Physics 202, Lecture 7

Today’s Topics
- Capacitance (Ch. 24)
- Review
- Energy storage in capacitors
- Dielectric materials, electric dipoles
- Dielectrics and Capacitance

Summary: Combination of Capacitors

\[ C = C_1 + C_2 + C_3 + \ldots \]

\[ \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots \]

Capacitors: Summary

- **Definition:**
  \[ C = \frac{Q}{\Delta V} \]

- **Capacitance depends on geometry:**
  - Parallel Plates
    \[ C = \frac{\varepsilon_0 A}{d} \]
  - Cylindrical
    \[ C = \frac{2\pi \varepsilon_0 L}{\ln \left( \frac{b}{a} \right)} \]
  - Spherical
    \[ C = 4\pi \varepsilon_0 \frac{ab}{b - a} \]

\( \varepsilon_0 \) has units of F/m

Energy of a Capacitor

- **How much energy is stored in a charged capacitor?**
  - Calculate the work provided (usually by a battery) to charge a capacitor to +/- \( Q \):

  Incremental work \( dW \) needed to add charge \( dq \) to capacitor at voltage \( V \):

  \[ dW = V(q) \cdot dq = \left( \frac{q}{C} \right) \cdot dq \]

- **The total work** \( W \) to charge to \( Q \) (\( W = \text{potential energy of the charged capacitor} \)) is given by:

  \[ W = \frac{1}{C} \int q dq = \frac{1}{2} \frac{Q^2}{C} \]

  Two ways to write \( W \):

  - In terms of the voltage \( V \):
    \[ W = \frac{1}{2} CV^2 \]
Where is the Energy stored?

- Claim: energy is stored in the electric field itself.
- Consider the example of a constant field generated by a parallel plate capacitor:
  \[
  U = \frac{1}{2} \frac{Q^2}{C} = \frac{1}{2} \left( \frac{Q}{A} \right) \left( \frac{1}{\varepsilon_0} \right)
  \]
- The electric field is given by:
  \[
  E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A} \implies U = \frac{1}{2} \varepsilon_0 E^2 Ad
  \]
- The energy density \( u \) in the field is given by:
  \[
  u = \frac{U}{\text{volume}} = \frac{U}{Ad} = \frac{1}{2} \varepsilon_0 E^2
  \]
  Units: \( \text{J/m}^2 \)

Examples: 24.42, 48, 51

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Example (I)

- Suppose the capacitor shown here is charged to \( Q \). The battery is then disconnected.
- Now suppose the plates are pulled further apart to a final separation \( d_i \),
- How do the quantities \( Q, C, E, V, U \) change?
  
  - \( Q \): remains the same.. no way for charge to leave.
  - \( C \): decreases.. capacitance depends on geometry
  - \( E \): remains the same.. depends only on charge density
  - \( V \): increases.. since \( C \downarrow \), but \( Q \) remains same (or \( V \uparrow \), but \( E \) the same)
  - \( U \): increases.. add energy to system by separating

\[
C_i = \frac{d}{d_i} C \quad V_i = \frac{d}{d_i} V \quad U_i = \frac{d}{d_i} U
\]

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Example (II)

- Suppose the battery (\( V \)) is kept attached to the capacitor.
- Again pull the plates apart from \( d \) to \( d_1 \),
- Now what changes?
  
  - \( C \): decreases (capacitance depends only on geometry)
  - \( V \): must stay the same - the battery forces it to be \( V \)
  - \( Q \): must decrease, \( Q=CV \) charge flows off the plate
  - \( E \): must decrease (\( E = \frac{V}{d}, E = \frac{Q}{A} \))
  - \( U \): must decrease (\( U = \frac{1}{2} CV^2 \))

\[
C_1 = \frac{d}{d_1} C \quad E_1 = \frac{d}{d_1} E \quad U_1 = \frac{d}{d_1} U
\]

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Dielectrics

- Empirical observation:
  Inserting a non-conducting material (dielectric) between the plates of a capacitor changes the VALUE of the capacitance.
- Definition:
  The dielectric constant \( \kappa \) of a material is the ratio of the capacitance when filled with the dielectric to that without it:
  \[
  \kappa = \frac{C}{C_0}
  \]
  permittivity: \( \varepsilon \equiv \kappa \varepsilon_0 \)

\( \kappa \) values are always > 1 (e.g., glass = 5.6; water = 80)
Dielectrics INCREASE the capacitance of a capacitor
More energy can be stored on a capacitor at fixed voltage:

\[
U' = \frac{CV'^2}{2} = \frac{\kappa C_0 V^2}{2} = \kappa U
\]
Dielectric Materials

Dielectrics are electric insulators:
- Charges are not freely movable, but can still have small displacements in an external electric field
- Atomic view: composed of permanent (or induced) electric dipoles:

\[ \begin{align*}
\text{Permanent dipole} & : \quad \begin{array}{c}
\text{O}^- \\
\text{H}^+ \quad \text{H}^+ \\
\end{array} \\
\text{Induced dipole} & : \quad \begin{array}{c}
\text{O}^- \\
\text{H}^+ \quad \text{H}^+ \\
\end{array}
\end{align*} \]

\[ \text{p} \quad \text{(dipole moment vector)} \]

Dielectrics In External Field

Alignment of permanent dipoles in external field (or alignment of non-permanent dipoles)

\[ \begin{align*}
\text{Zero external field} & : \quad E_0 = 0 \\
\text{Applying external E field} & : \quad E_{\text{ind}} = \frac{\sigma_{\text{ind}}}{\varepsilon_0} \\
\text{Equilibrium} & : \quad E = E_0 - E_{\text{ind}} = \frac{E_0}{\varepsilon_0} \\
\end{align*} \]

Note: induced field always opposite to the external field \( E_0 \)

Parallel Plate Example (I)

- Deposit a charge \( Q \) on parallel plates filled with vacuum (air)—capacitance \( C_0 \)
- Disconnect from battery
- The potential difference is \( V_0 = \frac{Q}{C_0} \)

Now insert material with dielectric constant \( \kappa \).

Charge \( Q \) remains constant
Capacitance increases \( C = \kappa C_0 \)
Voltage decreases from \( V_0 \) to:

\[ V = \frac{Q}{C} = \frac{Q}{\kappa C_0} = \frac{V}{\kappa} \]

Electric field decreases also:

\[ E = \frac{V}{d} = \frac{V_0}{d} = \frac{E_0}{\kappa} \]

Note: The field decreases even when the charge is held constant!

Parallel Plate Example (II)

- Deposit a charge \( Q_0 \) on parallel plates filled with vacuum (air)—capacitance \( C_0 \)
- The potential difference is \( V = \frac{Q}{C_0} \)
- Leave battery connected.

Now insert material with dielectric constant \( \kappa \).

Voltage \( V \) remains constant
Capacitance increases \( C = \kappa C_0 \)
Charge increases from \( Q_0 \) to:

\[ Q = CV = \kappa C_0 V = \kappa Q_0 \]

Net Electric field stays same (\( V, d \) constant)