Physics 202, Lecture 19

Today’s Topics
- Power in RLC
- Resonance in RLC
- Wave Motion (Review ch. 16)
  - General Wave
  - Transverse And Longitudinal Waves
  - Wave Function
  - Wave Speed
  - Sinusoidal Waves
  - Wave and Energy Transmission

Course Plan
- Today: Power and Resonances in RLC
- Today: General Waves
  - A review of some Phy201 contents. (Chapter 16 of text)
- Next week: Electromagnetic Waves
  - Tuesday: Section 34.1-3
  - Thursday: Section 34.4-7
- Thereafter: Light and Optics (Chapter 35-38)

Note: Exam 3 will cover chapters 31-34.

Please preview text before each lecture.

Exam 2 Result

Exam 2 Curve

Note: Boundaries for letter grades are for reference only. (A=15%, B=50%...)
(subject to final curve at the end of semester)

Review: Summary of Phasor Relationship

I_R and ΔV_R in phase

I_L 90° behind ΔV_L

I_C 90° ahead of ΔV_C

I_R = |ΔV_R|/R

I_L = |ΔV_L|/X_L

I_C = |ΔV_C|/X_C
Current And Voltages in a Series RLC Circuit

\[ i = I_{\text{max}} \sin(\omega t) \]

\[ \Delta v_R = (\Delta V)_{\text{max}} \sin(\omega t) \]

\[ \Delta v_L = (\Delta V)_{\text{max}} \sin(\omega t + \pi/2) \]

\[ \Delta v_C = (\Delta V)_{\text{max}} \sin(\omega t - \pi/2) \]

\[ \Delta V_{\text{max}} \sin(\omega t + \phi) \]

\[ Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + (\omega L - \omega C)^2} \]

\[ \phi \text{ when } \omega L = \omega C \]

Lowest impedance \rightarrow \text{largest current} \rightarrow \text{resonance}

For a general AC circuit, at resonance:

\[ \begin{align*}
& \text{Impedance is at lowest} \\
& \text{Phase angle is zero. (I is "in phase" with } \Delta V) \\
& \text{I}_{\text{max}} \text{ is at highest} \\
& \text{Power consumption is at highest}
\end{align*} \]

Power in AC Circuit

\[ P(t) = i(t) \Delta V(t) \]

True for any circuit, AC or DC

In an AC circuit, current and voltage on any component can be written in general:

\[ \begin{align*}
& \Delta V(t) = (\Delta V)_{\text{max}} \sin(\omega t) \\
& i(t) = I_{\text{max}} \sin(\omega t - \phi) \\
& P(t) = I_{\text{max}} \sin(\omega t - \phi) \cdot (\Delta V)_{\text{max}} \sin(\omega t) = I_{\text{max}} (\Delta V)_{\text{max}} \sin(\omega t - \phi) \sin(\omega t) \\
& P_{\text{average}} = \frac{1}{2} I_{\text{max}} (\Delta V)_{\text{max}} \cos(\phi) \\
& \text{(see board)}
\end{align*} \]

For resistor: \( \phi = 0 \) \( P_{\text{average}} = \frac{1}{2} I_{\text{max}} (\Delta V)_{\text{max}} \cos(\phi) \)

For inductor: \( \phi = \pi/2 \) \( P_{\text{average}} = \frac{1}{2} I_{\text{max}} (\Delta V)_{\text{max}} \cos(\phi) = 0 \)

For Capacitor: \( \phi = -\pi/2 \) \( P_{\text{average}} = \frac{1}{2} I_{\text{max}} (\Delta V)_{\text{max}} \cos(\phi) = 0 \)

(Ideal inductors and capacitors NEVER consume energy!)

For a general AC circuit:

\[ P_{\text{average}} = \frac{1}{2} I_{\text{max}} (\Delta V)_{\text{max}} \cos(\phi) = \frac{1}{2} I_{\text{max}} Z \cos(\phi) \]

\[ Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + (\omega L - \omega C)^2} \]

Wide used definitions:

\[ I_{\text{rms}} = \frac{I_{\text{max}}}{\sqrt{2}} , \quad \Delta V_{\text{rms}} = \frac{\Delta V_{\text{max}}}{\sqrt{2}} \]

Resonances In Series RLC Circuit

\[ \text{The impedance of an AC circuit is a function of } \omega. \]

\[ \text{e.g. Series RLC:} \]

\[ \begin{align*}
& \text{I}_{\text{max}} \text{ is at highest} \\
& \text{Power consumption is at highest}
\end{align*} \]

General Waves (Review of Ch. 16)

\[ \text{Wave:} \]

\[ \text{Propagation of a physical quantity in space over time} \]

\[ q = q(x, t) \]

\[ \text{Examples of waves:} \]

\[ \text{Water wave, wave on string, sound wave, earthquake wave, electromagnetic wave, "light", quantum wave…} \]

Waves can be transverse or longitudinal.
Wave On A Stretched Rope

- It is a transverse wave
- The wave speed is determined by the tension and the linear density of the rope:

$$v = \sqrt{\frac{T}{\mu}}; \mu \equiv \frac{\Delta m}{\Delta l}$$

conceptual understanding only

Seismic Waves

- Longitudinal
- Transverse
- Transverse
- Transverse

Quiz: Electro-Magnetic Waves are Transverse or Longitudinal?

- Two polarizations possible (will explain next week)

Wave Function

- Waves are described by wave functions in the form:

$$y(x,t) = f(x-vt)$$

- $y$: A certain physical quantity (e.g., displacement in y direction)
- $f$: Can be any form
- $x$: space position
- $t$: time. Its coefficient $v$ is the wave speed.
- $v>0$ moving right
- $v<0$ moving left
- Coefficient arranged to be 1
Practical Technique: Identify Wave Speed in A Wave Function

- A wave function is in the form:
  \[ y(x,t) = \frac{2}{(x - 3.0t)^2 + 1} \]

  - What is the wave speed:
    - Answer: 3.0 m/s to the right

  - Illustrate wave form at t = 0s, 1s, 2s

Sinusoidal Wave

- A wave describe a function \( y = A \sin(kx - \omega t + \phi) \) is called sinusoidal wave. (Harmonic wave)
- Quick Quiz: can you see that its wave speed \( v = \omega / k \)?
- Quick Quiz: At each fixed position x, the element is undergoing a harmonic oscillation. Can you see it? What are the amplitude, frequency and phase constant of this oscillation? (|A|, \( \omega \), \(-kx + \phi\))
- If we fix t and take a snap shot, the wave form is periodic over space \( x \). The spacing period \( \lambda \) is called wavelength and \( \lambda = 2\pi / k \) (see next page)

Parameters For A Sinusoidal Wave

- Snapshot with fixed t:
  - wave length \( \lambda = 2\pi / k \)
- Snapshot with fixed x:
  - angular frequency = \( \omega \)
  - frequency \( f = \omega / 2\pi \)
  - Period \( T = 1/f \)
  - Amplitude = A
- Wave Speed \( v = \omega / k \)
  - \( v = \lambda / T \), or
  - \( v = 1 / T \)
- Phase angle difference between two positions \( \Delta \phi = -k \Delta x \)

Waves Transfer Energy

- As motion in propagating in the form of wave in a medium, energy is transmitted.

  - It can be shown that the rate of energy transfer by a sinusoidal wave on a rope is:
    \[ P = \frac{1}{2} \mu A^2 \omega^2 v \]
  - Note: power dependence on \( A, \omega, v \)
Linear Wave Equation

- Linear wave equation
  \[ \frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2} \]
  
  Certain physical quantity

- Sinusoidal wave
  \[ y = A \sin \left( \frac{2\pi}{\lambda} x - 2\pi ft + \phi \right) \]
  
  A: Amplitude
  f: Frequency
  \( \phi \): Phase

Wave speed

General wave: superposition of sinusoidal waves