\]

\( r = \) distance b/w source and obs. point

**Note:** scalar integral

Examples:
finite line, ring, disk…
Potential due to Ring

- Imagine a homogenously charged ring

\[
V = \int dV = k \int \frac{dq}{r} = k \int \frac{dV}{\sqrt{x^2 + R^2}} = k \frac{1}{\sqrt{x^2 + R^2}} \int dq = k \frac{Q}{\sqrt{x^2 + R^2}}
\]

Continuous Charge Distributions: Gauss’s Law

Uniformly charged sphere:

Use

\[
\Delta V = - \int \vec{E} \cdot d\vec{l} = V_B - V_A
\]

since from Gauss’s law:

- \[E = \frac{kQ}{r} \quad r < R\]
- \[E = \frac{kQr}{R} \quad r > R\]

Compute E from V

- Imagine an electric potential of the following form

\[
V(x, y, z) = 2x^2 + 8y^2z + 2z^2
\]

Units of V

\[
E_x = -\frac{\partial V}{\partial x} = -4x, \quad E_y = -\frac{\partial V}{\partial y} = -16yz, \quad E_z = -\frac{\partial V}{\partial z} = -8y^2 - 4z
\]

Units of V/m