DEMONSTRATION OF RYDBERG BLOCKADE AND A NEUTRAL ATOM CNOT GATE

by

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Chapter 1

Introduction

This thesis covers the research we have done to produce a working neutral atom quantum logic gate which is an essential component of a neutral atom quantum computer. To achieve this goal many diverse techniques and innovative ideas were combined to reach the first demonstration of a quantum Controlled-NOT (CNOT) gate using neutral atoms. These include elements from the fields of quantum mechanics, optics, lasers, radio frequency (RF) devices, electronic feedback and control, and are accompanied with theoretical work in all these areas to understand the many interactions that we see in our experiment on a daily basis. This chapter will discuss the motivations of our work and include summaries of other work done in developing working quantum computers. The second chapter covers the necessary theoretical background to understand the interactions we use between neutral atoms, lasers, and the environment. The third chapter covers the experimental apparatus we use to manipulate and control the target atoms. The final chapter covers the results we have achieved to this point and will also describe the focus of our future work.

1.1 Quantum Computing

1.1.1 What is Quantum Computing

Quantum Mechanics has led to many revolutions in physics and technology over the last century. Developments over the last 30 years have led to direct observation of macroscopic quantum effects such as those seen in Bose-Einstein Condensates and superconductors. Quantum mechanics has become ever more important as researchers push technology and devices to smaller sizes. This means that quantum mechanical descriptions are required to explain many of the effects seen in these new technologies. However only recently have any of these effects been directly used to try to make a quantum computer. A quantum computer uses some quantum system to store information in a quantum state called a qubit. The quantum nature of these states allows them to be in superpositions of the computational states usually labeled $|0\rangle$ and $|1\rangle$. This effectively allows quantum computers to perform parallel operations because all possible states can be represented during a calculation. This parallelism enables a quantum computer to solve certain problems far more efficiently than a classical computer. For example algorithms have been developed that allow impressive speed improvements in searching unsorted lists [Grover, 1997] and factoring very large numbers [Shor, 1994]. Also quantum computers allow for simulations of quantum systems which is valuable as these simulations become intractable on classical computers as the number of simulated particles increases. All these plus the high likelihood of future applications are driving the extremely rapid developments in quantum computing and quantum information processing.

The requirements for a quantum computer were formally put together in [DiVincenzo, 2001]. There are five criteria a system needs to meet:

- 1. Scalable system with well characterized qubits
- 2. Ability to initialize the qubits to some simple state
- 3. Decoherence times longer than any relevant gate times
- 4. Universal set of quantum gates

5. Qubit specific state measurement

The first point relates to the physical system that will be chosen to represent the $|0\rangle$ and $|1\rangle$ states used in computations. The system chosen needs to be able to scale to large numbers of qubits without putting any exponential demand on necessary resources. If this is not possible then the benefits of quantum computation may be cancelled by the costs to implement a system large enough to perform the desired computations. Points 2, 3, 4, and 5 all relate to the operations performed on the qubits. To perform computations it is necessary to setup a desired input state followed by operating a set of gates that implement the quantum algorithm and finally perform a measurement of the system to record the output of the algorithm. Initialization is done by first putting all the qubits into some initial state, usually $|0\rangle$ or $|1\rangle$, followed by individual single qubit gates to put each qubit into the desired initial state. Next the quantum algorithm is implemented, which requires a universal set of gates. A universal set of gates can be made from just a two qubit gate, such as the CNOT gate, and from single qubit gates. There are other sets as well which allow the implementation of any quantum algorithm or provide advantages by reducing the total number of gates needed to be performed in an algorithm [Nielsen and Chuang, 2000]. The long decoherence time implies that multiple gate operations can be performed without loss of the quantum information so that it can either be readout or corrected using error correction schemes after performing some algorithm. Finally any computation needs to have a way to record the output. So a procedure that can measure a specific qubit state, either $|0\rangle$ or $|1\rangle$, is needed. Then the algorithm is repeated several times or run in parallel on many qubits to measure the probability for the qubit to be in each state.

It is likely that a quantum network of some type will be beneficial, if not necessary, to scale a specific quantum computer to sizes that can perform the computations needed for some quantum algorithms. This is essentially the same thing that is done with current classical computers where many individual computing cores are connected to a memory system or network effectively combining the computing ability of each core to make a

single much larger computer or supercomputer. There are two additional criteria given by DiVincenzo related to implementing a quantum network. These are:

- 6. Ability to convert stationary qubits to "flying" qubits
- 7. Ability to transfer "flying" qubits to a specific location

A "flying" qubit is a qubit that can be moved between two areas. Producing a "flying" qubit could be done either using one of the quantum computer's qubits such as has been suggested for ion quantum computing in segmented ion traps [Home et al., 2009] or by copying the information of a qubit onto a different physical qubit such as a photon and then transfer that between the quantum computers. This also implies that the ability to convert the information in this "flying" qubit back to information stored in a computing or memory qubit is necessary. The basics of such a quantum network have been demonstrated in various teleportation experiments [Olmschenk et al., 2009]. Many different physical implementations exist which satisfy some or all of these quantum computing criteria to some degree. Each system has its own strengths and weaknesses in regards to these criteria. Some of the leading systems will be briefly described in the next section.

1.1.2 Overview of current QC implementations

Many quantum systems have been pursued in the search for a quantum computer. Initially molecules manipulated using nuclear magnetic resonance (NMR) techniques were used to demonstrate many of the control schemes necessary for quantum computing. Other systems followed quickly such as: ions, neutral atoms, quantum dots, superconducting circuits, defects in certain crystals, linear optics, etc. When looking at a specific system to implement, important aspects to balance include how to create or trap a qubit, what resources are needed to manipulate the internal states and reduce external perturbations, what the timescales for gate operations are, what can be done to reduce the decoherence mechanisms, how the qubit states are measured, and what the associated costs are for each of these decisions. Each of the systems have their own advantages and disadvantages. For example: scalability is well understood for quantum dots and superconducting circuits, ions have shown the highest fidelities for state detection and preparation and have also demonstrated gates above 99% fidelity [Benhelm et al., 2008], and neutral atoms and ions have shown long coherence times due to their small interactions with the environment [Langer et al., 2005]. At this point no single system has been shown to have complete advantage over the others. Each system has weaknesses that need to be reduced before a fault-tolerant quantum computer can be built.

In the ion and neutral atom qubits, the computational basis is represented using specific electronic levels in the atoms. These are usually the hyperfine ground states or other long lived states. Many different ions and atoms can be used and each provides some advantage but also some disadvantage. These electronic states are manipulated using lasers or microwave radiation. Several implementations for both ions and atoms have shown that gate speeds can be on the order of 10³ times faster than the decoherence time [Yavuz et al., 2006] [Langer et al., 2005]. This means that many gate operations should be possible before needing to readout and re-initialize the qubits or perform error correction. Both ions and atoms have shown the ability to perform nearly perfect state detection. For example the Boulder group has achieved readout fidelities of 99.94% with ions by using a repeated quantum non-demolition measurement [Hume et al., 2007]. In addition ions have been used to perform high fidelity quantum gates and to perform several quantum gates in sequence while maintaining good fidelity [Hanneke et al., 2010] [Benhelm et al., 2008].

Superconducting qubits have more recently become successful due to improved fabrication techniques. Superconducting qubits rely on Josephson junctions but can operate in several different modes depending on their design. These different modes are related to charge, flux, or phase and each type has a different circuit design. The charge qubit uses an island of superconductor called a Cooper Pair Box that is connected to a nearby relatively large superconductor called a resevoir by a Josephson junction. Nearby electrodes change voltages or bias magnetic fields to control the tunneling of Cooper pairs into or out of the Cooper Pair Box. To measure the charge state a single electron transistor is used after a readout pulse is applied to move the Cooper pair from the box to the measurement electrode [Pashkin et al., 2009]. The flux qubit is based on the quantized flux in a superconducting loop that contains one or more Josephson junctions. Here other loops nearby vary magnetic fields to manipulate the flux in the qubit loop. This flux in the qubit loop is usually measured using a SQUID device located near the qubit [Friedman et al., 2000]. The phase qubit is based on an inductor-capacitor resonator but adds in a Josephson junction to allow for non-linear behaviors. This non-linearity provides a shift to the energy levels causing the spectrum to be anharmonic. This way the resonator can be an effective two level system as long as the anharmonicity is large enough [Martinis, 2009]. In addition attempts to combine the best characteristics of these basic qubits have led to new designs. For example, a design based on a charge qubit but taking some of the good behaviors from phase qubits was introduced and called a transmon qubit. This name comes from the fact that the qubit sits in a transmission line. This design nearly eliminates the sensitivity to charge noise and also improves the coupling of the qubit to a photon due to the qubit being part of a transmission line [Koch et al., 2007].

A transmon qubit has been used to perform a two qubit gate and even to implement a simple quantum algorithm [DiCarlo et al., 2009], and a phase qubit has been used to generate entanglement with a fidelity of 88% and show the violation of Bell's inequality [Ansmann et al., 2009]. All of these superconducting Josephson qubits have decoherence times that are on the order of 1 μ s and gates times that are about 20 ns. The ratio of these timescales gives a figure of merit of current implementations on the order of 10². This does not yet match the ion or neutral atom qubits' figure of merit of better than 10³ but is still high enough to demonstrate simple quantum algorithms.

Reasearch into other solid state systems is also showing significant progress. Some examples are quantum dots and defects in crystal structures [Hanson and Awschalom, 2008]. Quantum dots attempt to isolate a single electron in a specific location and manipulate the spin using lasers or nearby electrodes. Systems using defects use techniques from NMR and lasers to manipulate the spin of unbound electrons around a location where a different element has replaced an atom in a crystal lattice causing a defect. Both single and two qubit gates have been shown in these systems, but typically these solid state systems have short coherence times that are currently limiting the gate fidelities. Just as for the Josephson qubits new designs and fabrication techniques are making very rapid progress and other candidates are being found which may lead to better systems. Specifically the nitrogen vacancy centers in diamond look promising as they have very long coherence times compared to other solid state systems and have already been used for single and two qubit gates [Jelezko et al., 2004].

The last major system being pursued for quantum computing is in optics [Kok et al., 2007]. In this system the spatial or polarization states of photons are used to carry the qubit information. Through the use of phase shifters, waveplates, and beamsplitters all of the single qubit gates are fairly straightforward to implement. The difficulty lies in making a 2 qubit gate. Photons interact very weakly with each other making 2 qubit gates difficult. Two major approaches have been suggested. One uses a non-linear effect in a crystal to directly produce the phase shifts necessary to make a controlled phase or similar quantum gate. The second method relies on the quantum behavior of photons to perform a quantum gate probabilisticly and make measurements on ancilla qubits to know when the gate operation succeeded. The problem with the direct method is that a system with a non-linear effect large enough to be practical has not been found yet. The probabilistic method has been used to perform controlled phase and controlled NOT gates but there are still concerns with the costs of scaling this to a large number of qubits. Another difficulty with optical quantum computing is the on demand production of qubits. Single photon sources exist but they do not always produce a photon on demand in the desired mode. A recent result demonstrated a measured efficiency of .72 and that can theoretically reach >.9[Claudon et al., 2010]. So very soon this problem may no longer be important.

Another problem is the losses inherent to the many optical elements needed to run a large scale calculation may be too high to have a practical success rate when scaling to a large number of qubits.

As each of these various quantum systems are developed work is being done to find ways to combine the best aspects of each system into a single computer that may actually allow for faster implementation and better scaling of quantum computers. For instance a computer taking advantage of the very fast gate times of superconducting qubits could be combined with the long lifetimes of ions or atoms to store information until it is needed similar to the differences between processor caches and RAM in current classical computers. It would also be possible to build a quantum network between smaller quantum processors to combine them into a single super quantum computer [Kim and Kim, 2009]. For example: a superconducting qubit coupled to a microwave cavity can transfer that information to an ion or atom which then transfers the information to an optical photon with the aid of an optical cavity. This photon is sent through an optical fiber where the reverse process can transfer the information to a separate quantum processor.

These diverse approaches to quantum computing show the very broad interest in achieving quantum computation. This research has given many techniques to other researchers just as techniques found in other experiments have been adopted by quantum computing groups. The current progress towards a fault-tolerant quantum computer will allow simple quantum simulations and algorithms in the near future and these should grow ever more complex as devices and techniques improve.

1.2 Error Correction

Quantum computers all have sources of error that cannot be completely removed. In classical computers and information processing many different methods of correcting these errors have been developed. These methods do not directly translate to the quantum world due to the differences between quantum and classical mechanics. However, some methods can be adapted to a quantum system and many ideas from classical error correction are helpful in designing quantum error correction. The basic goal for quantum error correction is to be able to correct any errors accumulated during a computation so that the final result is reliable. To perform error correction it is necessary to encode the information. This usually involves storing the information for a single logical qubit in multiple individual qubits. With this a measurement of the individual qubits can be used to guess the correct state and if the error rate is low enough this guess will be accurate. A good introduction to this can be found in [Nielsen and Chuang, 2000] with more recent developments [Steane, 2003, Knill, 2005, Aliferis et al., 2008, Aliferis and Preskill, 2009].

Many versions of error correcting codes exist. A simple code that can correct for arbitrary single qubit errors uses nine qubits to encode a single logical qubit and was introduced in [SHOR, 1995]. This code actually looks like a double encoding. One layer encodes an initial qubit as three qubits to allow for corrections of phase flip errors, where the phase of a single qubit is flipped e.g. from $|0\rangle + |1\rangle$ to $|0\rangle - |1\rangle$. Then each of these encoded qubits are also encoded into another set of three qubits to allow for correction of bit flip errors where the state of a qubit is flipped. This encoding scheme is shown in Figure 1.1 and results in logical qubits given by the states

$$\begin{split} |0\rangle|0\rangle|0\rangle & \underline{\text{phase encoding}} \frac{(|0\rangle + |1\rangle)(|0\rangle + |1\rangle)(|0\rangle + |1\rangle)}{2\sqrt{2}} \equiv |+++\rangle \\ |+\rangle|0\rangle|0\rangle & \underline{\text{bit flip encoding}} \frac{(|000\rangle + |111\rangle)(|000\rangle + |111\rangle)(|000\rangle + |111\rangle)}{2\sqrt{2}} \equiv |0\rangle_L \\ |1\rangle_L \equiv \frac{(|000\rangle - |111\rangle)(|000\rangle - |111\rangle)(|000\rangle - |111\rangle)}{2\sqrt{2}}. \end{split}$$

The error correction is performed by first correcting the bit flip errors in the second layer and then correcting any phase flip errors in the first encoding layer. To see how an error can be detected and corrected, we can look at just the bit flip correction. A logical qubit of the bit flip code is represented in this code as $\frac{|000\rangle+|111\rangle}{\sqrt{2}}$. A bit flip error would flip the state of a bit i.e. $\frac{|000\rangle+|111\rangle}{\sqrt{2}} \Rightarrow \frac{|010\rangle+|101\rangle}{\sqrt{2}}$ if the second bit was flipped. To detect this a measurement of Z_1Z_2 and Z_2Z_3 where Z is the Pauli matrix $\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ is performed. These measurements written as an operator are

$$Z_1 Z_2 = (|00\rangle \langle 00| + |11\rangle \langle 11|) \otimes I - (|01\rangle \langle 01| + |10\rangle \langle 10|) \otimes I$$
$$Z_2 Z_3 = I \otimes (|00\rangle \langle 00| + |11\rangle \langle 11|) - I \otimes (|01\rangle \langle 01| + |10\rangle \langle 10|)$$

which just compares the state of the first and second qubits or the second and third qubits. If the states are the same then this measurement is equal to 1 if they are opposite then the result is -1. From this it is easy to find the bit that flipped and to correct the error by flipping the state of that bit. The phase errors are checked using $X_1X_2X_3X_4X_5X_6$ and $X_4X_5X_6X_7X_8X_9$ and corrected by applying a phase rotation to the appropriate group of qubits. There are many different encodings using many different numbers of qubits to encode a logical qubit. A seven qubit encoding originally described in [Steane, 1996] is one of the most popular. It turns out that five qubits are the minimum number needed to encode a logical qubit that is able to correct all single qubit errors [Nielsen and Chuang, 2000].

Another question for error correction is how well it can perform. It turns out that applying the same error correction code in multiple layers allows an arbitrary precision to be reached assuming that the individual gates exceed a certain error threshold [Nielsen and Chuang, 2000]. What the actual value for this threshold is depends on the encoding used, calculation capabilities, and noise assumptions that are used. Converting an operation to a fault tolerant model requires adding error correction after every step in a calculation. This includes steps where the qubit is just stored for some time. The qubit encoding and operations to a quantum algorithm. So it is important to ensure that this additional overhead needed to reach the desired accuracy is reasonable. It is found that the number of gates needed to simulate an algorithm requiring p(n) gates with a precision of ϵ is on the order

$$O\left(\operatorname{poly}\left(\log\left(\frac{p(n)}{\epsilon}\right)p(n)\right)\right)$$

where poly means a polynomial scaling. Therefore, the number of gates only scales polylogarithmically [Nielsen and Chuang, 2000]. Recent results suggest that the minimum



Figure 1.1 Shor nine qubit encoding for error correction of arbitrary single qubit errors.

error threshold for a scalable architecture lies between 10^{-4} and 10^{-2} [Steane, 2003, Knill, 2005, Aliferis et al., 2008, Aliferis and Preskill, 2009]. The final threshold really depends on what is considered scalable. Handling gate errors above 10^{-3} requires a very large number of qubits to encode the multiple levels of error correction needed which may make this high error rate impractical. For example using the scheme in [Knill, 2005], a computation needing 10^8 gates for a perfect quantum computer needs: four levels of encoding, about 10^5 times more quantum gates to perform a fault tolerant computation, and about 10^5 physical qubits for every logical qubit if using maximum parallelism. Fortunately if the computational system used has a good quantum memory so that only a moderate amount of parallelism is needed then the number of qubits can be reduced to be only about 400 per logical qubit. If the gate error is reduced to 10^{-4} , then only a level 3 encoding is needed, the gate overhead is drops to 10^{-3} , and the minimum number of qubits is about 100. If the error per gate is lower there are more efficient schemes than this one in terms of the required overheads.

Unfortunately current experiments are only capable of holding enough qubits for one or two logical qubits. Even so, error correction and some simple quantum algorithms have been successfully demonstrated [Cory et al., 1998, Chiaverini et al., 2004, Gulde et al., 2003]. As new ideas and techniques are found, these systems will steadily improve, and the future looks very promising for quantum computing.

Chapter 2

Neutral Atom Quantum Computing

2.1 Neutral atom qubit

Many options exist for the states to use to represent a qubit in neutral atoms. Many different choices need to be weighed to choose what parameters are most important for a certain experiment. Some are based purely on the element and specific isotope chosen such as the number of ground states, laser wavelengths and linewidths needed to cool the atoms and manipulate the electronic states, and how easily that isotope can be cooled and trapped. At the moment the alkali's are the most commonly chosen elements due to their single valence electron and well developed cooling techniques, but many proposals and some work has been done on many other elements that offer their own specific electronic states that will be used to represent the computational basis need to be selected. This leads to balancing things such as field sensitivites, lifetimes, and resources needed to perform state manipulations. An example of this kind of analysis can be found in [Saffman and Walker, 2005].

In the alkalis the qubits are generally a pair of states in the ground state hyperfine manifolds. A common choice is a set of states that have no first order Zeeman shift. This leads to choosing between the $m_F = 0$ clock states which have no shift at zero magnetic field, and states that have $\Delta m_F = 1$ or 2 work at various bias magnetic fields. The criteria used to choose between these possibilities depends on how the single and multiple qubit gates will be implemented. For example coherent Rabi oscillations can be performed by



Figure 2.1 ⁸⁷Rb ground state hyperfine level structure. The two computational states we have used in our experiments are circled.

two photon Raman transitions up to $\Delta m_F = 2$, but single photon excitation can only be used for $\Delta m_F = 0$ or 1 transitions. It is not necessary to limit the choice to these insensitive states, but work to shield the qubits from the environment or encode the qubits into decoherence free subspace will probably be necessary to allow high fidelity gates and qubit storage.

In our system we have used two different computational basis states from the ⁸⁷Rb ground states: the clock states $|F, m_F\rangle = |1, 0\rangle$ and $|2, 0\rangle$ and the stretched states $|1, 1\rangle$ and $|2, 2\rangle$. These states are shown in the Rb level diagram in Figure 2.1. For some time we used the stretched states because initializing the qubit to the $|2, \pm 2\rangle$ state is simple and relatively insensitive to slight experimental errors in polarization or bias magnetic field. This basis allowed for studies of single qubit gates between the two ground states and also with studies of coherent excitation and blockade of Rydberg levels which we use for our two qubit gate [Johnson et al., 2008, Urban et al., 2009]. However when trying to actually perform a two qubit gate in our system, the stretched state is too sensitive to small magnetic field fluctuations which leads to decoherence and prevented us from seeing any significant gate operation. Therefore we changed to the clock states where we were able to demonstrate a CNOT gate [Isenhower et al., 2010].



Figure 2.2 Bloch Sphere examples. a) A quantum state $\frac{|0\rangle+|1\rangle}{\sqrt{2}}$ depicted in the Bloch sphere representation. b) The red dashed circle shows a rotation of a state about the polar axis of the Bloch sphere which is equivalent to a phase gate.

2.2 Single Qubit gates

Now that we have defined our qubit, we will discuss how we manipulate the states of a single qubit. Our qubit can be modeled well as a two level system. To help understand the evolution of quantum states in a two level system the concept of the Bloch sphere was introduced. The Bloch sphere places the $|0\rangle$ and $|1\rangle$ states at the south and north poles of a unit sphere. Then a vector pointing to a point on the surface can represent all possible quantum states of the system. Single qubit quantum gates then simply become rotations around some axis of this sphere. Several methods exist to perform these rotations. The rotations can be driven directly with electromagnetic radiation at the transition frequency or can be driven through higher order processes such as two photon Raman transitions. In both cases control of the phase, intensity, and length of a manipulation pulse are needed to generate an arbitrary quantum state.

Furthermore any arbitrary rotation can be broken down into two rotations about specific axes which can simplify the system used to manipulate the qubit. One rotation is about the polar axis which is usually labeled the z axis and is performed by changing the relative phase between the two qubit states. A rotation by an angle ϕ about this axis can be written in matrix form as

$$\mathbf{U}_{z}\left(\phi\right) = \left(\begin{array}{cc} 1 & 0\\ 0 & e^{i\phi} \end{array}\right)$$

in this representation the state of the qubit is represented by a vector $|\psi\rangle = a|0\rangle + b|1\rangle \equiv \begin{pmatrix} a \\ b \end{pmatrix}$. The second rotation can be considered a rotation about an axis pointing through the equator of the Bloch sphere. A rotation by an angle θ is given by

$$\mathbf{U}_{R}\left(\theta\right) = \left(\begin{array}{cc} \cos\left(\theta\right) & -i\sin\left(\theta\right) \\ -i\sin\left(\theta\right) & \cos\left(\theta\right) \end{array}\right).$$

Fortunately instead of needing to perform two arbitrary single qubit rotations, any arbitrary rotation can be made from a set of only two discrete rotations [Nielsen and Chuang, 2000]. An example set of two rotations is the so called $\pi/8$ gate and the Hadamard gate which are represented as

$$\mathbf{U}_{\pi/8} = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\frac{\pi}{4}} \end{pmatrix}$$
$$\mathbf{H} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

The Hadamard gate can be made by performing first a rotation $U_R(\pi/4)$ followed by a phase rotation of $U_z(\pi/2)$. So if we can demonstrate these two gates we can implement any arbitrary single qubit rotation.

To implement these two rotations we take advantage of the interaction between light and a two level system. The system evolution is given by the Schrodinger equation

$$i\hbar\frac{\partial}{\partial t}|\psi\left(t
ight)
angle=\hat{H}\left(t
ight)|\psi\left(t
ight)
angle$$

where $|\psi(t)\rangle = c_0(t) e^{-i\omega_0 t} |0\rangle + c_1(t) e^{-i\omega_1 t} |1\rangle$ is the wavefunction for a two level system and $\hat{H}(t) = \hat{H}_0 + \hat{V}_d(t)$ is the system Hamiltonian plus a perturbation due to the light. These two terms are written as

$$\begin{split} \hat{H_0} &= \hbar \omega_0 |0\rangle \langle 0| + \hbar \omega_1 |1\rangle \langle 1| \\ \hat{V_d}\left(t\right) &= -\hat{d} \cdot \vec{E}\left(t\right) |0\rangle \langle 1| - \hat{d} \cdot \vec{E}\left(t\right) |1\rangle \langle 0| \end{split}$$

where $\hbar \omega_i$ is the energy of the *i*th state, \hat{d} is the dipole operator, and $\vec{E}(t)$ is the electric field of the light. We assume the light interacting with the two level system only has a single frequency

$$\vec{E}(t) = \frac{\mathcal{E}_0}{2}\vec{\epsilon}e^{-i\omega t} + \frac{\mathcal{E}_0^*}{2}\vec{\epsilon}e^{i\omega t}$$

Putting these into the Schrodinger equation we find

$$\dot{c_0} = i \frac{d_{01}E_0}{\hbar} e^{-i\omega_{10}t} c_1$$
$$\dot{c_1} = i \frac{d_{10}E_0}{\hbar} e^{-i\omega_{01}t} c_0$$

where we defined $d_{ij} = \langle i | \hat{d} \cdot \vec{\epsilon} | j \rangle$ with $\vec{\epsilon}$ the polarization vector of the electric field and $\omega_{ij} = \omega_i - \omega_j$. We now put in the electric field and make the rotating wave approximation to find

$$\dot{c_0} = i \frac{\Omega^*}{2} c_1 e^{i\Delta t}$$
$$\dot{c_1} = i \frac{\Omega}{2} c_0 e^{-i\Delta t}$$

where $\Omega^* = \frac{d_{01}\mathcal{E}_0^*}{\hbar}$ is the Rabi frequency and $\Delta = \omega - \omega_{10}$ is the detuning of the light from the transition frequency between the two states. This set of differential equations can be solved to find the evolution matrix $\mathbf{c}(t) = \mathbf{U}(t-0) \mathbf{c}(0)$

$$\mathbf{U} = \begin{pmatrix} e^{\frac{i\Delta t}{2}} \left(\cos\left(\frac{\Omega' t}{2}\right) - i\frac{\Delta}{\Omega'} \sin\left(\frac{\Omega' t}{2}\right) \right) & ie^{\frac{i\Delta t}{2}} \frac{\Omega^*}{\Omega'} \sin\left(\frac{\Omega' t}{2}\right) \\ ie^{-\frac{i\Delta t}{2}} \frac{\Omega}{\Omega'} \sin\left(\frac{\Omega' t}{2}\right) & e^{-\frac{i\Delta t}{2}} \left(\cos\left(\frac{\Omega' t}{2}\right) + i\frac{\Delta}{\Omega'} \sin\left(\frac{\Omega' t}{2}\right) \right) \end{pmatrix}$$

where $\Omega' = \sqrt{|\Omega|^2 + \Delta^2}$. If we rewrite this when $\Delta = 0$ we find

$$\mathbf{U}_{\Delta=0} = \begin{pmatrix} \cos\left(\frac{|\Omega|t}{2}\right) & i\sin\left(\frac{|\Omega|t}{2}\right) \\ i\sin\left(\frac{|\Omega|t}{2}\right) & \cos\left(\frac{|\Omega|t}{2}\right) \end{pmatrix}$$

which is a rotation in the opposite direction of U_R shown earlier. So by controlling Ω and t we can actually perform arbitrary rotations about this axis.

To perform a phase gate we need to change the relative phase of the two qubit states. Recalling the wavefunction $|\psi(t)\rangle = c_0(t) e^{-i\omega_0 t} |0\rangle + c_1(t) e^{-i\omega_1 t} |1\rangle$ we can move to a frame rotating with the $|0\rangle$ state to get $e^{i\omega_0 t} |\psi\rangle = c_0 |0\rangle + c_1 e^{-i(\omega_1 - \omega_0)t} |1\rangle$. This shows that the $|1\rangle$ state rotates with a frequency $\omega_1 - \omega_0$ relative to the $|0\rangle$ state. If we perturb the energy difference between the two states then this frequency would become $\omega_1 - \omega_0 + \delta$. So controlling the perturbation allows for a known phase shift to be introduced. This energy shift can be performed by letting light interact with the system to produce an AC Stark shift given by $\frac{|\Omega|^2}{4\Delta}$.

Unfortunately for our chosen qubit, the hyperfine ground states of ⁸⁷Rb, the transition frequency is at ~6.8 GHz. This microwave radiation has a large wavelength of 4.4 cm so it cannot be focused in a way to give site specific rotations at spacings that are relevant to our experiment. So some other method is needed to spatially select which qubit will interact with the light. One method is to use some other field to shift just a single qubit into resonance at a time. This could be done by applying a gradient magnetic or electric field to the whole trapping region or by using a laser beam to selectively Stark shift a single qubit [Schrader et al., 2004]. Another option, which is our chosen method, is to use a higher order process such as a two photon Raman process where the ground states are coupled using radiation that is slightly detuned from some third level. This allows optical wavelengths to be used which gives the high resolution needed to resolve single sites at reasonable separations. Unfortunately, this method does introduce an error due to spontaneous emission from the excited state. To minimize this error the radiation from the Raman beam is detuned far enough from the excited state to reduce the probability of spontaneous emission to an acceptable level.

The changes due to using a two photon excitation require revisiting the earlier derivation. We need to use a third level to couple our chosen qubit states. Since we want to use light with optical wavelengths this third level is at higher energy than the two qubit states, which is called a Λ system. Here I will give a brief review of a full derivation of the interaction between a Raman beam and an atom which can be found in [Pedersen, 2006]. The relevant levels, laser fields, and detunings are shown in Figure 2.3. The Raman beam is a laser with two frequencies, ω_a and ω_b , that are separated by approximately the hyperfine splitting. These frequencies are detuned from the excited state by Δ_a and Δ_b giving a two photon detuning of $\Delta_{ab} = \Delta_a - \Delta_b$ when ignoring any AC Stark effects. We start the derivation for this system just as we did for the two level system. First we write out the Hamiltonian for this system with the Hamiltonian split into two parts $H = H_0 + V_d(t)$

$$H_0 = \hbar\omega_0 |0\rangle \langle 0| + \hbar\omega_1 |1\rangle \langle 1| + \hbar\omega_p |p\rangle \langle p|$$

$$\hat{V}_{d}(t) = -\hat{d} \cdot \vec{E}_{a}(t) |0\rangle \langle p| - \hat{d} \cdot \vec{E}_{a}(t) |p\rangle \langle 0| - \hat{d} \cdot \vec{E}_{b}(t) |1\rangle \langle p| - \hat{d} \cdot \vec{E}_{b}(t) |p\rangle \langle 1|$$

This perturbation comes from the light field

$$\vec{E} = \vec{E}_a(t) + \vec{E}_b(t) = \frac{\mathcal{E}_a}{2}\vec{\epsilon_a}e^{-i\omega_a t} + \frac{\mathcal{E}_a^*}{2}\vec{\epsilon_a}e^{i\omega_a t} + \frac{\mathcal{E}_b}{2}\vec{\epsilon_b}e^{-i\omega_b t} + \frac{\mathcal{E}_b^*}{2}\vec{\epsilon_b}e^{i\omega_b t}.$$

Since this system has three levels the wavefunction becomes

$$|\psi(t)\rangle = c_0(t) e^{-i\omega_0 t} |0\rangle + c_1(t) e^{-i\omega_1 t} |1\rangle + c_p(t) e^{-i\omega_p t} |p\rangle.$$

Again these are put into the time dependent Schrodinger equation to give a set of differential equations

$$\dot{c_0} = i \frac{d_{0p}E_a}{\hbar} e^{-i\omega_{p0}t} c_p$$
$$\dot{c_p} = i \frac{d_{p0}E_a}{\hbar} e^{-i\omega_{0p}t} c_0 + i \frac{d_{p1}E_b}{\hbar} e^{-i\omega_{1p}t} c_1$$
$$\dot{c_1} = i \frac{d_{1p}E_b}{\hbar} e^{-i\omega_{p1}t} c_p$$

where $d_{ij} = \langle i | \hat{d} \cdot \vec{\epsilon} | j \rangle$ with $\vec{\epsilon}$ the polarization vector for the light and $\omega_{ij} = \omega_i - \omega_j$. Just as for the two level system we define $\Omega_a = \frac{\mathcal{E}_a d_{p0}}{\hbar}$ and $\Delta_a = \omega_a - \omega_{p0}$ as well as the similar terms. Making these substitutions and applying the rotating wave approximation leads to

$$\dot{c_0} = i \frac{\Omega_a^*}{2} e^{i \Delta_a t} c_p$$

$$\dot{c_p} = irac{\Omega_a}{2}e^{-i\Delta_a t}c_0 + irac{\Omega_b}{2}e^{-i\Delta_b t}c_1$$
 $\dot{c_1} = irac{\Omega_b^*}{2}e^{i\Delta_b t}c_p$

To simplify the solutions we change to writing the equations in matrix form and rotate to a new frame where $\tilde{c} = Rc$ with

$$\mathbf{R} = \begin{pmatrix} e^{-i\frac{\Delta_{ab}}{2}t} & 0 & 0\\ 0 & e^{i\frac{\delta}{2}t} & 0\\ 0 & 0 & e^{i\frac{\Delta_{ab}}{2}t} \end{pmatrix}$$

where $\Delta_{ab} = \Delta_a - \Delta_b$ and $\delta = \Delta_a + \Delta_b$. To rewrite the differential equation in this new frame we find

$$\frac{d}{dt}\mathbf{c} = \mathbf{A}\mathbf{c} \Rightarrow \frac{d}{dt}(\mathbf{R}\mathbf{c}) = \mathbf{R}\mathbf{A}\mathbf{R}^{-1}\mathbf{R}\mathbf{c}$$
$$\Rightarrow \mathbf{\tilde{c}} = \mathbf{R}\mathbf{A}\mathbf{R}^{-1}\mathbf{\tilde{c}} - \frac{d\mathbf{R}}{dt}\mathbf{c}$$

where **A** is the coefficient matrix for the set of differential equations. This rotation gives a new coefficient matrix which is time independent.

$$\tilde{\mathbf{A}} = \mathbf{R}\mathbf{A}\mathbf{R}^{-1} - \frac{d\mathbf{R}}{dt}\mathbf{R}^{-1} = \begin{pmatrix} -\frac{i\Delta_{ab}}{2} & \frac{i\Omega_{a}^{*}}{2} & 0\\ \frac{i\Omega_{a}}{2} & \frac{i\delta}{2} & \frac{i\Omega_{b}}{2}\\ 0 & \frac{i\Omega_{b}^{*}}{2} & \frac{i\Delta_{ab}}{2} \end{pmatrix}$$

We would like to remove the intermediate level from the calculation so we assume that $\Delta_a, \Delta_b \gg \Omega_a, \Omega_b$ which means that c_p will always remain near zero. This allows us to say that $\dot{c_p} = 0$. This is sometimes called adiabatic elimination of the intermediate state or slaving the intermediate state. This assumption allows us to write $c_p = -\frac{\Omega_a}{\delta}c_0 - \frac{\Omega_b}{\delta}c_1$ and reduce the system to an effective two level system

$$ilde{\mathbf{A}}' = \left(egin{array}{cc} -rac{i\Delta_1}{2} & -rac{i\Omega_R^*}{2} \\ -rac{i\Omega_R}{2} & rac{i\Delta_2}{2} \end{array}
ight)$$

where $\Delta_1 = \Delta_{ab} + \frac{|\Omega_a|^2}{\delta}$, $\Delta_2 = \Delta_{ab} - \frac{|\Omega_b|^2}{\delta}$, and $\Omega_R = \frac{\Omega_a \Omega_b^*}{\delta}$. This system can be solved to find the evolution matrix in the rotating frame

$$\tilde{\mathbf{U}} = e^{-i\frac{|\Omega_a|^2 + |\Omega_b|^2}{4\delta}t} \times \begin{pmatrix} \cos\left(\frac{\Omega't}{2}\right) - i\frac{\Delta'}{\Omega'}\sin\left(\frac{\Omega't}{2}\right) & -i\frac{\Omega_R^*}{\Omega'}\sin\left(\frac{\Omega't}{2}\right) \\ -i\frac{\Omega_R}{\Omega'}\sin\left(\frac{\Omega't}{2}\right) & \cos\left(\frac{\Omega't}{2}\right) + i\frac{\Delta'}{\Omega'}\sin\left(\frac{\Omega't}{2}\right) \end{pmatrix}$$

where we defined $\Omega' = \sqrt{\Delta'^2 + |\Omega_R|^2}$ and $\Delta' = \Delta_{ab} + \frac{|\Omega_a|^2 - |\Omega_b|^2}{\delta}$. Finally this solution can be rotated back to the normal frame using $\mathbf{U}(t, 0) = \mathbf{R}(t)^{-1} \tilde{\mathbf{U}}(t, 0) \mathbf{R}(t = 0)$ giving

$$\mathbf{U} = e^{-i\frac{|\Omega_a|^2 + |\Omega_b|^2}{4\delta}t} \times \begin{pmatrix} e^{i\frac{\Delta_ab^t}{2}}\left(\cos\left(\frac{\Omega't}{2}\right) - i\frac{\Delta'}{\Omega'}\sin\left(\frac{\Omega't}{2}\right)\right) & -ie^{i\frac{\Delta_ab^t}{2}}\frac{\Omega_R^*}{\Omega'}\sin\left(\frac{\Omega't}{2}\right) \\ -ie^{-i\frac{\Delta_ab^t}{2}}\frac{\Omega_R}{\Omega'}\sin\left(\frac{\Omega't}{2}\right) & e^{-i\frac{\Delta_ab^t}{2}}\left(\cos\left(\frac{\Omega't}{2}\right) + i\frac{\Delta'}{\Omega'}\sin\left(\frac{\Omega't}{2}\right)\right) \end{pmatrix}$$

In order to get full excitation we need to have $\Delta' = 0 = \Delta_{ab} + \frac{|\Omega_a|^2 - |\Omega_b|^2}{\delta}$ which means that the correct frequency difference for the Raman beams is not equal to the hyperfine splitting. A correction must be made to account for the differential AC Stark shift of the qubit states. In addition since $\Delta_{ab} \neq 0$, this interaction gives both a rotation about the polar axis and a rotation about an equitorial axis in the Bloch sphere picture. To show this explicitly we assume $\Delta' = 0$ and then rewrite the evolution matrix as

$$\mathbf{U} = e^{-i\frac{|\Omega_a|^2 + |\Omega_b|^2}{4\delta}t} e^{i\frac{\Delta_{ab}t}{2}} \times \begin{pmatrix} 1 & 0 \\ 0 & e^{-i\Delta_{ab}t} \end{pmatrix} \times \begin{pmatrix} \cos\left(\frac{\Omega_R t}{2}\right) & -i\sin\left(\frac{\Omega_R t}{2}\right) \\ -i\sin\left(\frac{\Omega_R t}{2}\right) & \cos\left(\frac{\Omega_R t}{2}\right) \end{pmatrix}$$

which shows that there is a global phase shift of $\phi_{global} = \frac{\Delta_{ab}t}{2} - \frac{|\Omega_a|^2 + |\Omega_b|^2}{4\delta}t$, a single qubit phase gate with $\phi = -\Delta_{ab}t$, and a rotation about an equitorial axis of $\theta = \frac{\Omega_R t}{2}$.

It is important to note that $\Delta' = 0$ only when the AC Stark shifts due to the Raman beams have been corrected. This can be done in several ways: first the frequency difference between the two Raman beams with equal Rabi rates can be chosen to correct the shift, a different laser with the same spatial profile but a different detuning can be used to cancel the shift [Haffner et al., 2003], or Raman beams with unequal Rabi rates can be used [Haze et al., 2009]. For our current implementation we have chosen the first method


Figure 2.3 The relevant energy levels for ⁸⁷Rb shown with the two photon Raman beams we use.

which means we need to account for this phase shift when we perform single qubit gates. This choice has further implications for when the Raman beam is off. The frequency difference of the Raman beams only matches the qubit difference when the beams are interacting with the qubits. This means that the Raman beam and qubit will accumulate a relative phase shift when the beams are not interacting with the qubits. This fact provides an advantage in that the implementation of a phase gate is simple, but a disadvantage that the phase shift for every qubit must be calculated in order to perform the desired rotations. To describe how we implement the phase gate we consider the frequency difference of the Raman beams to be the reference clock of our system then by turning the beams off for a certain length of time the qubit will accumulate a phase shift relative to this clock. By keeping track of this phase accumulation we can implement any phase gate we consider the Raman beams we can perform arbitrary single qubit rotations.

2.2.1 Single qubit gate errors

There are several sources of error for our single qubit gates. These include: spontaneous emission from the intermediate state, intensity or phase noise on the Raman beams, crosstalk errors from beams targetting neighboring sites, and errors due to atomic motion in the Raman beams. The errors from all of these can be reduced to levels that allow for fault tolerant quantum computing. This discussion follows that found in [Saffman and Walker, 2005]. First we will discuss the errors due to spontaneous emission. To find the probability for spontaneous emission we just need to evaluate

$$P_{se}\left(t\right) = \frac{1}{\tau} \int_{0}^{t} \left|c_{p}\right|^{2} dt$$

where τ is the lifetime of the intermediate state which is 26.2 ns [Volz and Schmoranzer, 1996]. In the previous section we assumed that $c_p = -\frac{\Omega_a}{\delta}c_0 - \frac{\Omega_b}{\delta}c_1$ when we have $\Delta_a, \Delta_b \gg \Omega_a, \Omega_b$. Using this and the expressions for c_0 and c_1 we found in the previous section we find that the probability of spontaneous emission for a π pulse is at most

$$P_{se}\left(\pi\right) = \frac{4\pi}{\Delta_{a}\tau}$$

for the case that the initial qubit state was $\frac{|0\rangle+|1\rangle}{\sqrt{2}}$. Our experiment uses $\Delta_a = -100$ GHz giving $P_{se}(\pi) = 8 \times 10^{-4}$. The next largest errors are due to the intensity noise and atomic motion. The error due to intensity noise as given in [Saffman and Walker, 2005] is $E = (1/4) (\delta I/I)^2$. The intensity noise of our Raman beams over the time for a single pulse, ~ 1 μs , is close to the shot noise limit but over longer time periods of a few minutes needed to make a measurement the beams have a measured intensity noise around 10^{-2} which gives an error of about 10^{-4} . Similarly the atomic motion in the FORT sites leads to intensity variations. This error is found to be $E = (1/4) \langle \epsilon \rangle^2$ where

$$\langle \epsilon \rangle^2 = \frac{\pi^2}{4} \left(\frac{T_a}{T_m} \right)^2 \left(\frac{w_f}{w} \right)^4$$

where T_a is the atom temperature, T_m is the depth of the FORT site, w_f is the beam waist of the FORT, and w is the beam waist of the Raman beams. Our typical parameters have

Error Source	Value
Spontaneous Emission	$8 imes 10^{-4}$
Raman Beam Intensity Noise	10^{-4}
Atomic Motion	$5 imes 10^{-5}$
Crosstalk	$3 imes 10^{-4}$
Total	$1.2 imes 10^{-3}$

Table 2.1 Table of the main errors for our single qubit operations.

 $T_a/T_m \simeq .05$ and $w_f/w \simeq .44$ resulting in an error from atomic motion that is about 5×10^{-5} . The final significant error is due to crosstalk when addressing a neighboring site. This has a fidelity error given by $E = \frac{\pi^2}{16} e^{-4\frac{d^2}{w^2}} = 3 \times 10^{-4}$ for our typical parameters.

We also need to understand how well the qubit states can be stored in our apparatus. The decoherence that limits the storage time is caused by variations in the hyperfine splitting of the qubit states. These variations are mainly due to magnetic field fluctuations and atomic motion withing the FORT trapping potential. The decoherence due to the atomic motion in the FORT was modeled in [Kuhr et al., 2005] where they found that

$$T_{2,ls}^* = .97 \frac{2U_0}{\delta_0 k_B T}$$

where U_0 is the peak FORT potential, δ_0 is the peak differential light shift of the FORT, and *T* is the atom temperature. In our system we have measured *T* to be 200 μ K and have calculated that $\delta_0 = U_0 \times (1.5 \text{ kHz/mK})$. With these values the calculated decoherence time is $T_{2,ls}^* = 6.5$ ms. The decoherence time due to magnetic field fluctuations depends upon the qubit basis that is chosen. The hyperfine splitting for the clock states is given by

$$\omega_{hf} = \omega_0 \sqrt{1 + \left(rac{\left(g_s - g_I\right)\mu_B B}{\hbar\omega_0}
ight)^2}$$

where ω_0 is the zero field hyperfine splitting, g_s and g_I are the electron and nuclear g factors, μ_B is the Bohr magneton, and *B* is the magnetic field. To find the sensitivity at a slight bias field this is expanded about a small bias field value. Doing this for our bias

field of 3.7 G and finding the frequency shift due to a magnetic field fluctuation δB

$$\delta_{hf} = 4.2\delta B + .57 \left(\delta B\right)^2$$

where δB is in gauss and the resulting δ_{hf} is in kHz. The decoherence time for this process is given by $T_{2,B}^* = 1/\delta_{hf}$ [Saffman and Walker, 2005]. As shown in Figure 4.4, we measured a decoherence time of 2.4 ms for single atoms in our apparatus. Using this and the calculated decoherence time due to the FORT differential light shift, we can estimate the decoherence time due to magnetic field fluctuations

$$\frac{1}{T_{2,B}^*} = \frac{1}{T_{2,ls}^*} - \frac{1}{T_{2,exp}^*}$$

giving $T_{2,B}^* = 3.8$ ms. This decoherence time corresponds to a magnetic field fluctuation of .06 G. To improve this decoherence time we will need to cool the atoms further and add circuits to reduce the magnetic field fluctuations.

2.2.2 Composite pulses and Spin echos

There are several mechanisms that can cause errors in the single qubit rotations. These can be put into two broad categories: amplitude noise and phase noise. The amplitude noise can be caused by intensity noise of the Raman beams, beam pointing noise, and the random motion of the atoms in the FORT due to their non-zero temperature. Phase noise can be caused by phase noise in the RF source used to drive the Raman laser, magnetic or electric field fluctuations, interactions with other laser fields, or differences in optical paths traveled by the two Raman beams. To correct for these error sources the idea of composite pulses was introduced. These were first performed in NMR experiments and have also been used by the ion quantum computing research groups [Haffner et al., 2008, Khaneja et al., 2005]. The idea behind composite pulses is to convert a single desired rotation into several different rotations designed to reduce the sensitivity to a specific type of error. Since we have the ability to implement arbitrary rotations with our Raman beam system, using composite pulse sequences may help to reduce our sensitivity to some errors.

A very simple and commonly used example of this is the spin echo technique where a π pulse is used to swap the two qubit state set of experimental pulses. This allows errors that are slower than the experiment time to accumulate for the first half and then cancel back out during the second half. In this case the noise is considered phase noise that is constant over a single experiment cycle but varies as the cycles are repeated to gather statistics. Without using the spin echo pulse the result of an experiment can be modeled as

$$\rho = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\phi} \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & e^{-i\phi} \end{pmatrix} = \begin{pmatrix} a & be^{-i\phi} \\ ce^{i\phi} & d \end{pmatrix}$$

where ϕ will vary from measurement to measurement. Now if instead we apply a π Rabi oscillation pulse to swap the qubit states half way through the experiment we find

$$\rho = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\frac{\phi}{2}} \end{pmatrix} \begin{pmatrix} 0 & -i \\ -i & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & e^{i\frac{\phi}{2}} \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & e^{-i\frac{\phi}{2}} \end{pmatrix} \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & e^{-i\frac{\phi}{2}} \end{pmatrix}$$
$$= \begin{pmatrix} d & c \\ b & a \end{pmatrix}$$

which is the original state with the states swapped but no errors. More robust pulse sequences exist and they only require that the pulse amplitude and phase be constant for the duration of the sequence rather than be constant over a measurement cycle like the spin echo sequence.

2.3 Methods for multiple qubit neutral atom gates

Many different methods have been suggested for implementing two qubit gates in neutral atoms: dipole-dipole interactions [Brennen et al., 1999], collisions [Jaksch et al., 1999], photon mediated gates [PELLIZZARI et al., 1995], and Rydberg state interactions [Jaksch et al., 2000]. So far only the Rydberg blockade method has produced a gate between individually addressed atoms. A method based on collisions has been used in ensembles to demonstrate a \sqrt{SWAP} gate and entanglement [Anderlini et al., 2007]. All of these gates have specific sources of error that must be taken into account. These errors



Figure 2.4 Schematic representation of the photon mediated gate setup. Based on the schematic in [PELLIZZARI et al., 1995]

also generally place a limit on the speed of the gate to achieve the necessary fidelities for fault tolerant computation.

The collisional gate for two qubits is performed by first loading the atoms into an optical lattice or array of dipole traps and cooling them into the ground motional state. Next the atoms are initialized into the desired input states. Then the gate is performed by moving the chosen atoms so that their trapping regions overlap for a controlled length of time and then moved apart again. During the interaction time the two qubits coherently swap their states. By choosing the correct interaction time a SWAP operation can be performed. To perform entanglement an interaction that only does a partial SWAP, called a \sqrt{SWAP} gate, is performed. For the interaction time to match the desired length the motion of the atoms must be completely controlled. This requires that the atoms be in the motional ground state of the traps and the motion of the traps used to perform the gate must be adiabatic in regards to the trap frequencies so the atoms are not excited out of the ground state. These requirements limit the gate frequency to be lower than the trap oscillation frequency. This method lends itself to generating many entangled pairs in a single operation. The \sqrt{SWAP} operation was demonstrated just this way using many atoms in an optical lattice by the NIST group [Anderlini et al., 2007]. Collisions are also the chosen interaction for several similar schemes such as spatially delocalized qubits [Mompart et al., 2003].

The photon mediated gate is performed by using a photon to transfer information from one qubit to another qubit. Typically this is envisioned as a chain of atoms held in a linear chain of dipole traps which sits in a high finesse cavity [PELLIZZARI et al., 1995]. Addressing beams come from another direction to select the atoms for the gate. The atoms have a transition between a qubit state and an excited state strongly coupled to the cavity and the other qubit state is coupled to the excited level by the addressing beam. Using carefully chosen pulses of the addressing beams allows a gate to be performed between the chosen atoms with high fidelity. Photon mediated gates need a strong coupling between the atoms and a single photon in the cavity mode. To avoid errors due to spontaneous emission and cavity decay, the pulses implement an adiabatic transfer using a cavity dark state. This choice requires that the pulses need to meet the conditions: gT, $\Omega T \gg 1$ and g, $\Omega \gg \kappa$, Γ , where g is the coupling strength between the atoms and the cavity, T is the pulse length, Ω is the addressing laser Rabi frequency, κ is the cavity decay time, and Γ is the spontaneous emission rate. The value of g can be on the order of 100's of MHz [Gehr et al., 2010] and the same is true for Ω so this method looks promising and should be capable of MHz speed gates.

The Rydberg blockade mediated gate is performed by attempting to excite the control qubit to the Rydberg level performing a 2π rotation on the target qubit and returning the control qubit to the ground state. This results in the $|1\rangle$ state of the target atom getting a π phase shift depending on the state of the control atom. This controlled phase is then converted to a CNOT by placing a Hadamard gate on the target atom before and after the controlled phase operation as shown in Figure 2.5. Other methods also exist to implement a CNOT and are discussed further in Section 2.5. This gate is limited by the lifetime of the Rydberg level and the amount of blockade shift between the two qubits. The gate speed must find a balance between running fast enough to avoid the Rydberg decay and running slow enough to have good blockade. Using higher Rydberg levels increases the Rydberg lifetime and blockade but also increases the sensitivity to the environment and reduces the Rabi flopping rate. This method has demonstrated a CNOT gate with a measured population probability fidelity of .73 and gate time of 10 μ s [Isenhower et al., 2010] and recent improvement has reached F = .85 with only 6 μ s needed for the gate.



Figure 2.5 Hadamard and C_z CNOT gate pulse sequence.

2.4 Rydberg Atoms and Blockade

We chose to pursue the Rydberg mediated two qubit gate. The higher gate speeds and low sensitivity to the atoms' temperature seems to offer a better platform than using a collision mediated gate, and the scalability is more obvious than the photon mediated method. To discuss this version in more detail first a description of the relevant Rydberg atom properties will be given.

A Rydberg atom is one where an electron has been excited to a high energy level, usually greater than 20. The wavefunctions and energy levels can be calculated using quantum defect theory [Seaton, 1958]. Rydberg levels have a limited lifetime which introduces errors to the gate. The lifetime is governed by the radiative lifetime, $\tau_{nl}^{(0)}$, and an additional term due to blackbody radiation, $\tau_{nl}^{(bb)}$, which can photoionize or transfer the Rydberg state to other states [Gallagher, 1994].

$$\frac{1}{\tau_{nl}} = \frac{1}{\tau_{nl}^{(0)}} + \frac{1}{\tau_{nl}^{(bb)}}$$

The states we use are $nD_{5/2}$ states in ⁸⁷Rb because we use two photon excitations and these states have higher Rabi frequencies than the nS or $nD_{3/2}$ states we can access. For this case the lifetime is given by

$$\frac{1}{\tau_{nl}} = \frac{1}{\tau_l \left(n^*\right)^{\alpha_l}} + \frac{4\alpha^3 k_B T}{3\hbar n^2}$$

where $\tau_l = 2.09$ ns, $n^* = n - \delta = n - 1.35$ is the quantum defect corrected principal quantum number, $\alpha_l = 2.85$, α is the fine structure constant, k_B is Boltzmann's constant, and *T* is the surrounding system temperature [Gallagher, 1994]. Calculating the lifetime for the 97D_{5/2} level gives a lifetime of ~300 μ s.

The Rydberg interaction occurs because of either a direct dipole-dipole interaction between two different Rydberg levels or by the Forster mechanism. The Forster mechanism involves couplings to nearby Rydberg levels. In this case a pair of Rydberg excited atoms are coupled to different Rydberg levels by the dipole interaction. This interaction leads to strong couplings because these nearby states have a total energy close to the initial



Figure 2.6 a) Energy diagram for the Rydberg energy levels near n=90. From [Saffman et al., 2010]. b) Plot of the blockade shift for $97D_{5/2}$ and double excitation probability. The solid line is the blockade shift and $y_1 - y_2$ is the distance between the atoms along the FORT axis. The long dashed line is a calculation of the probability distribution for $y_1 - y_2$ due to the thermal distribution of the atoms. The short dashed line is the double excitation probability.

pair of states. For example a pair of *nd* Rydberg levels are nearly degenerate to a set of (n + 2) p + (n - 2) f and (n + 1) p + (n - 1) f energy levels. This interaction can be thought of as a multiple level system being coupled by the dipole interaction.

$$\begin{pmatrix} \delta \cdot I_{\chi} & V_{dd} \\ V_{dd}^{\dagger} & 0 \cdot I_{\varphi} \end{pmatrix} \begin{pmatrix} |\chi\rangle \\ |\varphi\rangle \end{pmatrix} = \Delta \begin{pmatrix} |\chi\rangle \\ |\varphi\rangle \end{pmatrix}$$

where $|\chi\rangle$ represents the nearby Rydberg states with $n_a l_a j_a + n_b l_b j_b$ and $|\varphi\rangle$ represents the initially excited Rydberg states nlj + nlj. This problem leads to an eigenvalue problem given by

$$rac{V_{dd}^{\dagger}V_{dd}}{\Delta-\delta}|arphi
angle=\Delta|arphi
angle$$

The dipole interaction has several terms that are common to all of them which we separate out. First is the matrix elements $C_3 = e^2 \langle r \rangle_{nl}^{n_a l_a} \langle r \rangle_{nl}^{n_b l_b}$ and the other is the $1/R^6$ radial dependence. So with these substitutions we have

$$V_{dd}^{\dagger}V_{dd}|\varphi
angle=rac{C_{3}^{2}}{R^{6}}D_{arphi}|arphi
angle$$

where D_{φ} is the remaining part of the eigenvalues which typically vary between 0 and 1. Finally using all this we can find the energy eigenvalues

$$\Delta(R) = \frac{\delta}{2} - \operatorname{sign}(\delta) \sqrt{\frac{\delta^2}{4} + \frac{C_3^2}{R^6}} D_{\varphi}.$$

The theory behind these interactions can be very involved because of the large number of states that must be considered in a full description of this interaction. Many of these intricacies are covered in a recent paper [Walker and Saffman, 2008]. However, many of the consequences of this interaction can still be discussed by just defining an average blockade shift

$$\frac{1}{B^2} = \frac{2}{N\left(N-1\right)} \sum_{\varphi, k < l} \frac{\left|\kappa_{\varphi kl}\right|^2}{\Delta_{\varphi kl}^2}$$

where *N* is the number of atoms in the region, the $\Delta_{\varphi kl}$ are the energy eigenvalues of the Forster Hamiltonian just given, and $\kappa_{\varphi kl} = \frac{4}{\hbar^2 \Omega^2} \langle \varphi kl | H_k H_l | g \rangle$ is the overlap of the states

being excited with the Forster eigenstates. The blockade shift depends strongly on the geometry of the excitation beams, atoms, and any external fields as well as the choice of level used. A calculation of the average blockade shift after accounting for the thermal distribution of the atoms gives B = 4.8 MHz. With this average blockade shift we can look at the probability to excite two atoms in an ensemble to the same Rydberg state

$$P_2 = \frac{N-1}{N} \frac{\left|\Omega_N\right|^2}{2B^2}$$

where $\Omega_N = \sqrt{N}\Omega$ is the collectively enhanced Rabi frequency. For the case of our blockade gate between two single atoms the double excitation probability is

$$P_2 = \frac{|\Omega|^2}{2B^2} = .018$$

2.5 CNOT Based on Blockade

The large Rydberg atom interactions allow for mulitple qubit gates to be implemented. There are two regimes in which these gates are typically discussed. First is the large interaction regime where a single Rydberg excitation blocks completely any other attempted Rydberg excitations. Second is the small interaction regime where two qubits are excited to a Rydberg level and allowed to evolve for a controlled length of time before being returned to the ground state. Both methods allow for controlled phase shifts of the qubits which can be converted into a full CNOT quantum gate. The blockade regime provides one significant advantage: low sensitivity to the strength of the blockade. This means that the qubits can be at different distances from each other, and achieve a high fidelity gate as long as the interaction is still strong enough. This feature allows for a large number of directly coupled qubits and removes the need to be in the ground state of the optical traps. These benefits plus the ability to perform gates at MHz rates is why we chose to pursue the Rydberg blockade CNOT gate.

The Rydberg blockade CNOT gate starts with a controlled phase gate performed using the following steps: one state of the control qubit, e.g. the $|1\rangle$ state, is coupled to a Rydberg level sending that part of the population to the Rydberg level using a π pulse, the target qubit receives a 2π pulse on the same Rydberg level transition, the control qubit is returned to the ground state with a second π pulse. If the control qubit was excited to the Rydberg level the 2π rotation is blocked and the target qubit picks up no phase shift. If the control qubit was not excited then the target gets the 2π rotation resulting in a phase shift. This gives a controlled phase gate which can be represented in matrix form as7

$$C_z = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

where the state vector is written as $(|00\rangle, |01\rangle, |10\rangle, |11\rangle)^T$ with the control qubit state given first. This controlled phase operation can be converted to a CNOT gate by adding Hadamard single qubit gates to the target qubit before and after the controlled phase operation as shown in Figure 2.5.

The fidelity of this gate is limited by the blockade shift and the lifetime of the Rydberg atoms. The gate error due to these effects has been calculated in [Saffman and Walker, 2005, Saffman et al., 2010] and the leading terms are:

$$E \simeq \frac{7\pi}{4\Omega\tau} \left(1 + \frac{\Omega^2}{\omega_{10}^2} + \frac{\Omega^2}{7B^2} \right) + \frac{\Omega^2}{8B^2} \left(1 + \frac{6B^2}{\omega_{10}^2} \right)$$

where Ω is the Rabi frequency of the Rydberg excitations, τ is the Rydberg lifetime, ω_{10} is the qubit computational state splitting, and *B* is the average blockade shift. In the limit that $\omega_{10} \gg B$, Ω the optimal Rabi frequency to minimize gates errors is

$$\Omega_{opt} = (7\pi)^{1/3} \frac{B^{2/3}}{\tau^{1/3}}$$

which gives a minimum error in this limit of

$$E_{min} = \frac{3 \left(7\pi\right)^{2/3}}{8} \frac{1}{\left(B\tau\right)^{2/3}}$$

For our average blockade shift with the 97D_{5/2} level of 4.8 MHz and Rydberg state lifetime of ~300 μ s we find the optimum Rabi frequency to be $2\pi \times .645$ MHz. The Rabi

Error Source	Value
Intrinsic Gate Error	.008
Spontaneous Emission	.02
Doppler Broadening	.02
Other Sources	10^{-4}
Total	.048

Table 2.2 Summary of the errors for the CNOT gate due to the Rydberg excitation.

frequency used in our most recent gate measurement was .9 MHz which is very close to the optimal frequency. The error we expect to see from blockade leakage is only .008 with the settings we used which is well below the errors from other sources.

We also need to add in two additional errors because we use a two photon excitation to the Rydberg level that allows for spontaneous emission from the intermediate level and accounts for the residual doppler shift from the unequal wavelengths. To calculate the spontaneous emission error we repeat the same steps as in Section 2.2.1 but for the ladder system seen with the Rydberg excitation, we find

$$P_{se}(\pi) = \frac{\pi}{2\Delta\tau} \frac{(\Omega_{780} + \Omega_{480})^2}{\Omega_{780}\Omega_{480}}$$

where Δ is the intermediate state detuning, τ is the intermediate state lifetime, and Ω_{780} and Ω_{480} are the individual beam Rabi rates. For the gate measurements we had $\Delta = 2\pi \times$ 2.1 GHz, $\Omega_{780} = 2\pi \times 135$ MHz, and $\Omega_{480} = 2\pi \times 32$ MHz which gives a spontaneous emission probability of 2% for a π pulse. A calculation for the excitation error from the doppler broadening also gives a 2% excitation error.

In our system the Hadamard gates are implemented by $\pi/2$ Rabi oscillation pulses which adds trivial single qubit phases to the qubits. If the two $\pi/2$ pulses have the same

relative phase this results in an equivalent CNOT gate matrix given by

$$CNOT_0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & -i & 0 \end{pmatrix}$$

It's also possible to have the $\pi/2$ pulses π out of phase which also gives a CNOT gate but with the $|0\rangle$ state acting as the state that causes the NOT operation. For our implementation this matrix is:

$$CNOT_{\pi} = \begin{pmatrix} 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

Another method can use blockade to generate a CNOT gate directly and is based on work in [Ohlsson et al., 2002]. This method only uses π pulses to swap the populations between the appropriate states. This requires the ability to perform either a π pulse between the two computational states and between a computational state and a Rydberg state or the ability to perform π pulses between a Rydberg state and both computational states. The use of only π pulses should reduce the sensitivity to phase errors in applying the pulses; However, the larger number of pulses and increased time spent in the Rydberg level may override this gain.

2.5.1 CNOT gate with weak Rydberg interaction

Another method to generate a CNOT gate was mentioned in the original paper [Jaksch et al., 2000]. This method operates in the regime where the Blockade interaction is weak, $\Delta \ll \Omega$. In this case both qubits are excited to the Rydberg state and allowed to evolve. This evolution gives a different phase shift depending on whether the control atom was excited to the Rydberg state or not. By controlling the time the target qubit spends in the Rydberg state this phase difference can be set to π producing a controlled phase gate



Figure 2.7 Amplitude Swap CNOT pulse diagram.

similar to the blockade case. This method requires that the interaction strength be well known in order to get the correct phase shift and remains very sensitive to the atomic motion. However even with these difficulties, this technique does look capable of creating high fidelity gates at separations of over 100 μm [Saffman et al., 2010].

2.6 Parity Oscillations

One of the important uses for a two qubit quantum gate is to perform deterministic entanglement of the two qubits. This also serves as a good way to verify the quantum nature of a gate. Generating entanglement using a CNOT gate is done by placing the control atom in a superposition state and then operating the gate. For example if the input state is $\left(\frac{|0\rangle+|1\rangle}{\sqrt{2}}\right) \otimes |0\rangle$ for the control and target qubits respectively then the output for a CNOT operation is $\frac{|00\rangle+|11\rangle}{\sqrt{2}}$ which is one of the maximally entangled Bell states. Unfortunately just measuring the populations after performing the gate does not prove that the state was entangled. What makes this an entangled state is the fact that it is inseparable. This can be verified by measuring the coherences of the density matrix of the output of the CNOT gate. There are several ways to measure these coherences. One method is by performing a state tomography measurement which involves measuring all projections of the two qubits onto orthoganal axes and using this information to estimate the full density matrix [Altepeter et al., 2006, James et al., 2001]. Another method is to

measure the parity of the state after performing a special analysis pulse and verfying that there is enough coherence to show the state is inseparable. This second method is discussed here as it is what we used to verify entanglement.

To see how we can measure these coherences we will work with a general density matrix

$$\rho_{gen} = \begin{pmatrix} P_{00} & b_1 & a_1 & c_1 \\ b_1^* & P_{01} & c_2 & a_2 \\ a_1^* & c_2^* & P_{10} & b_2 \\ c_1^* & a_2^* & b_2^* & P_{11} \end{pmatrix}$$

The parity of this state is defined as $P \equiv P_{00} + P_{11} - P_{01} - P_{10}$. If we apply a ground state rotation of angle θ with a relative phase of ϕ to both qubits and measure the parity of the result we find

$$P' = P \cos^{2}(\theta) - 2\operatorname{Re}\left[c_{1}e^{2i\phi}\right] \sin^{2}(\theta) + 2\operatorname{Re}\left[c_{2}\right] \sin^{2}(\theta) + \operatorname{Im}\left[\left(a_{1} - a_{2} + b_{1} - b_{2}\right)e^{i\phi}\right] \sin(2\theta)$$

Looking at this we can isolate c_1 and c_2 if we choose to use $\theta = \pi/2$ and simplifying we find

$$P' = 2\text{Re}(c_2) - 2|c_1|\cos(2\phi + \xi)$$

where c_1 is the coherence between the $|00\rangle$ and $|11\rangle$ states, ξ is the phase of c_1 , and c_2 is the coherence between the $|01\rangle$ and $|10\rangle$ states. This shows that to measure c_1 which is the desired coherence for the $\frac{|00\rangle+|11\rangle}{\sqrt{2}}$ state, we just vary the phase of the $\pi/2$ pulse and measure the amplitude of the oscillation in the parity signal. In addition this same experiment allows us to measure c_2 to make sure it is zero as expected. A key thing to notice with this analysis is that this curve oscillates at 2ϕ . In our experiment we vary ϕ by using a time delay which accumulates phase at the differential AC Stark shift of the ground state Raman beam as discussed in Section 2.2. Since we know this phase accumulation rate, we can verify that we see this speed enhancement.

Unfortunately we do not have the ability to apply a pulse to both qubits at the same time with our apparatus. This means that we have a slight phase difference between the two ground pulses applied to the two qubits due to the phase accumulation during the different delays. If we perform the same analysis and again let the pulse area be $\pi/2$ and define the phase difference between the two pulses to be ϕ_2 we find

$$P' = 2 |c_2| \cos (\phi_2 + \zeta) - 2 |c_1| \cos (2\phi + \phi_2 + \zeta)$$

where ξ is the phase of c_1 and ζ is the phase of c_2 . Just as in the previous case by varying the phase ϕ of both pulses while keeping the phase between the two pulses ϕ_2 constant we still recover a signal that oscillates with an amplitude $2 |c_1|$ and oscillates at 2ϕ . However in this case, any measured offset is not simply related to c_2 . This distinction has implications for trying to measure this c_2 coherence which is important for other states such as a different maximally entangled Bell state $\frac{|01\rangle + |10\rangle}{\sqrt{2}}$.

In order to measure c_2 we need to change the ground state rotation used as an analysis pulse. To see an oscillation we need to vary the phase between the two ground state rotations, ϕ_2 . Unfortunately this means we will see oscillation if either c_1 or c_2 is non-zero and this will oscillate at an unenhanced rate of ϕ . To measure c_2 unambiguously we would need to perform the measurement where ϕ is varied to measure c_1 first. Performing both of these measurements actually requires taking more data than is needed to perform a state tomography measurement and with the data in a state tomography measurement we can measure the entire density matrix instead of just c_1 and c_2 .

We need to quantify what value of the coherence is necessary to prove that the state is inseparable and therefore entangled. To find this limit we look at the most general separable state possible. This state is $|\psi\rangle = (A_0|0\rangle + A_1|1\rangle) \otimes (B_0|0\rangle + B_1|1\rangle)$ which when written as a density matrix is

$$\rho = \begin{pmatrix} |A_0|^2 |B_0|^2 & \cdots & \cdots & A_0 B_0 A_1^* B_1^* \\ \cdots & |A_0|^2 |B_1|^2 & \cdots & \cdots \\ \cdots & \cdots & |A_1|^2 |B_0|^2 & \cdots \\ A_0^* B_0^* A_1 B_1 & \cdots & \cdots & |A_1|^2 |B_1|^2 \end{pmatrix}$$

We can ignore the · · · terms for this analysis. By recalling that each state is normalized, i.e. $|A_0|^2 + |A_1|^2 = 1$ and $|B_0|^2 + |B_1|^2 = 1$, we can write

$$|A_0|^2 + |A_1|^2 + |B_0|^2 + |B_1|^2 = 2$$

which can be shown to be equivalent to

$$(|A_0| - |B_0|)^2 + (|A_1| - |B_1|)^2 + 2|A_0||B_0| + 2|A_1||B_1| = 2$$

after a little algebra. Now we can see that the first two terms are positive definite so this allows us to write

$$|A_0| |B_0| + |A_1| |B_1| \le 1$$

We can square this and then after grouping terms we have

$$|A_0|^2 |B_0|^2 + |A_1|^2 |B_1|^2 + 2 |A_0| |B_0| |A_1| |B_1| \le 1$$

These terms are equivalent to certain values in the general density matrix used earlier allowing us to write

$$P_{00} + P_{11} + 2|c_1| \le 1$$

which gives the limit for a separable state. This is usually rewritten as $F = \frac{1}{2} (P_{00} + P_{11}) + |c_1|$ where a value of *F* larger than 1/2 is an entangled state. This technique was used in [Turchette et al., 1998, Sackett et al., 2000]. So if we measure these three values of the density matrix we can verify if the state we created is entangled or not. A similar analysis can find $F = \frac{1}{2} (P_{01} + P_{10}) + |c_2|$ with the same entanglement limit of 1/2.

2.7 Scalability

The final thing we need to show is that this system is scalable without placing exponential demands on resources. Scaling the number of trapping sites requires looking at the demands placed on spacing due to the Rydberg blockade. This is well studied in [Saffman and Molmer, 2008]. The number of fully interconnected sites was found to be

$$N_{max,vdW}^{(2D)} = C_{2D} E^{1/3} n^{2/3}$$

$$N_{max,vdW}^{(3D)} = C_{3D} E^{1/2} n$$

where C_{2D} and C_{3D} are constants that depend on the atom chosen, *E* is the error desired from the gate, and *n* is the principal quantum number for the Rydberg level used. For example this gives $N_{max,vdW}^{(2D,3D)} = 460$, 7600 when n = 100 and $E = 10^{-3}$. Since that is the number of fully interconnected qubits this could be scaled higher by chaining to other areas that are further away as would be done for systems with only nearest neighbor interactions. Unfortunately the story does not end with having the ability to operate high fidelity gates. This large number of sites requires a technique to either accomplish complete loading of the array or a way to know which sites have been loaded to know which sites to use during a calculation. The use of a Mott-insulator transition in a Bose-Einstein condensate, direct loading or removal of empty sites using a movable trap, or Rydberg blockade techniques are examples of how complete loading of an array could be accomplished [Saffman and Walker, 2002, Saffman et al., 2010].

Maintaining a large number of sites also becomes difficult due to losses from the sites. This means some technique to reload a single site will probably be necessary. Detecting the loss of an atom is possible by performing a non-destructive measurement using an ancilla qubit and if this measurement and subsequent loading process is done fast enough it is possible to maintain a fully loaded array. Even assuming a very good vacuum maintaining more than 1000 qubits looks very difficult [Saffman et al., 2010]. Other possibilities that are not as sensitive to single atom losses have been considered such as collective encoding [Saffman and Molmer, 2008, Brion et al., 2007]. This technique uses an ensemble of atoms to hold the information of many qubits encoded in the hyperfine ground states of many atoms. By storing the information in an ensemble the loss of a single atom is not as detrimental and is still correctable.

Chapter 3

Experimental Apparatus

This chapter will cover the experimental apparatus we use to cool, capture, and manipulate the atoms. For many of the subsystems I will give just a basic description as the full descriptions are available in the recent theses of Todd Johnson and Erich Urban. Several of the subsystems have seen significant upgrades and changes and these will receive a more detailed description. The description starts with how we cool and trap the atoms using a Magneto Optical Trap, Section 3.1, and far off resonance dipole trap, Section 3.2. Next I discuss how measurements of the qubit states are made in Section 3.3. And finally I cover the systems we use to control the atoms to perform initialization, Section 3.4, state manipulation of the computational states, Section 3.5, and excitation of our atoms to Rydberg levels, Section 3.6.

3.1 Magneto Optical Trap

The first stage in capturing our qubits is to cool them to very low temperatures so that they may be held in a shallow and very small far off resonance dipole trap. To perform this initial cooling we use a Magneto Optical Trap (MOT). This uses the force due to photon scattering to both cool and localize a small cloud of atoms. In a simple setup, 3 beams are aligned along mutually perpendicular axes and are circularly polarized. Then they are retro-reflected with the opposite polarization. These beams are slightly red detuned from a cycling transition, which causes the atoms to scatter more light from a beam it is moving towards due to the doppler shift. However, with just the light there is no additional force to return an atom back to the center of all the beams. To get this extra spatial trapping, a pair of coils is used in an anti-Helmholtz configuration to produce a gradient magnetic field around a zero field point. This gradient causes the atoms to be optically pumped into a different hyperfine state by the beams as they move from the zero field point leading to preferential scattering by one or more of the beams. If the polarization of the beams is chosen correctly this enhanced scattering will push the atom back towards the center of the trap. For general MOT and atomic cooling theory see [Metcalf and van der Straten, 1999]. Our MOT typically cools about 10^5 atoms to a temperature of 100-200 μK with a density of about 10^9 atoms/cm³. For more details about our specific implementation refer to [Urban, 2009].

3.2 Far Off Resonance Dipole Traps

In order to manipulate atoms in a controlled fashion, we must keep them well localized. To do this we make use of optical dipole traps. This is a trap based on the interaction between an induced dipole moment of an atom and a laser's oscillating electric field. This trap generates a potential that depends on the gradient and detuning of the laser beam. For a beam of red detuning, the atoms are attracted to a region of high intensity, and for a beam of blue detuning, the atoms are repelled by a region of high intensity. The dipole force is a conservative force, but this trap also has a non conservative force due to the photon scattering. To minimize photon scattering, these traps are usually operated in a far detuned regime. The parameters that are important to consider when trapping atoms are the trap depth, trap size, and heating rate. In addition, we also care about the differential AC Stark shift of our two computational states as fluctuations in this shift will lead to decoherence of these states limiting how long the qubits can be stored.

We call these traps dipole traps because the interaction is modeled by the dipole force. From this we define the trap depth as

$$U = -\alpha \frac{|\mathcal{E}|^2}{4} = -\frac{\alpha}{2c\epsilon_0}I$$



Figure 3.1 a) Diagram of the energy levels of ⁸⁷Rb used for the MOT and state measurements. b) Schematic of MOT beams and gradient coil in our experiment.

where α is the polarizability of the atom and \mathcal{E} is the electric field strength which we relate to the intensity by $|\mathcal{E}|^2 = \frac{2I}{c\epsilon_0}$. α depends on the transition strength of the nearest atomic transitions, the detuning of the field from these transitions, and the natural linewidth of these transitions. In the case where we have a large detuning, only a single transition responsible for the majority of the effect, and use the rotating wave approximation, we find

$$\alpha = \frac{3\pi\epsilon_0 c^3}{\omega_0^3} \left(\frac{\Gamma}{\omega - \omega_0}\right)$$

where ω_0 is the atomic transition frequency, ω is the frequency of the laser, and Γ is the natural linewidth of the transition. This shows that α will change signs depending on the detuning of the laser beam and that the trap depth goes as $\frac{1}{\omega - \omega_0}$. This simplified equation gives a polarizability for our FORT of $\alpha = -1 \times 10^{-38} \text{ s}^4 \text{A}^2/\text{Kg}$ when using only the D1 line of ⁸⁷Rb compared to the actual value of $\alpha = -1.27 \times 10^{-38} \text{ s}^4 \text{A}^2/\text{Kg}$ [Saffman and Walker, 2005] which is quite close considering all the assumptions used to derive this equation. The difference in values is due mainly to including only one transition and from applying the rotating wave approximation.

The size and depth of the trap depends on the geometry of the laser's electric field. In our case of a simple focused Gaussian beam, the trap size is on the order of the beam waist in the transverse direction and the Rayleigh range in the axial direction. As the atoms get hotter they will fill more of the volume of the trap until they get hot enough to escape from the trap. The trap depth is proportional to the beam intensity. For a Gaussian beam this intensity is

$$I = I_0 \left(\frac{w_0}{w_z}\right)^2 e^{\frac{-2r^2}{w_z^2}}$$

with $w_z = w_0 \sqrt{1 + \left(\frac{z\lambda}{\pi w_0^2}\right)^2}$ where w_0 is the $1/e^2$ beam waist, λ is the wavelength, $I_0 = \frac{2P}{\pi w_0^2}$ is the peak intensity, which is related to the beam power P, and r and z are respectively the transverse and axial coordinates. This potential can be expanded about the

origin and results in a harmonic potential in both the transverse and axial directions. Using this expansion the characteristic trap frequency can be found.

$$\omega_{r,trap} = \frac{2}{w_0} \sqrt{\frac{|U_m|}{m}}$$
$$\omega_{z,trap} = \frac{\sqrt{2}}{z_R} \sqrt{\frac{|U_m|}{m}}$$

where U_m is the maximum trap depth, *m* is the mass of the atoms, and $z_R = \frac{\pi w_0^2}{\lambda}$ is the Rayleigh length of the FORT beam. For our typical parameters of $w_0 = 3.4 \,\mu\text{m}$ and $\lambda = 1064 \,\text{nm}$ we find: $|U_m| \simeq 4.5 \,\text{mK}$, $\omega_{r,trap} \simeq 60 \,\text{kHz}$, and $\omega_{z,trap} \simeq 6 \,\text{kHz}$.

We must also be concerned with the heating rate of the atoms due to the trapping laser. The atoms may be heated directly through photon scattering or can also be heated indirectly though noise on the laser intensity or beam position. This noise will cause parametric heating effects if it occurs at the correct frequencies. First we consider the direct heating due to the residual photon scattering rate. This heating rate is given by [Grimm et al., 2000] Based on the beam size, ~6.5 μm , and FORT site separations, ~9 μm , the theoretical amplitude of the crosstalk is .008 matching very well with what we observe.

$$\dot{T} = \frac{1}{3} T_{rec} U \frac{\Gamma}{\hbar |\omega - \omega_0|}$$

where ω is the laser frequency, ω_0 is the atom's transition frequency, U is the trap depth, and $T_{rec} = \frac{\hbar^2 k^2}{m}$ is the recoil energy. The heating rate goes as $\frac{1}{\omega - \omega_0}$. Therefore, by moving to a larger detuning the heating rate is reduced even if you increase the intensity to maintain a constant trap depth.

The noise on the laser intensity and beam position will also heat the atoms due to parametric effects [Savard et al., 1997]. These effects require a matching of the frequency in the noise and the frequency of the atom moving in the trap. Intensity fluctuations at twice the trap frequency can add energy to the atom on every oscillation. For red detuned FORTs the intensity noise has a heating rate given by

$$R_{int} = \pi^2 \nu_{trap}^2 S\left(2\nu_{trap}\right)$$

where $S(2\nu_{trap})$ is the one sided power spectrum of the fractional intensity noise evaluated at twice the trap frequency ν_{trap} . Position fluctuations at the trap frequency are also able to add energy to the atoms as they oscillate in the trap. Similar to the intensity noise heating the beam position noise heating rate is given by

$$R_{pos} = \frac{\pi^2 v_{trap}^2}{\langle x^2 \rangle} S\left(v_{trap}\right)$$

where $S(v_{trap})$ is the one sided power spectrum of the position fluctuations evaluated at the trap frequency and $\langle x^2 \rangle$ is the mean square position for an atom in the trap.

Another source of loss from our FORT is collisions with residual background atoms in our vacuum chamber. The lifetime for an atom in a FORT can be written as [Saffman and Walker, 2005]

$$\frac{1}{\tau_c} = \sqrt{\frac{3k_B T_b}{m}} n_b \sigma_{Rb-Rb}$$

where k_B is Boltzmann's constant, T_b is the background atom temperature, n_b is the background atom number density, and $\sigma_{Rb-Rb} = 2.5 \times 10^{-13} \text{ cm}^2$ is the collisional cross section for two Rubidium atoms [Bali et al., 1999]. We have measured a FORT lifetime of about 3.5 s which corresponds to a background pressure of 1.2×10^{-9} Torr.

Finally we must address any decoherence the trapping laser may have on our chosen computational states. The decoherence mechanism we must worry about is the differential AC Stark shift of the two hyperfine ground states. The differential shift is due to the slightly different detunings for the F=1 and F=2 levels that we use for our computational basis states. This shift by itself does not lead to decoherence but just to a phase accumulation that could be corrected using single qubit operations. However, the atoms do not see a constant intensity while in the FORT. This is due mainly to their residual thermal motion but also to any intensity and position noise of the trapping laser. As discussed in Section 2.2.1, an effective decoherence time was found to be 6.5 ms for our parameters.

3.2.1 Modifications to our FORT

Our FORT has gone through several iterations. The early iterations may be read about in [Urban, 2009, Johnson, 2008]. The data collected up until the CNOT gates was all done using the setup described there. The CNOT data and all later data were taken using a slightly modified setup that will be described here. The FORT optical train was changed for a few reasons. First we needed to improve the consistency of the traps. The old setup which used a calcite crystal to produce two sites required setting the polarization to balance the power in the two sites. Unfortunately this position is the most sensitive to slight polarization drifts of the fiber output due to environmental changes. These small drifts would cause slow fluctuations in the depth of the sites which if not carefully monitored affected the loading and retention of atoms in these sites. The second reason to modify our old FORT setup was to allow for more trapping sites for use in future experiments. Finally we also slightly reduced the spacing between the sites in order to increase the Rydberg blockade shift between the sites.

To achieve this we use a design similar to the 6 site variant of the FORT described in detail in [Urban, 2009]. Two changes have been made to this setup shown in Figure 3.2. First the telescope used to focus the beam onto the diffractive optical element was changed to use a 60 mm and 50 mm set of achromatic doublet lenses giving a telescope with a magnification of .83 resulting in a slight reduction of the FORT site spacing. The other improvement is to the mirrors used in the rest of the optical train. We improved the mounts to reduce the clipping of the outermost sites from the diffractive element. These two changes allow for producing 5 FORT sites in the chamber from a single beam. In addition we can still use the calcite crystal to create a second beam before the diffractive element, allowing us to produce a second column of 5 FORT sites giving a total of 10 useable sites. The change of the laser wavelength to 1064 nm produced beams with a slightly larger waist than that found with the 1035 nm ELS laser used previously. The waists are about 3.4 μ m and the spacings between the sites are also slightly reduced by the change in telescope magnification to ~8.7 μ m.

Seed Laser	NP-Photonics Scorpio outputs over 50 mW at 1064 nm
Fiber Amplifier	NuFERN SFA-PM1064-10W-2 outputs up to 10 W at 1064 nm
Beam waist (µm)	3.4
Typical Depth (<i>mK</i>)	4.5, 3.2, and 2.3 (center to outside)
Central Trap Frequency	60 kHz radial, 6 kHz axial
Lifetime	3.5 s

Table 3.1 Typical parameters for the FORT sites used for the CNOT data.

The peak intensity for each site, which corresponds to the trap depth, is still not uniform. The central site nearest neighbors have an intensity which is 70% of the central site and the two outer sites have an intensity that is 50% of the central site. These differences do have slight effects on the loading and retention of atoms loaded into the sites. Tests done at the current atom temperatures of our MOT typically show loading rates that are reduced by about 5% compared to the central site and retention probabilities that are usually within 5% of the central site.

3.2.1.1 Blue Detuned Bottlebeam FORT

Our dipole trap will need to be improved to allow for quantum computing with more qubits and smaller spacings. We need to increase the retention time and reduce any decoherence rates induced by the dipole trap. To meet these goals we began working on a blue detuned dipole trap. A dipole trap made using blue detuned light requires generating a 3D intensity minimum. With care it is possible to generate a minimum that actually has zero intensity at the center of the trap. This provides several advantages when trying to manipulate atoms. First the near zero intensity that the atoms see means the AC Stark shift and photon scattering rate are greatly reduced. So the decoherence due to differential Stark shifts and the photon scattering heating rate are reduced. Another advantage is that a blue detuned trap will also trap an atom in a Rydberg state, allowing the trap to be left on which should reduce the probability of losing an atom during a gate operation.



Figure 3.2 Diagram of the optical train used to generate the FORT sites. The calcite crystal birefringence can be used to generate two beams travelling parallel and separated in the horizontal direction (into the page). The diffractive element produces five beams from each input beam which are collimated and reimaged into the experiment chamber. This system allows up to 10 sites in a 2 x 5 array with a site to site spacing of about 8.7 μ m and beam waists of about 3.4 μ m to be generated.



Figure 3.3 Image and associated line profile of our 5 site FORT as used in the CNOT datasets. From the line profile and total power in the beams we calculated the trapping potential to give trap depths of 4.5 mK for the central site, 3.2 mK for the nearest neighbors, and 2.3 mK for the outer pair.

The change to trapping at a zero intensity point surrounded by light changes the heating rate due to photon scattering. As the atoms heat up they will see a higher average intensity thus increasing their heating rate until they are lost from the trap. It was shown in [Grimm et al., 2000] that the heating rate for a blue detuned FORT is

$$\dot{T} = \frac{\kappa}{1+\kappa} T_{rec} \frac{\Gamma}{\hbar \left(\omega - \omega_0\right)} k_B T$$

where κ is a geometric factor which varies from 1 to 0 as the trap moves from a quadratic potential towards higher power potentials, $T_{rec} = \frac{\hbar^2 k^2}{m}$ is the recoil energy, and *T* is the atom temperature. The heating rates due to intensity fluctuations and position fluctuations should be the same as for a red detuned trap.

We demonstrated a system that used light blue detuned of the D2 line of Cesium 133 to trap a collection of Cs atoms. To generate the 3D intensity null we took a single beam, passed it through a Mach-Zehnder interferometer that contained telescopes of differing magnifications in each arm, and destructively interfered the output. The difference in magnification gave beams with different waists. Due to the different size and different Rayleigh lengths of these two beams the interference pattern can produce a 3D intensity null that resembles the cigar shape found in focused Gaussian beam red detuned traps. We call this resulting beam a bottlebeam since it is a single beam that can serve to hold atoms. See Figure 3.4 for a diagram of the optical setup and images of the resulting beam output. This interference pattern results in a potential that is quadratic in the axial direction but quartic in the radial direction when expanded about the intensity null. This difference leads to a slight reduction of the decoherence rate compared to a trap with a quadratic potential in all directions. This factor also can be seen in the heating rate where $\kappa = 2/3$ for the bottlebeam but $\kappa = 1$ for the FORT we are currently using since it is quadratic in all directions. Another important point to note is that the bottlebeam minimum depth comes at an angle to the beam propagation direction where there is a saddle point. This saddle point is roughly half the maximum depth of the trap in the radial direction or 30% of the maximum height in the axial direction.



Figure 3.4 a) Diagram of the optical setup needed to produce the Bottlebeam. b) Experimental images of the produced beam compared to theoretical calculations.



Figure 3.5 3D plot of the intensity of the bottlebeam trap. Note that the axes are on significantly different scales.



Figure 3.6 Atom lifetime measurement for the bottlebeam trap. The fit gives an exponential decay rate of 20 ms. The inset is an image of the trapped Cs atoms after being held for 40 ms. These data were taken July 9, 2008.

The trap we made had beam waists of 22 μ m and 12 μ m, a detuning of 40 GHz, and power of 40 mW [Isenhower et al., 2009]. This gave a trap with a 1.4 mK minimum depth. We overlapped this beam with a Cs MOT which had a density of about 10¹⁰ atoms per cm³ and an atom temperature of about 50 μ K. Due to the large AC Stark shift and scattering rate the trap could not be left on while loading the MOT. In addition limited computer controls of the trap intensity did not allow for the bottlebeam intensity to be turned on slowly. This led to a non-adiabatic transition which caused significant heating of the atoms. We estimated that the atom temperature after reaching thermal equilibrium was around 500 μ K. This relatively high initial temperature greatly increased the heating rate from photon scattering resulting in a short trap lifetime. This trap held several thousand Cs atoms with a lifetime measured to be about 20 ms. A much improved version of this trap using light at 532 nm and much smaller beam sizes is currently being implemented, and we expect it to achieve trapping conditions comparable to our current red detuned FORT.

3.3 Single Atom Readout and State Detection

To conduct our quantum computing experiments it is necessary to readout the state of the atoms after performing some operation on them. Also because our FORT sites are loaded stochastically we must make a measurement to know how many atoms a site starts with before any operation is performed. These measurements are made using the MOT lasers and an Electron Multiplying Charge Coupled Device (EMCCD) camera (Andor iXon). Most of our experiments use a measurement time of about 40ms but this time varied from 5 ms up to 50 ms due to changes in camera and laser settings. To make a measurement of the number of atoms loaded into a FORT site, we use the following steps. First we set the detuning of the MOT lasers to be about 1.5Γ , and we apply a chopping pulse scheme with a frequency of 1.25 MHz to the FORT and MOT lasers so the intensities are out of phase as shown in Figure 3.7. This chopping is necessary because of the large AC Stark shift, of order 200 MHz, due to the FORT potentials. Light scattered by the atoms is then collected and imaged onto the EMCCD camera using an optical system with a numerical aperature of about .4 that was able to capture about .53 steradians of solid angle. This optical train also included wavelength bandpass filters so that only the 780 nm light from the atoms would be imaged.

The very high sensitivity of this camera system allows us to distinguish between differing numbers of atoms. This is done by defining a region of interest around each imaged FORT site and summing the total counts in this region. The result of this is we see distinct bands for each atom number up to about n=3. This banding structure allows cuts to be set around a one atom signal with high confidence. Unfortunately this method by itself is unable to distinguish the two computational states. In order to have a state measurement we must add an additional phase to our experiment cycle. This phase we call a blowaway phase. We use light near the cycling transition propagating in a single direction without any repumping light to selectively heat atoms in the F = 2 ground state out of our FORT. Again a chopping sequence is necessary to remove the AC Stark shift of the



Figure 3.7 a) Pulse sequence used for an atom number measurement. The rise and fall times for the MOT beams are about 25 ns and the rise and fall times for the FORT is about 40 ns. b) Average of several images with all five sites loaded and showing the regions of interest surrounding each FORT site.

FORTs. However for this case the chopping frequency is set to .5 MHz to give more scattering events and allow faster heating. This is then followed by a normal atom number measurement sequence so that any atoms remaining in the F = 1 state are imaged. We also can make a direct measurement of the F = 2 population by first applying a rotation of the computational states to swap the F = 1 and F = 2 populations and then make a state measurement.

We need to quantify the effect of the blow-away used during state sensitive measurements on atoms in the two quantum states. This is done by preparing the atoms in either F=1 or F=2, performing the blow-away procedure, and measuring the atom number. Histograms of the results of such a test are shown in Figure 3.9. We can also measure the time it takes to perform the blow-away by stepping the length of the blow-away pulse as shown in Figure 3.10. For a typical experiment the blow-away phase lasts for .15 ms where we find on average that atoms in the F=2 state survive about 1% of the time and that atoms in F=1 have the same retention as measured without the blow-away procedure giving approximately 99% state detection fidelity without considering atom number assignment errors of the single atom cuts.

In the future we would prefer to make a state measurement without needing to destroy a specific state by heating it out of the FORT. This should be possible using a balanced pair of beams that are both σ_{\pm} polarized. This is necessary because using a 3D molasses, which does not have a well defined polarization in relation to the qubit quantization axis, allows for off-resonant excitation to the F' = 2 level which can decay to either F = 1 or F = 2, but with a well defined polarization this off-resonant excitation is not allowed. The off-resonant excitation limits the number of scattering events that can occur before the atom decays to the dark F = 1 state. As long as the two σ_{\pm} beams do not heat the atoms too quickly then enough photons should be scattered to give a detectable signal. Another possible solution is to increase the number of photons collected by the camera optics. Our current system only captures 4% of the scattered photons. A system


Figure 3.8 a) Histogram of the series shown in b) for the white and red dots. b) Sample series of the signal from a region of interest taken from many camera images. The white dots are the first camera shot and the red dots are the second camera shot. These show that the signal from a single atom is easily distinguished from the signal of other numbers of atoms. This is from a 20 ms exposure which has 9ms of time where the cycling light is active. The light was detuned by 1.5Γ from the cycling transition. These data were taken Feb. 9, 2010.



Figure 3.9 Effects of the blowaway procedure on measurements of atoms in a) F=1 and b) F=2. These data were taken Feb. 10, 2010.



Figure 3.10 Measurements of the needed blow-away time. a) Measurement for atoms in the F=2 state b) Measurement for atoms in the F=1 state. Note the different time scales for the two plots. These data were taken Feb 10, 2010.

with larger numerical aperture would greatly improve on this and possibly even reach an efficiency where a simple 3D molasses could be used.

3.4 Optical Pumping to the Computational Basis

After performing a readout to measure the atom number the atoms in the FORT sites will be distributed across all five states in the F = 2 manifold. Therefore a step to initialize the atoms into the computational basis is necessary. We achieve this initialization by optically pumping the atoms into one of the computational basis states. As mentioned earlier we have used two different bases: first the stretched state basis where $|0\rangle = |F, m_F\rangle = |1, 1\rangle$ and $|1\rangle = |2, 2\rangle$ and second the clock states where $|0\rangle = |1, 0\rangle$ and $|1\rangle = |2, 0\rangle$. In both cases we initialized the atoms into the $|1\rangle$ state. Each basis set required a slightly different method to obtain the desired initialization.

3.4.1 Stretched state pumping

The stretched state computational basis ($|0\rangle = |F, m_F\rangle = |1, 1\rangle$ and $|1\rangle = |2, 2\rangle$) is pumped into by first applying a bias magnetic field and then using a σ^+ beam that is tuned to the $F = 2 \rightarrow F' = 3$ D2 cycling transition and also using some repumping light on the $F = 1 \rightarrow F' = 2$ D2 transition to repump any atoms that happen to decay to F = 1. In this case the atoms are pumped into a bright state where they continue to scatter light from the optical pumping beam. This choice does have the downside that there will be additional heating of the atoms due to this continued scattering. However this scheme is more forgiving to state mixing and poor laser polarization. When using this state we verified that the length of the optical pumping phase was short enough that we did not noticably heat the atoms, and typically we achieved an optical pumping efficiency of greater than 95%.



Figure 3.11 Diagram of the atomic levels and laser detunings used for optical pumping. a) For the stretched state pumping using the D2 line of ⁸⁷Rb. b) For the clock state pumping after switching to the D1 line in ⁸⁷Rb.

3.4.2 Clock state pumping

To pump into the clock states ($|0\rangle = |1, 0\rangle$ and $|1\rangle = |2, 0\rangle$) we made use of the transition from $F = 2 \rightarrow F' = 2$ as shown in Figure 3.11. When using linearly polarized light the transition from $|2, 0\rangle$ to $|2', 0\rangle$ is forbidden by dipole selection rules making $|2, 0\rangle$, which is our $|1\rangle$ computational state, a dark state for this light. By scattering light on this transition atoms will accumulate in the dark state and with the appropriate settings this technique can achieve >.99 fidelity state initialization . Initially light on the D2 transition at 780 nm was used for this pumping. However, the applied bias magnetic field of 1.5 G needed to define our quantization axis causes slight mixing of the excited states allowing a small leak out of the $|2, 0\rangle$ state. Unfortunately since the hyperfine splitting is fairly small for the D2 transition there was significant mixing resulting in a limited optical pumping efficiency of about .95. This apparently small error is actually very significant to gate operations especially as the number of qubits used increases. To improve this we switched to a laser tuned to the D1 line at 795 nm. The larger hyperfine splitting of the $P_{1/2}$ level allows for higher fidelity pumping of greater than .98 for our settings.

The D1 laser is a Sacher TEC 300 Littrow style external cavity diode laser designed to give over 700 mW at 785 nm. We raise the laser temperature and use the diffraction grating in the external cavity to operate this laser at 795 nm. This gives more than 50 mW of power at the settings we typically use. This light is then passed through a spatial filter to help remove some of the spontaneous emission background and to give a better beam shape. Part of the light is sent to an FM saturated absorption lock nearly identical to that used in the master MOT laser lock described in [Johnson, 2008]. The laser is locked to the 1-2 crossover peak from the F = 2 ground state to the $P_{1/2}$ level. A double pass AOM setup is used to shift the frequency up to resonance with the $F = 2 \rightarrow F' = 2$ D1 transition. The complete optical layout is shown in Figure 3.13. This light is then fiber coupled to another board where we join this light with light from the MOT repumping laser into another fiber going to the main experiment. This path also includes a shutter to block any light that may leak from either setup when this beam is not needed. The final



Figure 3.12 Geometry for optical pumping and state manipulation experiments for a) the stretched state basis and b) the clock state basis.

optical train focuses the light onto the atoms is shown in Figure 3.14. It contains a 780 nm waveplate which we use to maximize the transmission through a broadband Glan-Taylor calcite polarizer, and a telescope which magnifies the beam to a 5 mm waist. This beam enters the chamber from the top and is perpendicular to the FORT beam axis which is also the quantization axis as shown in Figure 3.12 b). We originally used the old optical pumping train which had the order of the waveplate and polarizer reversed. However, the purity of the polarization after the 780 nm waveplate was not good enough to allow high fidelity pumping and limited the pumping efficiency to about .95 thus removing all benefits of moving to the D1 line.

To achieve high fidelity pumping using this dark state a lot of attention has to be given to make sure that the state stays dark. This is done by matching the bias magnetic field to the linear polarization of the optical pumping laser and ensuring that the purity of the polarization is very high. In order to achieve these goals we implemented new bias field controls to allow fast control on three perpendicular axes. This allows us to cancel any background magnetic fields that do not match the laser polarization. This extra control of the bias fields also allows for additional benefits. We can choose different settings for every phase of our experiment allowing us to improve the atom loading, atom number measurement, optical pumping efficiency, and state manipulation. This is necessary because we do not have every beam perfectly aligned to a single coordinate system and slight misalignments can have large effects on the efficiencies of the different experimental phases.

To align the optical pumping train we first made sure to align the polarization axis to the FORT beam direction. This was done using a polarizing beamsplitter to send light above the FORT train where cards were used to ensure good alignment. This method set the polarization to match the FORT axis to within 2 degrees. Because of the use of a Glan-Taylor polarizer the purity of the polarization should be in excess of 10^5 . Then to align the bias magnetic field to this laser polarization we first optically pump the atoms into the $|1\rangle$ state and then turn on the D1 laser by itself for a set length of time and measure the probability of the atoms to be pumped out of this state as we scan the transverse magnetic fields. The proper settings will decrease the probability of the D1 laser pumping the atoms out of the $|1\rangle$ state. As shown in Figure 3.15, this method allowed us to measure a depumping time of 20 ms which compared to the optical pumping time to the $|1\rangle$ state of .066 ms gives a pumping efficiency of .997.

3.5 Ground State Raman Laser

To perform single qubit operations we use a two photon Raman laser to perform coherent Rabi oscillations between the two computational basis states. This laser is an external cavity diode laser that has its current modulated at 3.4 GHz. This produces frequency sidebands which are at half the ground state hyperfine splitting of 6.8 GHz. Each sideband has about 5% of the intensity of the carrier. We then use a Fabry-Perot cavity with a free spectral range of 6.8 GHz and a finesse of about 80 to filter out the carrier. This gives a beam with two frequency components split by the hyperfine splitting and detuned from the D2 line by ~100 GHz. The detuning from the excited state varied over the experiments, but the most commonly used values were 55 GHz and 100 GHz. When we switched to the clock states, the ground state Raman laser and the 780 nm Rydberg laser were joined into the same fiber to ease the alignment process by reducing the number of beams we need to align to the FORT sites. Another improvement that was made at the same time as the improvements to the FORT train was a change in the final lens before joining the FORT beam to give a reduction of the beam size from 7 x 12 μ m to 6.2 x 9.5 μ m. This system allowed us to perform site specific two photon Rabi oscillations with rates up to 1 MHz being demonstrated. Typically we set the detuning and power to maintain a Rabi flopping rate around 600 kHz to allow finer control of the pulse time due to the limited timing resolution of computer controls.



Figure 3.13 Diagram of the 795 nm laser used for optical pumping using the D1 transition of 87 Rb.



Figure 3.14 Diagram of the optical pumping optical train that focuses the light onto the atoms.



Figure 3.15 a) Measurement of the time needed to optically pump into the $|1\rangle$ state. The fit is of the form $A - Be^{-t/\tau}$ with A = .88, B = .66, and $\tau = .066$ ms. b) Measurement of the depumping rate due to the optical pumping beam. The fit has values of A = 1, B = .99, and $\tau = 20$ ms. The ratio of these times gives an optical pumping efficiency of .997. These data were taken Dec. 3, 2009.

3.6 Two Photon Rydberg Excitation Lasers

The laser system used to excite atoms to the Rydberg levels consisted of two lasers. One at 780 nm and another at 960 nm which was then frequency doubled to 480 nm. The 780 and 960 nm lasers are external cavity diode lasers and both were locked to different longitudinal modes of a high finesse Fabry-Perot cavity. This cavity was made by optically bonding two very high reflectance mirrors to an ultra low expansion glass spacer. The cavity had a finesse in excess of 150,000, a free spectral range of 500 MHz, and was kept in a temperature controlled vacuum chamber to keep the longitudinal modes from drifting due to pressure or temperature changes. Through monitoring the changes of transition frequency of a certain Rydberg level we estimate that this cavity has long term absolute frequency stability of better than 100 Hz. The 780 and 960 nm lasers were measured to have Allan deviations of less than 100 Hz at times less than one second. The 780 nm laser was then sent through a double pass AOM setup which enabled us to scan over the frequencies between the longitudinal modes of the high finesse cavity. More details on the construction and stabilization of these two lasers are in [Johnson, 2008].

After passing to the experimental apparatus both beams passed through their own optical train that contained an AOM and telescopes to focus the beams onto the FORT sites. The AOM allows for 1D positioning of the beam. The 780 nm system could provide up to ~1 mW of power at the atoms and the 480 nm system produced up to 20 mW of power at the atoms. In order to better balance the individual Rabi rates of the 780 and 480 nm beams the 780 nm power was set to be around 2 μ W and the blue power was set to about 15 mW. The 480 nm laser uses an AOM based intensity noise eater to reduce any intensity noise from the cavity locks. This system is an upgraded version of the old design with a 3dB bandwidth of ~100 kHz. By setting the power below the peak power the noise eater could maintain a constant power for up to two weeks to allow for continued running without having to refind the frequency for the Rydberg transition and the Rabi flopping rate due to changes of the 480 nm Rabi frequency. We demonstrated Rabi flopping to Rydberg states from n=28 to n=102. The level that was chosen depended

upon the specific goals of the experiment. The data for the CNOT gate was taken at n=97 where we could get a large blockade shift without too much loss in Rabi flopping rate. As is done for the ground state Raman laser, we maintain a flopping rate between .5 and 1 MHz through our choice of intermediate detuning and 780 nm laser power. For the n=97 data typical values were 2.4 μ W of 780 nm light and 13 mW of 480 nm light with an intermediate state detuning of -2.1 GHz giving a two photon Rabi frequency of ~900 kHz.

3.6.0.1 Phase locked Rydberg B laser

To increase the capabilities for the Rydberg excitation system a new laser which allows for coupling between the Rydberg level and the other computational state is needed. This additional laser needs to be split by the hyperfine frequency of Rb and also have good phase coherence with the other Rydberg excitation lasers to enable well controlled Rabi flopping. To guarantee that these requirements are met a second laser at 780 nm is phase locked to the primary 780 nm laser already locked to the high finesse cavity. This additional laser allows for simplification of several pulse schemes for the CNOT gate or for deterministic single atom loading. Without this laser additional ground state rotations are needed to effectively connect the Rydberg level to the other computational state. For example the amplitude swap CNOT would require two fewer pulses to implement which should lead to higher fidelities due to the reduced time the control atom spends in the Rydberg state.

A second 780 nm Littman-Metcalf style external cavity diode laser is phase locked to the main 780 nm Rydberg excitation laser. To perform the phase lock a small portion of each beam is joined into an optical fiber that is connected to a high speed photodiode, Finisar HFD6x80-418. This allows for direct measurement of the 6.8 GHz beat note. Several steps are needed to convert this beat note into a useable error signal to phase lock the two lasers. First an ultra stable RF oscillator, SRS PRS10 Rb referenced 10 MHz source input into a Miteq DSZ-6.65-6.95-1K frequency multiplier, is used to mix this 6.8 GHz signal down to ~40 MHz to compare to another stable RF reference, HP 8662a, using a digital phase detector, Analog Devices 9901. The output of the phase detector is used to detect the frequency variations between the two lasers due to the phase changes they produce. The signal from the phase detector is then filtered using both active and passive filters and split into two components. A fast loop component is fed back directly to the current of the laser and a slow loop passes through additional filtering and is then connected to a piezo controlled mirror in the external cavity. Schematic diagrams of these are shown in figures 3.16 and 3.17.

Once these two lasers are phase locked the beat note between them can be analyzed to see the relative linewidth of these two lasers. By activating only the slow component of the feedback the beat note frequency would be stabilized but the linewidth would remain at around 1 MHz. With the high speed loop enabled the short term linewidth of the beat note is greatly reduced to be below the spectrum analyzer bandwidth limit of 100 Hz. With the lasers locked the feedback bandwidths can be measured by observing the feedback servo bumps in the beat note. These bumps correspond to where the feedback has switched from negative feedback to positive feedback. The high speed loop has a bandwidth measured to be about 1.5 MHz and the slow loop has a bandwidth of about 300 Hz. The stability of the lock is limited by environmental changes in the room which cause the main laser to hop from its cavity lock or cause the phase locked laser to mode hop. These changes are generally rare so the two lasers can be phase locked for many hours at a time. Relocking the phase lock is also generally very simple and can be reset in less than five minutes.



Figure 3.16 Block diagram of the phase locked 780 nm Rydberg laser used to drive Rabi oscillations between the F=1 state and a Rydberg state.



Figure 3.17 Block diagram of the entire feedback loop for the phase locked Rydberg B laser.



Figure 3.18 a) Schematic of the slow loop low pass filter circuit used to drive a piezo controlled mirror in the laser's external cavity. b) Schematic of the phase advance and bias tee circuits used to drive the laser.

Chapter 4

Experimental Results

This chapter covers the results we have had working to demonstrate a two qubit quantum gate. The results discussed follow a similar pattern to the order of the DiVincenzo criteria. We first demonstrated a good qubit system by performing single atom readout, coherent Rabi oscillations between the ⁸⁷Rb clock states, and demonstrated gate times significantly faster than the qubit decoherence time [Yavuz et al., 2006]. Next we demonstrated coherent Rabi flopping to various Rydberg levels [Johnson et al., 2008]. To move towards demonstrating a two qubit gate, we showed Rydberg blockade effects between atoms separated by ~10 μm [Urban et al., 2009]. Finally after putting all these components together we demonstrated the first CNOT gate between well separated, individually addressed neutral atoms [Isenhower et al., 2010]. This chapter will go into more detail about each of these results and cover what has been done to try to improve upon these and to continue moving towards a scalable quantum computer.

4.1 Single atom state measurement

As the first major milestone achieved in our lab, the demonstration of coherent Rabi oscillations between the clock states of ⁸⁷Rb showed that these states could be a very good basis for a qubit [Yavuz et al., 2006]. However this initial result was based on measuring the Rabi oscillations of an atomic ensemble of about 10 atoms. This paper also showed that we had achieved the ability to measure single atoms even though this was not used in demonstrating the Rabi oscillations. Single atom experiments were not used because

we did not have the ability to measure the state of a single atom using just fluorescence. To enable single qubit experiments we need a method to measure accurately the state of the qubit. Currently we employ a destructive state measurement where a specific qubit state, the state with F=2, is selectively heated out of the FORT, using light on the F=2 to F'=3 cycling transition, and the remaining atoms are measured by scattering light from both ground states.

To find the fidelity of our state measurement we need to look at several things. First we need to see how well we can distinguish a single atom from other atom numbers. This is done by fitting Gaussian peaks to each atom number peak in a histogram of many camera shot trials where atoms are randomly loaded into the FORT sites. Our system loads atoms in a stochastic manner with the number well described by a Poissonian distribution. So as an added constraint we force the Gaussian peaks fitting each atom number peak to have relative heights given by a Poisson distribution. The final fit function is

$$A\sum_{n=0}^{m} \frac{e^{-\bar{n}}\bar{n}^{n}}{n!} \frac{1}{\sqrt{2\pi (ns+b)}} e^{-\frac{(x-x_{0}-ns)^{2}}{2(ns+b)}}$$

where *A* is a multiplier to get the needed peak amplitude, *m* is the maximum number of atom peaks to use in the fit, \bar{n} is the average atom number, *s* is the number of photoelectrons produced by a single atom, and *b* is the variance of the zero atom peak. A typical histogram and corresponding fit for a 20 ms camera exposure using light that is chopped at 1.25 MHz with a 45% duty cycle and has a detuning of 1.5 Γ and intensity of ~4 I_{sat} is shown in Figure 4.1. In our experimental analysis we set cuts around the zero and one atom peaks as shown by the shaded bands around those peaks in Figure 4.1. We can then use this fit to find the probability for a certain count level to be zero, one, two, etc. atoms. This is done by integrating the Gaussian peak for each atom number over the cut values that we typically use. For typical settings this analysis shows that we select >98% of the single atom loads and have <1% chance to mislabel a zero or multiple atom count as a one atom count. However, another possibility of mislabeling a measurement as a single atom does exist. If the trap starts with multiple atoms there is some probability that a light



Figure 4.1 Single atom measurement with histogram fit. These measurements used a 20 ms camera exposure with light at a detuning of 1.5Γ , total intensity of ~4 I_{sat}, and a 1.25 MHz chopping frequency with a MOT light duty cycle of .45. These data were taken Mar. 26, 2010.

assisted collision will occur part of the way through the readout process thus ejecting one or both of the atoms involved. If this occurs in such a way that the multiple atoms scatter the same number of photons as a single atom it would be labeled as a single atom when in fact it is most likely not a single atom. We limit this process by keeping the average number of multiple loads below 15%.

We can estimate the probability of a double atom load becoming either a zero, one or two atom measurement in the second shot by selecting cuts where two atoms are loaded on the first shot and seeing what happens during the second shot. This analysis for the data shown in Figure 4.1 shows that ~25% of the double atom loads are read as two atoms in the second shot and that ~35% are labeled as a one. The rest are in the unknown bands or labeled as zero. Through repeating this analysis on several datasets, we estimate that on average about 1/3 of the double loads will turn into false single atom measurements. Even though some of these double atom loads end up as a single atom in the trap, the resulting atoms probably do not have the same thermal distribution as single atoms due to the collision with the other atom and would thus behave differently.

It is important to know how well we can measure a single atom and what the losses are for single atoms. During several parts of the experiment the FORT potential must be turned off to allow atom number measurements, optical pumping, and prevent loss of



Figure 4.2 Histogram of an atom number measurement on a set of double atom loads. The blue band marks the zero atom cut levels and the red band marks the single atom cut levels. Based on the data shown in Figure 4.1 and from other FORT sites in that same dataset.

atoms in the Rydberg state. This process plus mislabeling some double atom loads leads to about a 5% loss of single atoms between the two atom number measurements. The other major loss is from background gas collisions. The depth of our traps is very small compared to the 300 K thermal energy of the background atoms therefore any collision with a background atom will cause a loss. At our pressure of $\sim 1.2 \times 10^{-9}$ Torr, we estimate a trap lifetime of about 3.5 s as discussed in Section 3.2. This collision loss depends upon the time between the atom number measurements. In early experiments this time was typically on the order of 300 ms which gives a loss probability of about 10%. In current experiments we have improved the mechanical shutters to reduce this time to about 100 ms which should give less than 4% loss due to this mechanism. Accounting for all of these losses leads to a total single atom retention of about 90% in our current experiments. This loss is measured daily to ensure good operating conditions and is normalized out of any measurements.

4.2 Single qubit rotations

We have used the Raman laser described in Section 3.5 to drive coherent Rabi oscillations at up to 4 MHz with contrast above .98 using either of the basis states $|F, m_F\rangle = |1, 0\rangle$, $|2, 0\rangle$ or $|1, 1\rangle$, $|2, 2\rangle$. The $m_F = 0$ clock states do show longer coherence times, about 2400 μ s versus about 30 μ s for the stretched states see Figure 4.4, as expected from their lower magnetic field sensitivity. Typically we use a Rabi frequency between .5 and 1 MHz because of the limited timing resolution in our system. An example of typical Rabi oscillations is shown in Figure 4.3a). Since the beam that we use is focused to a $1/e^2$ intensity radius of about 6.5 μ m in the direction of the other FORT sites we have very little crosstalk as shown in Figure 4.3b). Based on the beam size and FORT site separations of 8.7 μ m the calculated Rabi frequency at the neighboring sites is $e^{-2(8.7/6.5)^2}\Omega = .028\Omega$. This can be used to calculate the expected amplitude of the crosstalk $\frac{(\Omega_R)^2}{(\Omega_R)^2 + \Delta'^2} = .012$ for our typical case $\Omega_R = .03\Omega \simeq 2\pi \times .014$ MHz and $\Delta' \simeq 2\pi \times .125$ MHz which is actually slightly larger than the observed value of .006.



Figure 4.3 a) Ground state Rabi oscillations at the targeted site. The fit is of the form $A(1 - \cos(2\pi ft)e^{-t/\tau})$ with values: A = .48, f = .50 MHz, and $\tau = 200 \ \mu$ s. b) Crosstalk at neighboring site. Fit parameters are A = .006, f = .33 MHz, and $\tau = \infty$. These data were taken Feb. 26, 2010.



Figure 4.4 Ramsey measurement for the clock states, a)-c), and for the stretched states d). This is done by applying a $\pi/2$ ground state rotation leaving a gap and applying a second $\pi/2$ pulse with a variable phase. As described in Section 3.5 the Raman laser accumulates a relative phase at the AC Stark shift of 125 kHz. So by using a small time variation we can vary the phase between the pulses. a) shows the Ramsey contrast measured near zero pulse gap. b) shows the Ramsey contrast near a 2 ms gap time. c) Shows the Ramsey contrast measured versus the pulse gap time. The fit gives an exponential decay constant of 2.4 ms. d) Shows a Ramsey measurement of the stretched states where the decay can be measured directly in a single curve due to the field sensitivity of the qubit states. The decay time measured was 30 μ s. The clock state data were taken Apr. 21, 2010, and the stretched state data were taken Nov. 4, 2008.

Since these Rabi oscillations are very well understood, we use them to help measure many aspects of our apparatus. The Raman beam is used to align itself and the Rydberg 780 nm beam to the FORT since they come through the same fiber. This way we know the 780 nm Rydberg beam is aligned before trying to align the 480 nm Rydberg beam. This was not possible for the stretched state computational basis because the Rydberg and the Raman beams required different polarizations. We also apply a bias field to provide a quantization axis for optical pumping and to Zeeman select a single Rydberg state to avoid interference due to exciting multiple states. The splitting of the hyperfine ground states can be measured precisely to give the bias field strength along the quantization axis for the different settings used during the experiment. Finally, by monitoring changes in the Rabi oscillation frequency and decoherence rate, we can detect small changes in beam alignment to the FORT sites to indicate when a realignment is necessary.

During quantum computing experiments, we use this system to perform state preparation, preparation of measurement states, and for other single qubit rotations needed in gate sequences. State preparation is performed by first using the optical pumping system to initialize the atoms in the $|1\rangle$ state followed by pulses from the Raman laser to create the desired input states. To measure the fidelity of this process we apply π pulses as needed to create the basis states. For a two qubit system these are the $|00\rangle$, $|01\rangle$, $|10\rangle$, and $|11\rangle$ states. Once one of these states is prepared we then measure the probability for the system to be in each of these four states. These measurements serve as a baseline for any quantum gates we want to run. The fidelity we can achieve with any quantum gate will at best be equal to the fidelity of these measurements. When we first changed to the clock states we were optically pumping using the D2 line which cannot reach complete pumping with our settings. A calculation using these settings estimates a pumping efficiency of about 95%. A typical two qubit state preparation measurement is shown in Figure 4.5

a). The population matrix values for this measurement are [Isenhower et al., 2010]

$$\begin{pmatrix} .93 \pm .07 & .08 \pm .03 & 0 & 0 \\ .04 \pm .02 & .81 \pm .07 & 0 & .01 \pm .01 \\ .07 \pm .03 & 0 & .81 & .04 \pm .02 \\ 0 & .01 \pm .01 & .04 \pm .02 & .77 \pm .07 \end{pmatrix}$$

with each row a specific input state in the order: $|00\rangle$, $|01\rangle$, $|10\rangle$, and $|11\rangle$ and the columns representing the measured output state in the same order. This matrix is an average of about 100 measurements per point. The fidelity of this matrix can be given by $F = \frac{1}{4}$ Tr [$\rho_{ideal}\rho$] = .83. Summing each row allows us to check for losses out of our computational system. On average each row sums to a value that matches well with the expected value of $.95^2 = .9$ due to imperfect optical pumping. To improve this we switched to optical pumping with the D1 transition which theoretically achieves better than 99% optical pumping with our settings. The results from this switch are shown in Figure 4.5 b). The population matrix based on an average of 50 measurements per point is

$$\left(\begin{array}{cccccc} .99 \pm .07 & .06 \pm .04 & 0 & 0 \\ 0 & .94 \pm .08 & 0 & 0 \\ 0 & 0 & .99 \pm .07 & 0 \\ 0 & 0 & .06 \pm .04 & .95 \pm .09 \end{array}\right)$$

which has F = .97 and an average low value of .01. The rows sum to 1.0 on average which shows that the optical pumping efficiency is comparable to that predicted theoretically. In addition other improvements allowing us to decrease the time between atom number measurements reduced the atom losses enough that this new matrix is even able to exceed the old corrected matrix without atom loss corrections. The uncorrected matrix is

$$\left(\begin{array}{cccccc} .90 \pm .06 & .06 \pm .04 & 0 & 0 \\ 0 & .86 \pm .07 & 0 & 0 \\ 0 & 0 & .90 \pm .06 & 0 \\ 0 & 0 & .06 \pm .04 & .86 \pm .08 \end{array}\right)$$



Figure 4.5 State preparation measurement for two qubits using a) the D2 transition for optical pumping b) the D1 transition for optical pumping. The data in a) were taken July 22, 2009, and the data in b) were taken Mar. 7, 2010.

which gives a fidelity of .88 and an average row sum of .91 which matches the measurement to measurement retention of .9 we typically measure in our current system.

In order to perform the Hadamard-Controlled phase CNOT gate we need to be able to control the phase of the Raman pulses we use to implement the Hadamard operation. The phase of the Raman laser can be controlled in two ways. First a simple delay can be used due to the difference in frequency between twice the laser current modulation and the qubits when the Raman beam is off as described in Section 3.5. This difference is measured to be ~120 kHz so a π phase shift requires a delay of ~4 μ s. The other method is to directly change the phase of the microwave source driving the current using a voltage controlled phase shifter. This allows for changes in the phase in about 150 ns which is fast enough that the sideband filter cavity lock does not notice the change. However these circuits decrease the stability of the Raman laser and require computer controlled resources we need for other systems. Therefore we have chosen to use the time delay method.



Figure 4.6 Measurement of the phase accumulation rate for the computational states relative to the ground state Raman laser. The curve is a sine fit with amplitude .44 and frequency .122 MHz. Comparing the frequency of the microwaves driving the Raman laser to the unshifted clock state splitting of ⁸⁷Rb predicts a frequency of .123 MHz. These data were taken Apr. 6, 2010.

In order to prepare for future work, we have implemented a basic composite pulse scheme to show the phase control that we have using the ground state Raman beam. This scheme is to perform some rotation and then perform the same rotation but with the opposite phase. This effectively cancels the rotation and should demonstrate a reduced sensitivity to amplitude errors that are slower than the pulse times. To show this improvement we purposely misaligned the Raman beam to give larger intensity variations due to the atomic thermal distribution and measured the results of two types of pulses. The first type consisted of two pulses that had the phase adjusted to have a multiple of 2π phase shift between them. The second scheme used the same pulse lengths but adjusted the phase so that the pulses had a π phase shift. The result of this test is shown in Figure 4.7. The in phase pulses show what looks like a normal Rabi oscillation curve but with larger decoherence as seen by the distance of the points from the curve fit around the 2π point at 1.2 μ s. The out of phase pulses stay near zero population transfer much better than the two in phase pulses showing the reduced sensitivity to amplitude noise.

We have also used the phase control of the Raman laser to perform single qubit state tomography. State tomography requires making measurements of a quantum state in three orthogonal bases. This can be performed by making a state measurement as is done normally, making a second state measurement after first applying a $\pi/2$ Rabi pulse, and finally by making a third state measurement after applying a $\pi/2$ pulse that is performed with a $\pi/2$ relative phase shift. These measurements correspond to making a measurement along the *z*, *x*, and *y* axes of the Bloch sphere. The data from these measurements are then fit to find the most probable physically realistic density matrix. This same idea can be scaled to higher numbers of qubits and enables the estimation of the full density matrix for a quantum state.

4.3 Rydberg excitations and blockade

With the good control of the single qubit rotations we could start moving towards implementing the Rydberg blockade CNOT gate. First we need to demonstrate the ability



Figure 4.7 Simple composite pulse experiment where the ground state Raman beam was purposely misaligned to increase the intensity noise due to the atom thermal distribution in the FORT sites. The red curve shows a two pulse Rabi flopping experiment with both pulses in phase and the blue curve shows the same pulses but with the phase changed by π between the two pulses. Notice the significant difference between the points at 1.2 μ s where they should both return to zero. The effects of decoherence prevent the in phase pulses from returning the atom to the $|1\rangle$ state. However, the out of phase pulses maintain the atom in the $|1\rangle$ state very well due to their lower sensitivity to this type of decoherence.



Figure 4.8 Reconstructed density matrix from a quantum state tomography measurement of the $|1\rangle$ state. The state fidelity is $F = \text{Re} [\text{Tr} [\rho_{ideal}\rho]] = .98$

to coherently excite atoms to a Rydberg state and bring them back to the ground state. We began by working at lower Rydberg levels due to their reduced sensitivity to environmental effects and higher Rabi frequencies. However, these lower levels do not have large enough interactions to cause a full blockade interaction, but decoherence when multiple atoms were present in a single FORT site showed that some interaction was occurring [Johnson et al., 2008]. To increase the Rydberg interaction we moved up to higher Rydberg levels where we were able to demonstrate Rydberg blockade with the 79D_{5/2} and 90D_{5/2} states [Urban et al., 2009].

We have continued to improve our experiment as we moved towards a CNOT gate. We now operate at the $97D_{5/2}$ Rydberg level, and we have changed to the clock states which required a change in the geometry of the bias field and excitation beams as shown in Figure 3.12. Figure 4.9 shows a typical Rabi oscillation curve to this level and an associated blockaded curve. The curve fits are done using the function $(1 - A) + A \cos(2\pi f t) e^{-\frac{t}{\tau}}$, where A is the fitted amplitude, f is the frequency, and τ is the decoherence time. We expect to see some residual oscillation in the blockaded case due to imperfect optical pumping (<2%), incomplete Rydberg excitation of the control atom ($\sim6\%$), control atom loss before the Rabi pulse (~4%), and residual blockade leakage (2%). The Rydberg Rabi oscillations typically have a decoherence time measured to be about 10 μ s. This short time is responsible for the majority of the incomplete Rydberg excitation as the excitation probability for a π pulse, about 650 ns, is about $e^{-\frac{.65}{10}} = .94$. We believe that the decoherence is caused by intensity noise in the excitation lasers, by the spatial distribution of the atoms in the FORT due to their temperature, and to a lesser degree by any bias or external field noise. Combining these we can get an estimate of the expected amplitude of the blockaded curve. Doing this we find $.98 \times .94 \times .96 \times .98 = .86$ giving an expected contrast of ~.14. This simple estimation is not far from the measured value of .2 for the data in Figure 4.9.

We have also tried to use the $102D_{5/2}$ state, however, the large sensitivity of Rydberg levels was demonstrated at this level. The nearest Rydberg level to $102D_{5/2}$ is $103P_{3/2}$



Figure 4.9 Rabi oscillations to the $97D_{5/2}$ level for a) single atoms and b) blockaded single atoms. The blockading atom is about 8.7 μ m away. The curves are fits of the form $1 - A + A \cos(2\pi f t) e^{-t/\tau}$. For a) the values are A = .46, f = .97 MHz, and $\tau = 5.3 \ \mu$ s and for b) A = .1, f = .97 MHz, and $\tau = \infty \ \mu$ s. These data were taken Dec. 23, 2009.



Figure 4.10 Rabi oscillations to the $102D_{5/2}$ level. The fast loss and decoherence is due to a nearby Rydberg level that is coupled by microwaves used by GSM cell phones. The fit is of the modified form $(1 - A + A \cos (2\pi ft)) e^{-t/\tau}$ with A = .38, f = .46 MHz, and $\tau = 2.8 \ \mu$ s. These data were taken May 13, 2009.

which is separated by 1.91 GHz which happens to be very close to the GSM cell phone spectrum. When trying to perform Rabi oscillations to $102D_{5/2}$ we see fast decoherence and loss of the atom as shown in Figure 4.10. This is due to microwaves emitted by GSM cell phones causing Rabi oscillations to the $103P_{3/2}$ state where they cannot be returned to the ground state and become photoionized by the FORT and lost. The fit gives an exponential decay time of 2.8 μ s. This is a nice example of the unexpected problems that can occur when using highly sensitive Rydberg levels and shows that care must be taken in choosing the Rydberg level to use.

4.4 Two qubit Rydberg blockade CNOT gate

With the demonstration of Rydberg blockade we had all the necessary components to put together a two qubit Rydberg blockade CNOT gate. Putting all these components together at the same time is not a trivial task. We first tried to combine these components using the stretched states, $|1, 1\rangle$ and $|2, 2\rangle$, as the computational basis and the 90D_{5/2} Rydberg level but soon found that the coherence time for these ground states was not long

enough to give high enough fidelity gates. This poor coherence time is evident when trying to find the correct delay time between the $\pi/2$ pulses which implement the Hadamard gates in the $H-C_z$ CNOT sequence. This measurement corresponds to a Ramsey fringe type measurement because the delay introduces a phase shift between the ground state Raman laser and the atoms. To measure the correct delay for the gate the Rydberg pulses were also implemented in this measurement but detuned from a Rydberg state to avoid any actual transition effects but keeping the phase shifts on the computational states due to the AC Stark shifts of these beams. The experimental sequence and results are shown in Figure 4.11. At that time we chose to use a time delay rather than use the phase control circuitry because we had evidence that those circuits decreased the stability of the ground state Raman laser. The delay needed for the correct phase plus the time needed for the Rydberg pulses combined to be about 9 μ s which was too long compared to the decoherence time of about 30 μ s resulting in low contrast Ramsey fringes. Even if using the electronic phase shifter the minimum delay was limited to about 5 μ s which was needed for the Rydberg pulses and beam pointing changes. So even in this best case the maximum contrast we would expect was $e^{-5/30} = .85$. This problem plus the additional errors we still had due to imperfect pulses and blockade leakage combined to give too many errors preventing the measurement of any significant gate operation.

To try to reduce our sensitivity to the poor ground coherence, we took an idea from [Ohlsson et al., 2002] and tried the amplitude swap gate discussed in Section 2.5. This version of the gate only uses π pulses which we thought would reduce the sensitivity to the pulse phases enough to perform a gate operation. In addition the removal of the need to get the proper phase matching of the pulses decreased the gate time from 8-10 μ s to about 7 μ s. However, these changes did not gain enough to override the other errors of atom loss, blockade leakage, and pulse errors we still had. At this point we decided to switch to the clock states, $|1, 0\rangle$ and $|2, 0\rangle$, to greatly reduce the errors we had due to the ground state coherence time and also increase the blockade shift by going to a higher Rydberg level. These changes led to demonstrating the CNOT gate with a fidelity of



Figure 4.11 a) The experiment sequence used when finding the correct delay for the $\pi/2$ pulses in the H-C_z CNOT gate. b) This plot shows the results of the experiment sequence shown in a). This curve was done by detuning the Rydberg 480 nm laser from the Rydberg transition to remove any Rydberg excitation effects while keeping any phase shifts due to differential AC Stark shifts of the qubit states. The contrast measured here, .46±.15 according to the fit, places an upper limit on the gate fidelity we would expect with these settings.



Figure 4.12 Measured population matrices for the CNOT gates: a) the amplitude swap style gate and b) the $H-C_z$ gate. The data in a) were taken July 21, 2009, and the data in b) were taken Oct. 19, 2009.

.73 [Isenhower et al., 2010]. The Amplitude-Swap gate had a measured gate population matrix of

$$.05 \pm .03$$
 $.71 \pm .07$ $.03 \pm .02$ $.11 \pm .04$ $.75 \pm .06$ $.01 \pm .01$ $.02 \pm .02$ $.02 \pm .02$ $.02 \pm .01$ $.04 \pm .02$ $.72 \pm .05$ $.08 \pm .02$ 0 $.01 \pm .01$ 0 $.73 \pm .07$

and the $H-C_z$ gate was measured to give

$$\begin{pmatrix} .05 \pm .03 & .74 \pm .09 & .02 \pm .02 & .04 \pm .03 \\ .73 \pm .09 & .06 \pm .03 & .02 \pm .02 & .02 \pm .02 \\ 0 & .02 \pm .02 & .79 \pm .09 & .12 \pm .05 \\ .02 \pm .02 & .03 \pm .03 & .06 \pm .04 & .63 \pm .09 \end{pmatrix}$$

These matrices are based on the average of about 100 measurements per entry. These measured gate populations are shown in Figure 4.12. The fidelity is given by $F = \frac{1}{4} \text{Tr} \left[\rho_{ideal}\rho\right]$ so $F_{AS} = .73$ and $F_{HC_z} = .72$.

These measurements showed promise because the ratio of the average peak to the average low value is about 20 showing that fixing the atom losses should give a rapid increase in gate fidelity. The errors in these measurements we attributed to several sources: atom loss before the gate operation, imperfect optical pumping, pulse errors, and Rydberg blockade leakage. The first two were the major contributors. The chance for one of the two atoms to be lost between the first atom number measurement and the gate is mostly due to background gas collisions. We have measured a lifetime of about 3.5 s for atoms in our FORT sites and the time before the gate was about .13 s which leads to a loss probability of ~4% per atom. The imperfect optical pumping from using the D2 transition also left about 5% of the atoms in the wrong state. Combined this means that the input state matches the desired state with only about $(.96 \times .95)^2 = .84$ probability due to one of the qubits being lost or in the wrong state. These problems are also seen in the state preparation measurements reported in Section 4.2. So assuming no other errors are introduced these problems limit the gate fidelity to ~.84.

In the effort to improve on these measurements we changed to optical pumping using the D1 transition as discussed in Section 3.4 and have improved the shutter setup for the MOT beams allowing for a smaller gap between the atom number measurement and the start of the gate. We estimate that the optical pumping efficiency is about .98 and that the atom losses before the gate are now only 1%. So combining these the expected probability that the gate starts with the desired state is $(.99 \times .98)^2 = .94$. This significant improvement is seen in the state preparation measurement shown in Figure 4.5 and in the Hadamard-C_z CNOT gate which has a measured population matrix of

$$\begin{pmatrix} .06 \pm .03 & .90 \pm .07 & .02 \pm .02 & .05 \pm .03 \\ .94 \pm .08 & .03 \pm .03 & .02 \pm .02 & .02 \pm .02 \\ 0 & .02 \pm .02 & .66 \pm .09 & .04 \pm .03 \\ .02 \pm .02 & .03 \pm .02 & .08 \pm .03 & .90 \pm .07 \end{pmatrix}$$

based on an average of 50 measurements per entry and is shown in Figure 4.13. The fidelity of the matrix is F = .85


Figure 4.13 H-C_z CNOT gate after the improvements to our apparatus. These data were taken Mar. 29, 2010

The remaining errors in our gate are due to pulse errors. The pulse errors have two major contributions. First the atom temperature causes errors due to doppler shifts of the excitation beams. This error causes incomplete Rydberg excitation and is calculated to add a 2% error per π pulse. The second major component is due to spontaneous emission from the intermediate state during the Rydberg pulses. For a π pulse it is calculated that the probability of spontaneous emission is also 2%. Incomplete optical pumping adds about 1% error per atom. Combining the known errors from state preparation, spontaneous emission, and doppler broadening gives a predicted fidelity of .99² × .98² × .98² = .9. This value matches well to what we would have if we can improve the one poor peak point (.66) to match the other three. So with improvements to these known errors we should be able to continue to increase our gate fidelity.

4.5 Entanglement Generation

To verify the quantum nature of this gate we attempt to generate entanglement by placing the control atom in a superposition state and then operating the gate. For example if we use our $CNOT_{\pi}$ gate on the input state $\left(\frac{|0\rangle+|1\rangle}{\sqrt{2}}\right) \otimes |1\rangle$ for the control and target qubits respectively then the output is $\frac{|00\rangle+|11\rangle}{\sqrt{2}}$ which is one of the maximally entangled Bell states. For our implementation we calculate the expected density matrix to be

$$\rho = \left(\begin{array}{rrrr} .5 & 0 & 0 & -.5 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -.5 & 0 & 0 & .5 \end{array}\right)$$

without accounting for the qubit phase shifts due to the Rydberg excitation pulses. We first measured the populations on the diagonal and found

$$\left(\begin{array}{c} |00\rangle\\ |01\rangle\\ |10\rangle\\ |11\rangle\end{array}\right) = \left(\begin{array}{c} .47 \pm .07\\ 0\\ 0\\ .50 \pm .06\end{array}\right)$$

As described in Section 2.6 we can measure the expected coherence by applying a variable phase $\pi/2$ analysis pulse to both qubits after the gate and measure the resulting parity.

$$P' = 2|c_2|\cos(\phi_2 + \zeta) - 2|c_1|\cos(2\phi + \phi_2 + \xi)$$

where c_2 is the coherence of the $|01\rangle$ and $|10\rangle$ states, ϕ_2 is the phase difference between the two $\pi/2$ analysis pulses, ζ is the phase of c_2 , c_1 is the coherence of the $|00\rangle$ and $|11\rangle$ states, ϕ is the phase of the analysis pulses, and ξ is the phase of c_1 . For this state we expect to have an oscillating function with no offset since $|c_2|$ should be zero. Because we know the time delay between the two pulses, .5 μ s, and the phase accumulation rate, $2\pi \times .125$ MHz, we find $\phi_2 = 2\pi \times (.125 \text{ MHz}) \times (.5 \ \mu s) = .4$.

This measurement for the $(|00\rangle + |11\rangle) / \sqrt{2}$ state is shown in Figure 4.14. The curve is a fit to the parity equation just given modified to set $\phi_2 = .4$ and $\phi = 2\pi$ (.125 MHz) twhich are known values from other measurements leaving only the coherences, c_1 and c_2 , and the phases of these coherences, ξ and ζ , as the free parameters. This fit resulted in the values: $c_1 = .25 \pm .02$, $\xi = -.77 \pm 2 \times 10^{-4}$, $c_2 = .03 \pm .01$, and $\zeta = 1.8 \pm .07$. From this value for c_1 we calculate a fidelity of $F = \frac{1}{2} (P_{00} + P_{11}) + c_1 = .73 \pm .10$ putting this state more than two standard deviations above the threshold for entanglement.

4.6 Deterministic single atom loading

Now that we have demonstrated that a Rydberg blockade CNOT gate works, we are focusing on the remaining quantum computer criteria: scalability and reaching the



Figure 4.14 a) Measured populations for the $\frac{|00\rangle + |11\rangle}{\sqrt{2}}$ state. b) Parity measurement of the $\frac{|00\rangle + |11\rangle}{\sqrt{2}}$ entangled state after applying a $\pi/2$ rotation with a varying phase to both qubits. These data were taken Mar. 26, 2010.

threshold for error correction. Scaling to many qubits will require some changes in our current setup but some of the work using more than two qubits can be done with the current apparatus. Our plan for scaling neutral atom quantum computing involves putting the qubits in an optical array. This will likely be an array of blue detuned FORTs based on the bottlebeam approach discussed in Section 3.2 with a site spacing of about 5 μm and FORT beam sizes of .5 and 1 μm for each individual bottlebeam. A system with 49 sites is currently being implemented in a Cs experiment. A major hurdle for this scheme is to fill each site with a single atom. This complete loading scheme is probably not necessary for scalability of a neutral atom quantum computer but would increase the number of qubits a computer could have. Currently our loading procedure gives Poissonian loading which limits the single atom loading probability to about .37. Schemes that take advantage of light assisted collisions have demonstrated a loading probability of .5 [Schlosser et al., 2001]. To use fractional loading for quantum computing a measurement of which sites had been loaded would need to be made to know which qubit positions to use during the computation, or a technique which selectively fills individual sites is needed.

To improve on this, Rydberg blockade has been suggested as a way to obtain a loading probability of close to unity. There are several ways this can be done, but the general idea is that a site is loaded with multiple atoms and then a pulse or series of pulses that pass through a Rydberg state can place one atom into a different state from the remaining atoms. Then a blow-away pulse is used to remove the unwanted atoms leaving a single atom in the site. One version was proposed in [Saffman and Walker, 2002] and performs this procedure by first applying a Rydberg π/\sqrt{N} pulse that takes advantage of the \sqrt{N} enhancement provided by Rydberg blockade to excite a single atom to the Rydberg state. Then a separate Rydberg π pulse with pulse area set for a single atom de-excites the atom to the other ground state thus putting one atom in one ground state, for example F=1 in ⁸⁷Rb, and the remaining atoms in another ground state, e.g. F=2 in ⁸⁷Rb, which then gets ejected from the trap. This method requires a fairly high number of atoms to be loaded

initially so that the \sqrt{N} enhancement does not vary as much for small variations in atom number in order to achieve high fidelities.

A second method which tries to avoid this problem was proposed by [Mølmer, unpublished]. This proposal uses many pulses to slowly take a few atoms down to just a single atom again by using the faster collective Rabi frequency due to blockade. This is done by applying a single atom 2π pulse. If more than one atom is in the trap then more than a 2π rotation is performed leaving some probability of a single atom to be in the Rydberg state. Next a step is performed that causes the loss of this Rydberg atom and these two steps are repeated. One initial problem with this method is there are certain numbers of atoms which would have zero probability to be in the Rydberg state such as for N=4, however, this can be avoided by using pulses that are detuned slightly from the Rydberg state which removes this degeneracy for reasonable values of N. This scheme does require that the 2π pulses have a high fidelity so that after many applications there is still a high probability for a single atom to be in the site.

4.7 Conclusions

The hard work of our group to understand and improve the systems we use to control the electronic states of ⁸⁷Rb has lead to the first demonstration of a CNOT quantum gate between two individually addressed qubits. This gate was used to generate entanglement on demand with a fidelity of .73. The techniques used scale for use in 2D arrays of qubits and should be capable of making a quantum register with up to several hundred fully interconnected qubits. Before this stage can be reached several aspects must be improved. The ability to load every site or to know which sites have been loaded is a key hurdle that needs to be passed. Several techniques for this look very promising. In addition improvements need to be made to reduce pulse errors and reduce atom losses to improve the fidelity of the gate operation. The atom losses can be significantly improved by moving to a system with a lower background pressure. This should also enable an increase in data collection as the current system can only handle one measurements per loading cycle due to the background collision losses and the destructive state readout.

The current apparatus should be able to perform operations on up to 5 qubits. This allows for studies of Toffoli gates as well. The current plan takes advantage of the trap geometry and the scaling of the Rydberg blockade shift to implement this gate. Future work will also include demonstrations of composite pulse schemes to help improve the gate fidelity, work towards achieving deterministic single atom loading using Rydberg blockade, and non-destructive state readout. In addition our group is continuing to work towards a larger scale apparatus which can handle up to 50 qubits in a 2D array and will take advantage of the many discoveries from this first apparatus.

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