Charge Motion in a Conductor

Electrons in a conductor have random motion ($\vec{v}_{\text{ave}} = 0$)

In an external electric field (e.g. as supplied by a source of potential difference such as a battery), electrons accelerate, producing current:

Average current:

$$I = \frac{\Delta Q}{\Delta t}$$

Instantaneous current:

$$I = \frac{dQ}{dt}$$

direct current (DC): $I$ constant

Current: Macroscopic View

Current: rate at which charge flows through surface:

Unit: 1 Ampere = 1 C/s

Current is directional: Follows positive charge (convention)

+q moving in +x direction $\leftrightarrow$ –q in moving –x direction

Charge conservation $\rightarrow$ Current conservation

$I_{\text{in}} = I_{\text{out}}$

Current: Microscopic View

Current: motion of charged particles

Current density: (vector)

$$J = \frac{I}{A} = nqv_d$$

$$\int \vec{J} \cdot d\vec{A} = I$$

$v_d$: average drift velocity

$n$: number density
Ohm’s Law: Resistance

- Ohm determined experimentally that the electric current through a wire is proportional to \( \Delta V \):

\[ I \propto V \]

- For a fixed material and geometry

\[ I = \frac{V}{R} \text{ or } V = RI \]

Conductivity, Resistivity, Resistance

Ohm’s Law (microscopic): \( \mathbf{J} = \sigma \mathbf{E} \)

Ohm’s Law (macroscopic): \( \Delta V = IR \)

Resistance \( R \) (unit: Ohm \( \Omega \))

Exercise: relate \( R \) to \( \rho \)

Resistivity (intrinsic)

Length & Cross-section (shape)

Resistance

Ohmic and non-Ohmic Materials

Ohmic:
Linear I-V relationship
(constant resistance over wide range of voltages)

non-Ohmic:
Nonlinear I-V relationship

Resistivity For Various Materials

<table>
<thead>
<tr>
<th>Material</th>
<th>Resistivity (( \Omega \cdot \text{m} ))</th>
<th>Temperature Coefficient (( \Omega \cdot \text{m} \cdot ^\circ \text{C}^{-1} ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silver</td>
<td>( 1.59 \times 10^{-8} )</td>
<td>( 5.8 \times 10^{-3} )</td>
</tr>
<tr>
<td>Copper</td>
<td>( 1.7 \times 10^{-8} )</td>
<td>( 5.9 \times 10^{-3} )</td>
</tr>
<tr>
<td>Gold</td>
<td>( 2.44 \times 10^{-8} )</td>
<td>( 5.6 \times 10^{-3} )</td>
</tr>
<tr>
<td>Aluminum</td>
<td>( 2.82 \times 10^{-8} )</td>
<td>( 5.5 \times 10^{-3} )</td>
</tr>
<tr>
<td>Tungsten</td>
<td>( 5.6 \times 10^{-8} )</td>
<td>( 4.5 \times 10^{-3} )</td>
</tr>
<tr>
<td>Iron</td>
<td>( 10 \times 10^{-8} )</td>
<td>( 5.0 \times 10^{-3} )</td>
</tr>
<tr>
<td>Platinum</td>
<td>( 11 \times 10^{-8} )</td>
<td>( 3.92 \times 10^{-3} )</td>
</tr>
<tr>
<td>Lead</td>
<td>( 22 \times 10^{-8} )</td>
<td>( 5.9 \times 10^{-3} )</td>
</tr>
<tr>
<td>Nichrome(^a)</td>
<td>( 1.50 \times 10^{-8} )</td>
<td>( 0.4 \times 10^{-3} )</td>
</tr>
<tr>
<td>Carbon</td>
<td>( 3.5 \times 10^{-8} )</td>
<td>( -0.5 \times 10^{-3} )</td>
</tr>
<tr>
<td>Germanium</td>
<td>( 0.46 )</td>
<td>( -48 \times 10^{-3} )</td>
</tr>
<tr>
<td>Silicon</td>
<td>( 690 )</td>
<td>( -75 \times 10^{-3} )</td>
</tr>
<tr>
<td>Glass</td>
<td>( 10^{10} ) to ( 10^{11} )</td>
<td></td>
</tr>
<tr>
<td>Hard rubber</td>
<td>( -10^{11} )</td>
<td></td>
</tr>
<tr>
<td>Sulfur</td>
<td>( 10^{10} )</td>
<td></td>
</tr>
<tr>
<td>Quartz (fused)</td>
<td>( 75 \times 10^{11} )</td>
<td></td>
</tr>
</tbody>
</table>

\(^a\) All values at 30°C.

\(^b\) See Section 27A.

\(^c\) A nickel–chromium alloy commonly used in heating elements.
**Resistance And Temperature**

Resistivity is usually temperature dependent.

- **Normal Metal**
- **Semiconductor**
- **Superconductor**

**Superconductivity**

Superconductors: temperature $T < T_c$, resistivity $\rho = 0$
(a quantum phenomenon!)

**Critical Temperatures for Various Superconductors**

<table>
<thead>
<tr>
<th>Material</th>
<th>$T_c$ (K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>HgBa$_2$CuO$_4$</td>
<td>134</td>
</tr>
<tr>
<td>Tl$_2$Ba$_2$Ca$_2$Cu$_2$O$_8$</td>
<td>125</td>
</tr>
<tr>
<td>Bi$_2$Sr$_2$CaCu$_2$O$_8$</td>
<td>105</td>
</tr>
<tr>
<td>YBa$_2$Cu$_3$O$_7$</td>
<td>92</td>
</tr>
<tr>
<td>Nb$_3$Ge</td>
<td>23.2</td>
</tr>
<tr>
<td>Nb$_5$Sn</td>
<td>18.05</td>
</tr>
<tr>
<td>Nb</td>
<td>9.46</td>
</tr>
<tr>
<td>Pd</td>
<td>7.18</td>
</tr>
<tr>
<td>Hg</td>
<td>4.15</td>
</tr>
<tr>
<td>Sn</td>
<td>3.72</td>
</tr>
<tr>
<td>Al</td>
<td>1.19</td>
</tr>
</tbody>
</table>

**Electrical Power**

Electrical Power:

$P = \frac{dU}{dt} = \frac{d(Q\Delta V)}{dt} = I\Delta V$

Ohmic:

$P = I^2R = \frac{(\Delta V)^2}{R}$

Unit: watts (W)

1 KWH = 3.6 MJ

(power delivered to resistor)

**Example: Battery Connected To A Resistor**

Energy flow of this battery-resistor set-up

- Chemical Process $\rightarrow \Delta V = 1.5V$
- $\Delta V$ on Resistor $\rightarrow$ Current $I = \Delta V/R$

Charge flow through the resistor in $\Delta t$:

$Q = I\Delta t = \Delta V/R\Delta t$

Electrical potential energy released:

$U = Q\Delta V = \Delta V/R\Delta t \Delta V = (\Delta V)^2/R\Delta t$

Power: $P = U/dt = (\Delta V)^2/R$

Energy Flow:
- Chemical $\rightarrow$ Electrical $U \rightarrow K_E \rightarrow$ thermal/light