The quantum liquid described above possesses a remarkable property known as superfluidity.

**Superfluidity** - flow of liquid without dissipation.

\[ E = \frac{mv^2}{2} \]

Viscosity means the dissipation of kinetic energy, i.e., particles constructing the current flow scatter on imperfections.

Superfluid liquid flows through narrow capillaries without viscosity.

Let us consider a coordinate system moving with the liquid. Then,

- liquid helium is at rest
- walls of the capillary move with velocity \( \mathbf{v} \).

When viscosity is present, the liquid at rest must also begin to move.

It is obvious that not the whole liquid starts moving due to interaction with walls. So the first step of beginning of moving is accompanied by appearing of elementary excitations.

Suppose that first elementary excitations appear with momentum \( \mathbf{p} \) and energy \( E(p) \).

Consider that just one excitation with \( E(p) \) appears.
Then, the energy and momentum of a liquid at the system coordinates at which the liquid is at rest

\[ E_0 = E_{\text{liquid}} = E(p) \]
\[ P_0 = P_{\text{liquid}} = P \]

Back to not moving system of coordinates

\[ E = E_0 + P_0 \cdot \nabla + \frac{1}{2} M v^2 \]
\[ \begin{aligned} \vec{p} = \vec{p}_0 + M \vec{v} \end{aligned} \]
\[ E = \frac{\vec{p}^2}{2m} \]

Initial kinetic energy \( M \)-mass of the liquid of the flowing liquid

\[ E = E_p + \frac{1}{2} M v^2 \]

the change in energy due to excitations

\( E_p + \frac{1}{2} M v^2 \leq 0 \) - the energy of moving liquid must decrease \( \equiv \) energetical advantage of excitations.

\( \Rightarrow P \) and \( v \) are antiparallel

\( \Rightarrow E_p - |P|M < 0 \)

\[ |M| > \frac{E}{P} \]

This is a condition of appearing excitations.

So we should find the minimum of \( (E) \). If \( \min (E) \) is not zero, then for velocities of flow below certain value, excitations cannot appear. This means that the flow will not become slower, i.e. that the liquid exhibits superfluidity.
4. If the spectrum of excitations is 
   \[ \epsilon_p = \frac{p^2}{2m}, \]
then
   \[ \frac{\epsilon}{p} = \frac{p}{2m} \sim p. \]

\[ \min (\frac{\epsilon}{p}) = 0 \Rightarrow \text{for any value of velocity it is} \]
advantageous to have excitations.
\[ \Rightarrow \text{we will always have viscosity} \]

5. If the spectrum is linear, \( \epsilon_p = up \),
then
   \[ \frac{\epsilon}{p} = u. \]

For \( v < u \) it is disadvantageous to create excitations. Liquid will flow without dissipation!

Any liquid which has \( \epsilon(p) \sim p \), i.e. low-energy excitations are phonons, will demonstrate superfluidity at sufficiently low-T. Note that the entropy of the S&L is \( \Omega = 0 \). All liquid is in a single macroscopic state.

\( T \neq 0 \)

Then, liquid contains quasiparticles:

\[ \tilde{\eta}_p = \frac{1}{e^{\frac{\epsilon}{T}} - 1} \quad \mu = 0 \]

However, previous arguments are still valid: the motion of a liquid with some small velocity \( v \) cannot create new excitations.
If temperature is low, the number of quasiparticles will be small. Thus we can consider qP as non-interacting and can use a quasiparticle gas approximation.

Let gas of qP move relative to a liquid with velocity \( V_{\text{gas}} \).

Then the distribution is

\[
n_p = \frac{1}{e^{\frac{\epsilon - pV}{kT}} - 1}
\]

Consider a reference frame which moves together with gas:
- a) gas is at rest
- b) liquid moves with \(-V\).

\[
E = \epsilon_0 - pV + \frac{1}{2} MV^2
\]

\[
E - pV < 0
\]

\[
E + pV < 0
\]

Can not be \( \Rightarrow \) liquid with qP can not produce new excitation.

We can understand this in a different way: qPs collide with walls, loose energy and thus after some time, gas of qP will not move!

Conclusion:

at \( T \approx 0 \), we have two parts of a liquid:

a) viscous gas of qP: \( n_n \) - density of qP

b) superfluid liquid: \( n_s \) - density of sf
The total density of the liquid is

\[ \rho = \rho_n + \rho_s \]

but \( T < T_0 \)

At \( T = T_0 \), there is no QPs and \( \rho_n = 0 \).

Accordingly, the liquid has only superfluid component.

At \( T = T_0 \),

\[ \rho_n = \frac{1}{3} \mathcal{V} \int \left[ -\frac{\partial n}{\partial \epsilon} \right] \rho^2 \, d\epsilon \]

\[ \downarrow \text{QPs carry some mass!} \]

\[ \epsilon = u_0 \phi \implies \rho_n = \frac{1}{3u} \int \frac{dn}{dp} \rho^2 \left( \frac{4\pi p^2 \, dp}{(2\pi h)^3} \right) \]

\[ = \left[ \text{after integration by parts} \right] = 4 \int \frac{\epsilon nd\epsilon}{3u^2} \]

\[ \rho_s = \rho - \rho_n \]

At \( T = T_0 \), \( \rho_s = 0 \) (there is no condensate)

\[ \rho = \rho_n \]

We can also compute the roton's contribution to \( \rho_n \). Remember that rotons are described by Boltzmann distribution.

At very low-\( T \), the phonon contribution to \( \rho_n \) is large compared with roton contribution.

\[ (\rho_n)_{\text{phon}} \approx (\rho_n)_{\text{rot}} \text{ at } T \approx 0.6 \text{ K} \]