

Creating single-atom and single-photon sources from entangled atomic ensembles

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We discuss the application of dipole blockade techniques for the preparation of single-atom and single-photon sources. A deterministic protocol is given for loading a single atom in an optical trap, as well as ejecting a controlled number of atoms in a desired direction. A single-photon source with an optically controlled beamlike emission pattern is described.

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Quantum-information science builds on the laws of quantum mechanics to transmit, store, and process information in varied and powerful ways. Advances in this field rely on our ability to manipulate coherently isolated quantum objects while eliminating incoherent interactions with the surrounding environment. It was proposed two decades ago that the availability of single-photon sources would enable secure transmission of information without risk of eavesdropping [1]. Single-photon sources have since been proposed and realized using a number of different approaches [2], and demonstrated for information transmission [3]. Another example of a physical embodiment of quantum information is provided by a single neutral atom, the internal states of which can be used to represent a quantum bit or “qubit.” Placing individual atoms on a lattice defined by optical beams and allowing them to interact is one of the approaches currently being explored for constructing a quantum computer [4].

In this Brief Report we describe some approaches for creating single-atom and single-photon sources for use in quantum information science. These ideas build on the availability of an entangled mesoscopic atomic ensemble that is prepared using dipole-dipole interactions of highly excited Rydberg atoms as described by Lukin *et al.* [5]. While Lukin *et al.* emphasized the application of mesoscopic many-atom qubits for quantum logic we discuss here how to combine many-atom entanglement with laser cooling and trapping techniques for loading a single-atom into an optical trap, as well as creating single-atom and single-photon sources. We show that both atoms and photons can be extracted with well-defined propagation directions.

Consider a collection of N atoms at positions \mathbf{r}_j each with nondegenerate ground states $|a\rangle$, $|b\rangle$, intermediate states $|e\rangle$, and highly excited Rydberg states $|r\rangle$ as shown in the inset of Fig. 1. We envision that the atoms have been laser cooled to μK temperatures, and confined to a volume of a few μm^3 defined by an optical trap created by far off-resonance optical beams (FORT trap) using standard techniques [6]. Following Ref. [5] we define the following collective atomic states: the ground state $|g\rangle = \prod_j |b_j\rangle = |b_1 b_2 \cdots b_N\rangle$, the singly excited state $|r_j\rangle = |b_1 \cdots r_j \cdots b_N\rangle$, and the doubly excited state $|r_j r_k\rangle = |b_1 \cdots r_j \cdots r_k \cdots b_N\rangle$. The interaction Hamiltonian governing the coupling of levels $|b\rangle$ and $|r\rangle$ assuming zero atom-field detuning is taken to be $\hat{V} = \hat{V}_d + \hat{V}_{dd}$ where $2V_d/\hbar = \sum_j \Omega_j |r_j\rangle \langle g| + \sum_{j,k>j} \Omega_k |r_j r_k\rangle \langle r_j| + \text{H.c.}$ describes the electric-dipole atom-field coupling

in the rotating wave approximation, and $V_{dd}/\hbar = \sum_{j,k>j} \Delta_{jk} |r_j r_k\rangle \langle r_j r_k|$ describes the dipole-dipole interaction of two excited atoms. Here $\Omega_j = -\langle r_j | \hat{d} | b_j \rangle \mathcal{E}(\mathbf{r}_j) / \hbar$, where \hat{d} is the dipole moment operator, and the position dependent optical field is $E(\mathbf{r}_j) = [\mathcal{E}(\mathbf{r}_j)/2] e^{-i\omega t} + \text{c.c.}$ The dipole-dipole shift in the case of dipole moments aligned parallel to the vector separating the atoms is given by $\Delta_{jk} = -f(n) e^2 a_0^2 / |\mathbf{r}_j - \mathbf{r}_k|^3$ with e the electronic charge and a_0 the Bohr radius. The numerical factor $f(n) \sim n^6$ for a level with principal quantum number n [7].

We will be interested in the limit where $\Delta_{jk} \gg |\Omega_j|$ so that we can safely neglect triply excited states and higher. An arbitrary N atom state vector can be written in this limit as $|\psi(t)\rangle = c_g(t) |g\rangle + \sum_j \tilde{c}_j(t) |r_j\rangle + \sum_{j,k>j} \tilde{c}_{jk}(t) |r_j r_k\rangle$. We can obtain analytical estimates of the dynamics by considering the atomic evolution due to a field that is pulsed on to excite a transition to a Rydberg state and has a spatially uniform intensity such that $\Omega_j = \Omega e^{i\phi_j}$. We neglect the atomic motion during a pulse so that ϕ_j is taken as constant. Incorporating the phases $\phi_j = \mathbf{k} \cdot \mathbf{r}_j$ for a traveling wave with wave vector \mathbf{k} into the amplitudes $c_j = \tilde{c}_j e^{-i\phi_j}$ and $c_{jk} = \tilde{c}_{jk} e^{-i(\phi_j + \phi_k)}$, assuming all coefficients c_j equal, and adiabatically eliminating the doubly excited state, the normalized singly excited symmetric state can be written as

$$|s\rangle = \frac{1}{\sqrt{N}} \sum_j e^{i\phi_j} |r_j\rangle \quad (1)$$

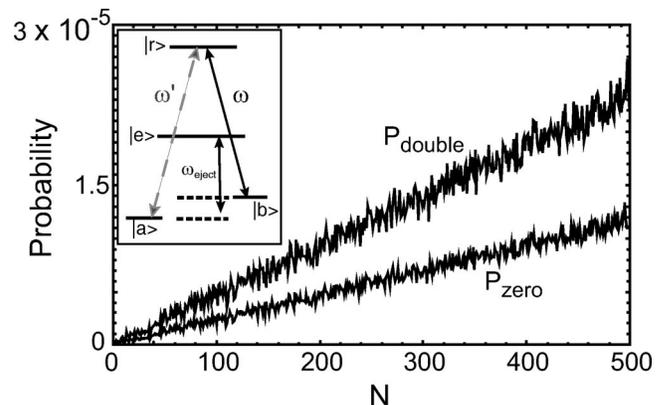


FIG. 1. Numerical calculation of the probability of nonexcited and doubly excited states after a Rydberg pulse as a function of the number of atoms. The inset shows the atomic level scheme.

and has amplitude $c_s = \sqrt{N}c_j$. The Schrödinger equation results in

$$\dot{c}_g = -i \frac{\sqrt{N}\Omega^*}{2} c_s, \quad (2a)$$

$$\dot{c}_s = -i \frac{\sqrt{N}\Omega}{2} c_g + i \frac{(N-1)|\Omega|^2}{2\bar{\Delta}_{dd}} c_s, \quad (2b)$$

with $\bar{\Delta}_{dd} = [N(N-1)/2][\sum_{j,k>j} 1/\Delta_{jk}]^{-1}$. Solving Eqs. (2) with the initial condition $c_g(0) = 1$ (this is readily achieved with optical pumping techniques) we find $|c_s(t)|^2 = l^{-1} \sin^2[(\sqrt{N}l|\Omega|/2)t]$ with $l = 1 + (N-1)^2|\Omega|^2/4N\bar{\Delta}_{dd}^2$. Thus at time $t = \pi/(\sqrt{N}l|\Omega|)$ we have rotated the ground state to the singly excited symmetric state $|s\rangle$ with probability $P_{\text{single}} = 1/l$ and since Eqs. (2) conserve probability $|c_g(t)|^2 = P_{\text{zero}} = 1 - P_{\text{single}}$. The unwanted leakage into the doubly excited states at the end of the π pulse is found by summing over the doubly excited probabilities resulting in

$$P_{\text{double}} \sim \sum_{j,k>j} |c_{jk}|^2 \sim \frac{N-1}{2l} \frac{|\Omega|^2}{\bar{\Delta}_{dd}^2}. \quad (3)$$

We can estimate the fidelity of preparation of a singly excited state starting with N trapped atoms using physical parameters of ^{87}Rb as an example. The results shown in Fig. 1 assume that the atoms are randomly distributed in a sphere of $5 \mu\text{m}$ diameter and that we are using the $n=50$ Rydberg levels. A quantum defect calculation shows that a pair of atoms with the maximum separation of $5 \mu\text{m}$ subject to a hybridizing dc field experience a dipole-dipole shift of $\Delta/2\pi \sim 100$ MHz. Using $|\Omega|/2\pi = 1$ MHz we see that the probability of creating a state with zero or two excitations grows linearly with N (the average interatomic spacing and $\bar{\Delta}_{dd}$ stay roughly constant as atoms are added) and remains less than 3×10^{-5} for up to $N=500$ atoms.

After creating the singly excited state we apply a π pulse at ω' to transfer the single-atom excitation to the lower ground state $|a\rangle$. The collective atomic state after this pulse sequence is proportional to $(1/\sqrt{N})\sum_{j=1}^N e^{i\phi_j} |b_1 \cdots a_j \cdots b_N\rangle$, which has the same functional form as Eq. (1). It can be shown that corrections to the N atom results due to spontaneous emission are $O(N\gamma_R/\bar{\Delta}_{dd})$ with γ_R the Rydberg level spontaneous decay rate. The correction is negligible for experimental conditions of interest with up to several thousand atoms. It should be noted that it is essential for high fidelity preparation that the pulses be applied sequentially. Applying both ω and ω' simultaneously leads to large amplitude multiple excited leakage from $|b\rangle$ to $|a\rangle$ due to multiphoton Raman processes.

One application of this many-atom entangled state is to load an array of optical traps with a single-atom in each one for use as a quantum computer [4]. In that application the dipole blockade mechanism can also be used between adjacent qubit sites to implement two-qubit logic gates [8]. With N atoms in a given lattice site we use the above procedure to prepare the state $|a\rangle$. To eject the $N-1$ atoms remaining in

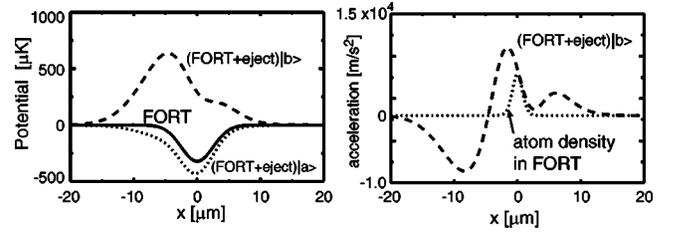


FIG. 2. Potentials(left) and acceleration(right) for atom ejection with the eject beam centered $3 \mu\text{m}$ to the left of the FORT beam. Calculations for ^{87}Rb with $w_{\text{FORT}} = 5 \mu\text{m}$, $P_{\text{FORT}} = 100$ mW, $\lambda_{\text{FORT}} = 1.06 \mu\text{m}$, $w_{\text{eject}} = 10 \mu\text{m}$, $P_{\text{eject}} = 9 \mu\text{W}$, $(\omega_{\text{eject}} - \omega_{eb})/2\pi = 1$ GHz, and $T_{\text{atomic}} = 30 \mu\text{K}$.

state $|b\rangle$ we apply an optical beam that causes strong mechanical forces on $|b\rangle$ but only weak forces on $|a\rangle$. One possibility is to use a beam with a waist a few times larger than the FORT beam waist w_{FORT} and with a frequency ω_{eject} that is tuned to the red of ω_{ea} and to the blue of ω_{eb} as shown in Fig. 1. This gives repulsive gradient forces on $|b\rangle$ and attractive forces on $|a\rangle$ as shown numerically in Fig. 2. In the left-hand plot we see that the combined potential from the FORT beam and the eject beam is strongly repulsive for $|b\rangle$, but attractive with only a small shift in the position of the minimum for $|a\rangle$. The net acceleration of atoms in state $|b\rangle$ due to the FORT and eject beams is positive causing motion to the right over the entire region occupied by the atom cloud. After a characteristic time t_1 determined by $\frac{1}{2}at_1^2 = w_{\text{FORT}}^2$ with $a = F/m$ the net acceleration, atoms in state $|b\rangle$ will be swept free of the FORT potential, leaving a single atom in state $|a\rangle$ behind. Using the parameters in Fig. 2 we get $t_1 \sim 40 \mu\text{s}$. Note that since the indistinguishable atoms are in an entangled superposition state we cannot speak of 1 atom left behind in the trap and $N-1$ ejected until a measurement or decoherence has caused projection of the wave function. Finally it should be emphasized that although several experiments have achieved single-atom loading in optical traps, they have relied on stochastic loading into extremely small volumes [9]. High fidelity single-atom loading is essential for filling a large number of traps in a neutral atom quantum processor [10].

As suggested in Ref. [5] a deterministic beam of atoms can also be generated using the entangled ensemble. We use a protocol very similar to that described above for single-atom loading but now start by optically pumping the trapped atoms to state $|a\rangle$. A sequence of π pulses on ω' and ω will transfer a single excitation to $|b\rangle$ which can be ejected from the trap as described above. Pulses containing several atoms can be created using multiply excited states. Numerical estimates using the parameters in the preceding paragraph show that kHz rate atomic pulses can be created.

The direction of the ejected atoms will be determined by the positioning of the eject beam with respect to the FORT, which provides a means of scanning the ejected atoms. The transverse spread of the atomic beam will be determined by gradients in the eject beam, as well as fluctuations due to spontaneous emission. For the 1 GHz detuning used in the numerical example of Fig. 2 the number of photons scattered from atoms in $|b\rangle$ during a $t_1 \sim 40 \mu\text{s}$ eject pulse is n_{scat}

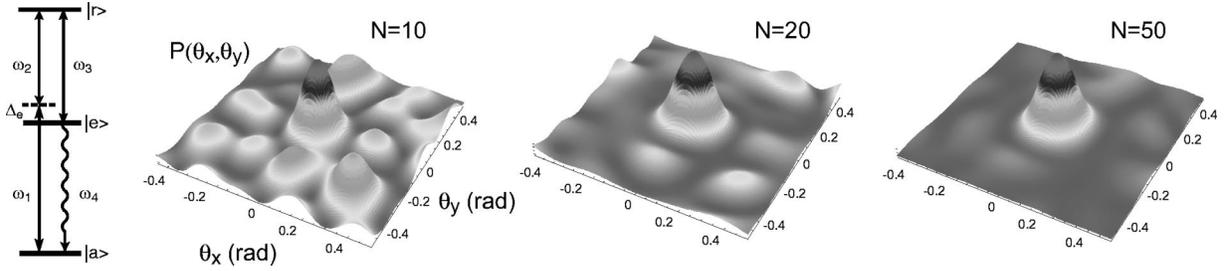


FIG. 3. Fields used for state preparation and angular emission probabilities (arb. units on linear scale) at $\lambda_4 = 0.78 \mu\text{m}$ for $N = 10, 20$, and 50 atoms randomly positioned within a $5 \mu\text{m}$ diameter sphere.

~ 21 . (The corresponding number of photons scattered from atoms in $|a\rangle$ for which the detuning is -5.8 GHz is only about 0.6.) This results in an rms momentum transfer due to spontaneous emission that is about a tenth of the coherent impulse after an eject time t_1 . The emitted beam will therefore be well collimated. It is also feasible to move atoms a controlled distance using traveling dipole force fields as demonstrated recently in Ref. [11]. The flexibility of optical control of the beam direction opens the potential of many applications in areas that include optical lattice neutral atom quantum computers, atomic interferometers, precision low-level current sources, and also nanofabrication tasks at the single-atom level.

We turn now to the creation of a phased array single-photon source with a diffraction-limited emission pattern. When the optical fields propagate through the sample, atoms will be excited with position dependent relative phases as in Eq. (1). This results in an entangled state with a phase structure that mimics the phase of the exciting beam and can be used to create a single-photon source with well-defined directionality. While previous work has achieved single-photon sources with a controlled emission direction by coupling to microcavities [12] our approach results in a source emission pattern that is reconfigurable and is defined by the structure of the preparation optical fields. Referring to Fig. 3, fields at ω_1 and ω_2 with Rabi frequencies Ω_1 and Ω_2 drive a two-photon transition $|a\rangle \rightarrow |e\rangle \rightarrow |r\rangle$ to the Rydberg level $|r\rangle$ in an N -atom ensemble. The effective Rabi frequency for the two-photon process acting on atom j is $|\Omega|e^{i\phi_1+i\phi_2} = (|\Omega_1|e^{i\phi_1(\mathbf{r}_j)}|\Omega_2|e^{i\phi_2(\mathbf{r}_j)})/2\Delta_e$, where ϕ_1 and ϕ_2 are the phases of fields ω_1 , ω_2 at the atomic position \mathbf{r}_j and $\Delta_e = \omega_1 - \omega_{ea}$. As long as P_{double} given by Eq. (3) is small only transitions to states with a single excited atom are energetically allowed and we have an effective dipole blockade. Under these conditions an ensemble of atoms in $|a\rangle$ subjected to a π -pulse applied on $|a\rangle \rightarrow |r\rangle$ produces the entangled symmetric superposition state $|\psi\rangle = (-i/\sqrt{N})\sum_j e^{i(\phi_{1j} + \phi_{2j})}|r_j\rangle$. The phases $\phi_{mj} = \mathbf{k}_m \cdot \mathbf{r}_j$ are simply the phase of the m th laser field ($\sim e^{i(\mathbf{k}_m \cdot \mathbf{r} - \omega_m t)}$) at the position \mathbf{r}_j of the j th atom.

We now apply a π -pulse with ω_3 tuned to the $|r\rangle \rightarrow |e\rangle$ single-photon transition. The wave function is then transformed into

$$|\psi\rangle = \frac{-1}{\sqrt{N}} \sum_j e^{i(\phi_{1j} + \phi_{2j} - \phi_{3j})}|e_j\rangle. \quad (4)$$

This state will radiate into a variety of modes with all the atoms in state $|a\rangle$ and a single-photon propagating in direction \mathbf{k}_4 . The amplitude for emission into state $|g, 1_{\mathbf{k}_4}\rangle = |a_1 \cdots a_j \cdots a_N, 1_{\mathbf{k}_4}\rangle$ is proportional to

$$\begin{aligned} & \langle g, 1_{\mathbf{k}_4} | (\mathbf{e}_{\mathbf{k}_4} \cdot \hat{\mathbf{d}}) \hat{a}_{\mathbf{k}_4}^\dagger e^{-i\mathbf{k}_4 \cdot \mathbf{r}} | \psi, 0_{\mathbf{k}_4} \rangle \\ &= - \sum_j \frac{\langle g | \mathbf{e}_{\mathbf{k}_4} \cdot \hat{\mathbf{d}} | e_j \rangle e^{-i(\mathbf{k}_4 - \mathbf{k}_1 - \mathbf{k}_2 + \mathbf{k}_3) \cdot \mathbf{r}_j}}{\sqrt{N}}, \end{aligned} \quad (5)$$

where $\mathbf{e}_{\mathbf{k}_4}$ is the polarization of the emitted photon and $\hat{a}_{\mathbf{k}_4}^\dagger$ is the creation operator for a photon in mode \mathbf{k}_4 . Assuming an isotropic angular distribution from the atomic matrix element gives for the angular distribution

$$P(\mathbf{k}_4) \propto \frac{1}{N} \left| \sum_j e^{-i(\mathbf{k}_4 - \mathbf{k}_1 - \mathbf{k}_2 + \mathbf{k}_3) \cdot \mathbf{r}_j} \right|^2. \quad (6)$$

Note that the emission is highly directional; in the direction $\mathbf{k}_4 = \mathbf{k}_1 + \mathbf{k}_2 - \mathbf{k}_3$ the phases are zero for each atom giving $P(\mathbf{k}_4) = N$ while for other directions the phase factors become random and $P(\mathbf{k}_4) \sim 1$. A calculation of the angular distribution, assuming an isotropic atomic matrix element, is shown in Fig. 3. The width of the central emission peak is determined by diffraction. For $N = 50$ the numerical value for the FWHM agrees to better than 10% with the diffraction estimate of λ/D , where D is the diameter of the phased atom cloud.

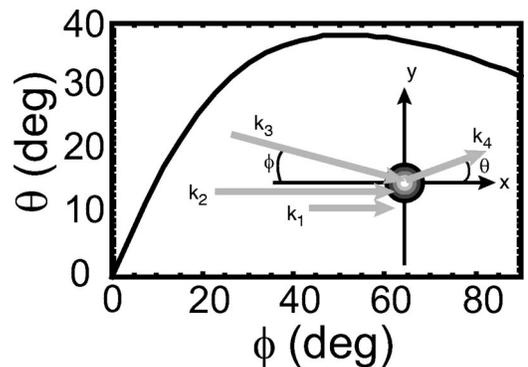


FIG. 4. Angular direction θ of the photon emitted along \mathbf{k}_4 for collinear Rydberg excitation beams \mathbf{k}_1 , \mathbf{k}_2 , and \mathbf{k}_3 tilted by an angle ϕ .

It is evident from Eq. (6) that the angular pattern of the emitted single-photon beam bears a close analogy with four-wave mixing. If we arrange for $\mathbf{k}_2 = -\mathbf{k}_1$ the photon will be emitted in a phase conjugate mode with $\mathbf{k}_4 = -\mathbf{k}_3$. For a single-photon source it is more convenient to have the emitted photon angularly separated from the other beams as shown in Fig. 4. By setting the waist size of the \mathbf{k}_1 , \mathbf{k}_2 , \mathbf{k}_3 beams to be much larger than the size of the atomic distribution all atoms will to a good approximation be driven by the same optical intensities so that the accuracy of the preparation Rabi pulses will be high. The angular divergence of the emitted pulse will be determined by the size of the atomic cloud, as seen in Fig. 3.

Several factors can contribute to angular broadening of the emitted mode. The atomic motion during a $3 \mu\text{s}$ sequence of preparation pulses, accounting for the characteristic speed of $30 \mu\text{K}$ ^{87}Rb atoms, is $\Delta x \sim 0.15 \mu\text{m}$, which is about 1/5 of the emission wavelength. The contribution of

this motional dephasing to broadening of the emission mode is a subject of current study. Imperfect dipole blockade leading to the doubly excited states can also degrade the single-photon fidelity. We note that this will be strongly suppressed by angular selection since the contribution to emission along \mathbf{k}_4 due to a doubly excited Rydberg state channel is proportional to $(1/N) |\sum_j e^{i(k_4 - 2(k_1 + k_2) + k_3)}|^2$, which is not phase matched along the direction θ selected in Fig. 4. Finally we note that by employing an excited level $|e\rangle$ that is not dipole coupled to the ground state it will be possible to create entangled two-photon pairs with similar beamlike emission properties.

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