PHYSICS 308

Experiment D: Field of a Magnetic Dipole

Motivation: The most commonly encountered static magnetic field is that of the magnetic dipole. We wish to describe the dependence of the dipole field on the distance from and orientation with respect to the dipole moment.

Theory:

For a static magnetic dipole moment $\vec{m}$, the field is analogous to the field of an electric dipole, and is given by:

$$\vec{B}(\vec{r}) = \frac{\mu_o}{4\pi} \frac{(3\hat{r}(\hat{r} \cdot \vec{m}) - \vec{m})}{r^3}$$

where $\hat{r}$ is the unit vector in the $\vec{r}$-direction. The components of $\vec{B}$ in spherical coordinates are then just:

$$B_r = (\mu_o/4\pi)(2m\, cos\theta/r^3) \quad (1.1)$$

$$B_\theta = (\mu_o/4\pi)(m\, sin\theta/r^3) \quad (1.2)$$

$$B_\phi = 0 \quad (1.3)$$

where $\hat{z}$ is chosen to be parallel to $\vec{m}$.

A coil of N loops, each of area A, through which a current I is passed, will produce a magnetic dipole moment $\vec{m} = \text{NIA}\hat{n}$ (where $\hat{n}$ is the unit vector normal to the plane of the loops), as well as other moments, which are insignificant if measurements are made far enough away from the coil. (How far is "far enough"?) The field produced by this arrangement, using ordinary coils and current levels, is quite weak, and presents two problems in measurements: first, the earth's magnetic field may be of comparable strength, and would have to be compensated for in some way; and second, it is difficult to measure small static magnetic fields very precisely. However, if we replace the static field with a varying field (produced by passing a varying current through the coil) we can easily detect this field by the voltage induced in a second coil, without interference from any stray static fields such as the earth's. (At what frequencies can we justify replacing the static field with a varying one? Think about the electric field experiment.)

Your main equipment is a pair of matched dipole and detector coils, tuned with a parallel capacitor to some particular resonant frequency. You will apply a voltage to the dipole coil and measure the voltage induced in the detector coil. It should be clear that the current I' in the detector coil produces a dipole moment $m' = N'I'A'n'$, exactly analogously to the moment m in the dipole coil. There then exists an interaction between the induced dipole m′ and the field B due to m, where the interaction energy is given by:

$$W = -\vec{m} \cdot \vec{B} = -(\mu_o/4\pi)[3(\vec{m} \cdot \hat{r})(\vec{m}' \cdot \hat{r}) - \vec{m} \cdot \vec{m}']/r^3 \quad (2.1)$$

$$= -N'I'A'\hat{n}' \cdot \vec{B} \quad (2.2)$$

From (2.2) it is clear that W varies as I', and thus as V', the voltage across the detector coil. Therefore:

$$V' \propto |cos(\theta - \theta') - 3\, cos\theta\, cos\theta'|/r^3 \quad (3)$$

where θ and θ' are the angles between $\vec{m}$ and $\vec{r}$ and between $\vec{m}'$ and $\vec{r}$, respectively.

PROCEDURE:

NOTE: Be especially careful to correct or account for zero-point errors, both in r and θ.

1) Connect the dipole coil to the oscillator through a 1kΩ resistor and find the resonant frequency. Then remove the resistor and find the resonant frequency of the dipole-detector combination. (It should be the same.)

2) Measure both the radial and angular dependence of both the radial and angular components of the $\vec{B}$ field produced by the dipole coil. Note that you will be using arbitrary units for $\vec{B}$, since you do not have enough information to determine the absolute field strength.

3) Determine (on paper) what conditions on θ and θ' lead to maxima and minima in W. Check your results experimentally for several values of θ'.

4) For some value of θ' (not 0 or $\pi/2$) measure the angular dependence of W.

ANALYSIS:

1) Why was the 1kΩ resistor used when finding the dipole coil's resonant frequency? Why wasn't it used for the rest of the experiment? (Refer back to your LRC experiment)

2) Verify that your results agree with equations (1.1) an (1.2). (An interesting way to do this in the case of r-dependence is to plot your data on log-log graph paper; feel free to use any method you want, though.)

3) Verify that your results agree with your predictions.

4) Verify that your results agree with equation (3).

FINAL QUESTION:

Since a voltage is induced by $\vec{B}$ in the detector coil, and this voltage drives a current through a resistance (where?), $\vec{B}$, and thus the circuit producing $\vec{B}$, is transferring energy into the detector circuit. What effect does this have on the dipole circuit? Was this effect noticeable in the experiment? Why or why not?