Today’s Topics
- Sources of the Magnetic Field (Ch. 30)
- Review of Biot-Savart Law
- Ampere’s Law
- Magnetism in Matter

Magnetic Fields (Biot-Savart): Summary

Current loop, distance \( x \) on loop axis (radius \( R \)):

\[
B_x = \frac{\mu_0 IR^2}{2(x^2 + R^2)^{3/2}}
\]

Center of arc (radius \( R \), angle \( \theta \)):

\[
B_{center} = \frac{\mu_0 I\theta}{4\pi R}
\]

Straight wire: finite length

\[
B = \frac{\mu_0 I}{4\pi a} (\cos \theta_1 - \cos \theta_2)
\]

infinite wire:

\[
B = \frac{\mu_0 I}{2\pi a}
\]

Ampere’s Law

Ampere’s Law:

\[
\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}
\]

- applies to any closed path, any static B field
- useful for practical purposes only for situations with high symmetry

Ampere’s Law: B-field of \( \infty \) Straight Wire

Use symmetry:

Choose loop to be circle of radius \( R \) centered on the wire in a plane \( \perp \) to wire.

Why?

Magnitude of \( B \) is constant (function of \( R \) only)
Direction of \( B \) is parallel to the path.

\[
\oint \vec{B} \cdot d\vec{l} = \oint Br d\theta = 2\pi rB = \mu_0 I_{encl} = \mu_0 I
\]

\[
B = \frac{\mu_0 I}{2\pi r}
\]
**B Field Inside a Long Wire**

Total current $I$ flows through wire of radius $a$ into the screen as shown.

What is the $B$ field inside the wire?

By symmetry -- take the path to be a circle of radius $r$:

$$\oint \vec{B} \cdot d\vec{l} = B2\pi r$$

- Current passing through circle:

$$I_{\text{encl}} = \frac{r^2}{a^2} I$$

$$\oint \vec{B} \cdot d\vec{l} = B2\pi r = \mu_0 I_{\text{encl}} \Rightarrow B_{\text{in}} = \frac{\mu_0 I r}{2\pi a^2}$$

**Ampere's Law: Toroid**

Toroid: $N$ turns with current $I$.

$B_\theta = 0$ outside toroid!

(Consider integrating $B$ on circle outside toroid: net current zero)

$B_\theta$ inside: consider circle of radius $r$, centered at the center of the toroid.

$$\oint \vec{B} \cdot d\vec{l} = B2\pi r = \mu_0 I_{\text{encl}} = \mu_0 NI$$

$$B = \frac{\mu_0 NI}{2\pi r}$$

**B Field of a Long Wire**

Inside the wire: $(r < a)$

$$B = \frac{\mu_0 I}{2\pi r}$$

Outside the wire: $(r > a)$

**B Field of a Solenoid**

Inside a solenoid: source of uniform $B$ field

Solenoid: current $I$ flows through a wire wrapped $n$ turns per unit length on a cylinder of radius $a$ and length $L$.

If $a \ll L$, the $B$ field is approximately contained within the solenoid, in the axial direction, and of constant magnitude.

In this limit, can calculate the field using Ampere's Law!
**Ampere’s Law: Solenoid**

The B field inside an ideal solenoid is:

\[ B = \mu_0 nI \]

\( n = \frac{N}{L} \)

**Magnetism in Matter**

The B field produced along the axis of a circular loop (radius R) by a current I is:

\[ \vec{B} = \frac{\mu_0 I}{2\pi z} \hat{z} \quad \text{typical dipole behaviour} \]

\( \mu \) is the magnetic moment = \( I \cdot \text{area} \)

and \( z \gg R \)

Materials are composed of particles that have magnetic moments -- (negatively charged electrons circling around the positively charged nucleus).

\[ \mu_i = \frac{q_i \hbar}{2m_e} = 9.27 \times 10^{-24} \frac{J}{T} \]

**Magnetization**

Apply external B field \( B_0 \). Field is changed within materials by these magnetic moments.

Magnetization: total magnetic moment per unit volume

\[ \vec{M} \equiv \frac{\mu_{\text{total}}}{V} \]

The B field in the material is

\[ \vec{B} = \vec{B}_0 + \mu_0 \vec{M} \]

Define H (magnetic field strength):

\[ \vec{B} = \mu_0 (\vec{H} + \vec{M}) \]

\( \vec{H} = \vec{B}_0 / \mu_0 \)

Magnetic susceptibility:

\[ \vec{M} = \chi \vec{H} \]

\[ \vec{B} = \mu \vec{H} = \mu_0 (1 + \chi) \vec{H} \quad \text{“permeability”} \quad \mu (= \kappa_m \mu_0) \]

Material is classified by magnetic susceptibilities:

- **Paramagnetic** (aluminum, tungsten, oxygen, etc.)
  Atomic magnetic dipoles line up with the field, increasing it. Only small effects due to thermal randomization: \( \chi \sim +10^{-5} \)

- **Diamagnetic** (gold, copper, water, etc. as well as superconductors)
  - Applied field induces an opposing field; usually very weak \( \chi \sim -10^{-5} \)

- **Ferromagnetic** (iron, cobalt, nickel, etc.)
  - Dipoles prefer to line up with the applied field (similar to paramagnetic), but tend to all line up the same way due to collective effects: very strong enhancements \( \chi \sim +10^{13} \cdot 10^{-5} \)

Magnetic susceptibility temperature dependent
(above range of typical values at T=20°C)
Ferromagnets

- Dipoles tend to strongly align over small patches – “domains” (even w/o external magnetic field). With external field, the domains align to produce a large net magnetization.

“Soft” ferromagnets
- Domains re-randomize when magnetic field is removed

“Hard” ferromagnets
- Domains persist even when the field is removed
- “Permanent” magnets
- Domains may be aligned in a different direction in a new external field
- Domains may be re-randomized by sudden physical shock
- If temperature is raised above “Curie point” (770 °C for iron), domains will also randomize (like a paramagnet)