Brief summary

\[ H = H_0 - \sum_{a} \frac{e^2}{8mc^2} (\vec{H} \times \vec{r}_a)^2 \]

\[ \downarrow \text{paramagnetism} \]

\[ \mu_B = \frac{e\hbar}{2mc} \]

\[ m = -\mu_B \left( \frac{L + S}{2} \right) \]

\[ \downarrow \text{diamagnetism} \]

Weak fields:

\[ \chi = \vec{B} + \frac{\vec{A}^2}{T} \]

\[ \vec{A} = \langle k' | M_2 | k \rangle = 0 \quad \text{no spontaneous moment} \]

\[ \vec{B} = \langle B_k \rangle, \quad B_k = \sum_k | \langle k' | M_2 | k \rangle |^2 \] \[ \frac{E^{(1)}_{k'} - E^{(1)}_k}{E^{(0)}_{k'} - E^{(0)}_k} \]

\[ -\frac{e^2}{4mc^2} \sum_a \langle k | x_a^2 + y_a^2 | k \rangle \]

Inert gases: \( m = 0 \)

\[ \chi = -\frac{e^2}{6mc^2} \sum_a \langle 0 | x_a^2 | 0 \rangle \quad \text{Langevin diamagnetism} \]

\( \chi \) is independent of \( T \).
\[ m \neq 0 : \]

\[ J_{\text{para}} = \begin{cases} \frac{\mu_B^2}{3T} (4\lambda (\lambda+1) + L(L+1)), & T \gg \Delta E_x \\ \frac{\mu_B^2}{3T} g^2 J(J+1), & T \ll \Delta E_x \end{cases} \]

- paramagnetic susceptibility

Magnetism of electron gas

We always suppose \( T \ll \varepsilon_F \).

Weak fields

\[ \frac{\mu_B}{\hbar} \mathbf{H} \ll T \]

\[ \mathbf{m} = - \left( \frac{\partial \mathbf{F}}{\partial \mathbf{H}} \right)_{T, \nu, \mathbf{n}} = - \left( \frac{\partial \mathbf{L}}{\partial \mathbf{H}} \right)_{T, \nu, \mathbf{n}} \]

Two contributions in susceptibility and magnetization:

1) diamagnetic - due orbital motion of electron in an external magnetic field

2) paramagnetic - due to spin of electron
Paramagnetic contribution

a) "High" - T - Boltzmann gas

\[ J_{\text{para}} = \frac{\mu_B^2 g^2 s(s+1)}{3T} = \frac{\mu_B^2}{3T} \frac{4 \cdot \frac{1}{2} \cdot \frac{3}{2}}{1} \]

\[ g = 2 \text{ for } s = \frac{1}{2} \]

- Curie paramagnetism

b) "Low" - T - Fermi gas

\[ \varepsilon = \frac{p^2}{2m} \pm \mu_B H \]

\[ \varepsilon_{1} \uparrow \]

- Zeeman splitting

In the distribution function

\[ \begin{aligned}
\varepsilon &\rightarrow \begin{cases} 
\frac{\varepsilon_0 - \mu + \mu_B H}{2} & \text{for spin } \uparrow \\
\frac{\varepsilon_0 - \mu - \mu_B H}{2} & \text{for spin } \downarrow 
\end{cases}
\end{aligned} \]

\[ \mu \rightarrow \mu \pm \mu_B H \text{ - renormalization of the chemical potential} \]

The potential \( \mathcal{N} \) of an electron gas in a magnetic field may therefore be written as

\[ \mathcal{N} = \frac{1}{2} \left( \mathcal{N}_0 (\mu + \mu_B H) + \mathcal{N}_0 (\mu - \mu_B H) \right) \]
\( \Sigma_0(\mu) \) is the thermodynamic potential in the absence of the field.

In a weak field we can expand \( \Sigma \) in terms of \( \mu_0 H \):

\[
\Sigma(\mu) = \Sigma_0(\mu) + \frac{1}{2} \mu_0^2 H^2 \frac{\partial^2 \Sigma_0}{\partial \mu^2} + \ldots \quad \text{(linear term cancels)}.
\]

\[
\Sigma(\mu) = \Sigma_0(\mu) - \vec{m} \cdot \vec{H} \rightarrow
\]

\[
- \left( \frac{\partial \Sigma}{\partial H} \right)_{\mu_0, N} = - \vec{m} = - \vec{H} \mu_0 \right\} \cdot \left( \frac{\partial^2 \Sigma_0}{\partial \mu^2} \right) = - \text{magnetic moment}
\]

The number of electrons

\[
N = - \frac{\partial \Sigma}{\partial \mu} \Rightarrow \frac{\partial^2 \Sigma_0}{\partial \mu^2} = \left( \frac{\partial^2 N}{\partial \mu} \right)_{\mu_0, N}
\]

\[
\Rightarrow \vec{m} = \mu_0 \left( \frac{\partial^2 N}{\partial \mu} \right)_{\mu_0, N}
\]

\[
\Rightarrow \chi_{\text{para}} = \frac{\mu_0^2}{V} \left( \frac{\partial^2 N}{\partial \mu} \right)_{\mu_0, N}
\]

or relative to unit volume

\[
\chi_{\text{para}} = \frac{\mu_0^2}{V} \left( \frac{\partial^2 N}{\partial \mu} \right)_{\mu_0, N}
\]

Neglecting the temperature effects, which are small when \( T \ll T_F \), we have
\[ \frac{N}{V} = \left( \frac{2m}{\hbar^2} \right)^{3/2} = \frac{P_E}{3 \hbar^2} \quad (T=0) \]

\[ J_{\text{para}} = \frac{\mu B^2}{3 \hbar^2} \left( \frac{2m}{\hbar^2} \right)^{3/2} \left( \frac{\mu B}{\hbar} \right)^{1/2} = \frac{\mu B^2}{3 \hbar^2} \left( \frac{2m}{\hbar^2} \right)^{3/2} \frac{\mu B}{\hbar} \]

\[ B_{\text{para}} = \frac{\mu B^2}{\hbar^2} \frac{P_E m}{\hbar^2} \]

- Paramagnetism
- Pauli, (1930)

\[ J_{\text{para}} = \frac{3}{2} \frac{\mu B^2}{\hbar} \frac{N}{V} \]

Diamagnetic contribution

The Hamiltonian of a nonrelativistic electron in an external magnetic field is

\[ \mathcal{H} = \frac{1}{2m} (\vec{p} - e \vec{A})^2 \]

\( \vec{A} \) - electromagnetic vector potential, which is related to magnetic field as

\[ \mathcal{H} = \vec{D} \cdot \vec{B} = \text{curl} \vec{A} \]

This doesn't define the vector potential uniquely, we can add curl-free component to vector potential without changing magnetic field.
\[ \vec{A} \to \vec{A} + \nabla \varphi \], i.e. adding the gradient of the scalar field to \( \vec{A} \) changes the overall phase of the wave function, but does not change the field.

\[ \varphi \to \varphi - \frac{1}{c} \frac{\partial \varphi}{\partial t} \]

We choose vector potential such that

\[
\begin{align*}
Ax &= -Hy \\
Ay &= A_z = 0
\end{align*}
\]

This is called "choosing the gauge".

Then, \[ H = \frac{1}{2m} \left[ (p_x - \frac{eH}{c} y)^2 + p_y^2 + p_z^2 \right] \]

The Hamiltonian does not depend on \( x \) and \( z \) \( \Rightarrow \) \( p_x = -i\hbar \frac{\partial}{\partial x} \) commutes with \( H \)

\[ [H, p_x] = 0 \]

\[ \varphi = \tilde{\varphi}(y) e^{i\frac{\hbar}{\sqrt{2}} (p_x \cdot x + p_z \cdot z)} \]

Now we solve Schrödinger equation:

\[ H \psi = E \psi \]

\[ (-\frac{\hbar^2}{2m} \nabla^2 + V) \psi = E \psi \]

\[ x'' + \frac{2m}{\hbar^2} \left( E - \frac{p_z^2}{2m} \right) - \frac{m}{\hbar^2} \omega_0^2 (y - y_0)^2 x = 0 \]

\[ y_0 = -\frac{c p_x}{e H} \]

\[ \omega_0 = \frac{e H}{mc} \] - cyclotron frequency.
This is a quantum harmonic oscillator.

\[ E(p_z, n) = \frac{p_z^2}{2m} + \frac{\hbar \omega_0}{2} (n + \frac{1}{2}) \quad n = 0, 1, 2, \ldots \]

- \( p_z \) is the momentum along the field.
- \( \omega_0 = \frac{eH}{mc} \) - cyclotron frequency

When the external field is turned on, the energy spectrum associated with the motion in \( xy \) plane changes from a continuous spectrum to a discrete one; and the level spacing and degeneracy increases with \( H \).

\[ H = 0 \quad n \geq 0 \]

Each set of energies with the same value of \( n \) is called Landau levels.

Landau quantization in QM is the quantization of the cyclotron orbits of charged particles in a magnetic field. As a result of this quantization, the electrons can occupy only orbits with discrete energy levels, called Landau levels.
\[ p_z = 0 \]

\[ E = \hbar \omega_0 (n + \frac{1}{2}) \]

The energies do not depend on \( p_z \) and thus are hugely degenerate.

In the absence of magnetic field

\[
D(\varepsilon) = \int \frac{d^2 k}{(2\pi)^2} \delta(\varepsilon - \frac{\hbar^2 k^2}{2m}) = \]

\[
= \frac{1}{2\pi} \int_0^\infty dk \kappa \delta(\varepsilon - \frac{\hbar^2 k^2}{2m}) = \]

\[
= \frac{1}{4\pi} \int_0^\infty dE \frac{2m}{\hbar} \left( \delta(\varepsilon - E) \right) = \frac{m}{2\pi \hbar^2} \quad \text{for } \varepsilon > 0
\]

We can obtain the degeneracy of Landau levels as follows: since the total number of states does not change, the Landau levels must accommodate these states. Thus the degeneracy of the first Landau level is
\[ N_L = \int_{\text{area}}^\text{two} \int_0^\infty dE D(E) = \frac{m \text{two} L^2}{2\pi \hbar^2} \frac{e\hbar}{2\pi \hbar} \frac{L^2}{2\pi \hbar} = \frac{e\hbar}{2\pi \hbar} \frac{L^2}{2\pi \hbar} \]

At low-T, the low-energy states are filled successively until all \( N \) electrons are accommodated. If \( N = 2nN_L \), with \( n = 1, 2, \ldots \)
the lowest \( n \) Landau levels are completely filled and others are empty. Factor 2 stays for spin. In the generic case, \( \frac{N}{2N_L} \) is not an integer, the highest Landau levels is partially filled.

The total energy
\[ E = \sum_{n=0}^{\hat{n}-1} 2N_L \text{two} (n + \frac{1}{2}) + (N - \hat{n} 2N_L) \text{two} (\hat{n} - \frac{1}{2}) \]

\[ \hat{n} = \frac{N}{2N_L} \]

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\( X \times X \) \( n = 4 \)

\( X \times X \times X \) \( n = 3 \)

\( X \times X \times X \times X \) \( n = 2 \)

\( X \times X \times X \times X \times X \) \( n = 1 \)