In this lab, we are going to build the circuit of Figure 1 to investigate propagation characteristics of transmission lines. From reflection measurements, we will determine the propagation speed of waves inside a transmission line, and measure its impedance.

Noting Figure 1, consider a transmission line of length $L$, impedance $Z_0$. Let’s denote the propagation speed of electromagnetic waves inside the line as $u_0$. We supply a voltage pulse with time duration of $T$ at point $C$ of the circuit. When the pulse hits point $B$, due to impedance mismatch, part of the wave is reflected. This reflected wave encounters a similar phenomenon when it reaches point $A$. As a result, multiple reflections can be observed at points $A$ and $B$.

Figure 1: The circuit diagram for the transmission line experiment
1) Assume that $T << \frac{L}{u_0}$, and $Z_T, Z_i >> Z_0$. Plot what you would expect to see on points A and B (if we look on a scope for example).

2) Repeat (1), but now assume that $Z_T = 0$ (short circuit). What would you expect to see on points A and B.

3) Repeat (1), but now let’s introduce some loss to our system (in reality every circuit is lossy). Let’s assume that, while propagating from A to B, 10% of the wave is attenuated. What would you expect to see on points A and B.
4) In the lecture, we have derived the telegraph equations for a transmission line. The plane wave solution for these equations is of the form

\[ V(z, t) = V_0 \cos(\omega_0 t - k_0 z), \tag{1} \]

where \( \omega_0 \) is the angular frequency of the wave. As we have also derived in the lecture, the phase velocity for a monochromatic wave is

\[ u_0 = \frac{\omega_0}{k_0} = \frac{1}{\sqrt{L_L C_L}}, \]

where \( L_L \) is the inductance per unit length, and \( C_L \) is the capacitance per unit length. When the wave contains multiple frequencies, its velocity can be different than the phase velocity. For the simple case of a two frequency wave,

\[ V(z, t) = V_0 \cos(\omega_0 t - k_0 z) + V_0 \cos(\omega_1 t - k_1 z), \tag{2} \]

show that the wave will propagate with a velocity of

\[ u_g = \frac{\omega_1 - \omega_0}{k_1 - k_0} \equiv \frac{\Delta \omega}{\Delta k} \tag{3} \]

The quantity, \( u_g \) is called the group velocity of the wave.

5) In the laboratory, we are going to measure the velocity of a square pulse, by measuring the time it takes for the pulse to travel from point A to point B. Are we measuring the phase velocity or the group velocity? (In our laboratory experiment these two velocities are almost identical \( u_g \approx u_0 \), and we will not worry about the difference between the two velocities).

6) (Optional) Rigorously prove your answer to question (5), by taking the Fourier transform of a time domain square pulse, propagating each Fourier component and then reconstructing the pulse. What is the condition that the pulse doesn’t change its shape while propagating through the transmission line?