1. Carry out a Monte Carlo simulation of a nearest-neighbor spin-$\frac{1}{2}$ Ising model on a square and a honeycomb lattices. Consider both ferromagnetic and antiferromagnetic interactions. In simulations, use periodic boundary conditions.

Your report should include:

a) Plots of the lattice averaged magnetization for the FM case and staggered magnetization for the AFM case as functions of temperature $T$ for 3 different lattice sizes, and also as functions of the Monte Carlo time for 3 different temperatures for the largest $L$ you have used for simulations.
b) Plots of the specific heat and susceptibility as functions of $T$ for at least 3 different sizes of the lattice.

c) Find the critical temperature using Binder’s fourth-order cumulant. Does the critical Temperature $T_c$ correspond to the peak in the specific heat and the susceptibility?

d) Find critical exponents characterizing the phase transition. Describe how you performed a scaling analysis. Do your results agree with critical exponents describing Ising transition?
(2) Now perform theoretical analysis of the ferromagnetic Ising model on a square lattice. Using mean field approximation calculate magnetization as a function of temperature and find the mean field value of $T_c$. Calculate the mean field expressions for the specific heat and susceptibility. Then find the mean field values of critical exponents: $\beta, \delta, \gamma, \alpha$ and compare them with your Monte Carlo simulations.