Physics 202, Lecture 21

Today’s Topics

- Electromagnetic Waves (EM Waves)
  - Review: Maxwell’s equations and EM waves
  - Propagation of E and B
  - Properties of EM Wave
  - Energy Carried by EM Wave, Poynting Vector
  - Momentum Carried by EM Wave

- About Exam 3
Maxwell’s Equations and EM Waves

Maxwell equations when there is no charge and current:

\[ \oint E \cdot dA = 0 \]

\[ \oint B \cdot dA = 0 \]

differential forms: (single polarization)

\[ \frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t} \]

\[ \frac{\partial B_z}{\partial x} = -\mu_0 \varepsilon_0 \frac{\partial E_y}{\partial t} \]

\[ \frac{\partial^2 E_y}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E_y}{\partial t^2} \]

\[ \frac{\partial^2 B_z}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 B_z}{\partial t^2} \]
Electromagnetic Waves

- **EM wave equations:**
  \[
  \frac{\partial^2 E_y}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E_y}{\partial t^2}
  \]
  \[
  \frac{\partial^2 B_z}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 B_z}{\partial t^2}
  \]

- **Plane wave solutions:**
  \[
  E = E_{\text{max}} \sin(kx - \omega t + \phi) \quad B = B_{\text{max}} \sin(kx - \omega t + \phi)
  \]

- **Properties:**
  - No medium is necessary.
  - E and B are normal to each other.
  - E and B are in phase.
  - Direction of wave is normal to both E and B (EM waves are transverse waves).
  - Speed of EM wave: \( c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 2.9972 \times 10^8 \text{ m/s} \)
  - \( E/B = E_{\text{max}}/B_{\text{max}} = c \)
  - Transverse wave: two polarizations possible

\( \)
The EM Wave

Two polarizations possible
Energy Carried By EM Waves

- Recall: energy densities \( u_E = \frac{1}{2} \varepsilon_0 E^2 \), \( u_B = \frac{1}{2} \frac{B^2}{\mu_0} \)

- For an EM wave, at any time/location,
  \( u_E = \frac{1}{2} \varepsilon_0 E^2 = \frac{1}{2} \frac{B^2}{\mu_0} = u_B \) (using \( E/B = c \))
  - In an electromagnetic wave, the energies carried by electric field and magnetic field are always the same.

- Total energy stored (per unit of volume):
  \( u = u_E + u_B = \varepsilon_0 E^2 = \frac{B^2}{\mu_0} \)

- Power transmitted per unit of area (flux) is equal to \( uc \) in the direction of wave

- Poynting vector: \( S = \frac{1}{\mu_0} E \times B \), \( S = \frac{1}{\mu_0} EB = uc \)
  - Averaging over time:
    \( u_{av} = \frac{1}{2} \varepsilon_0 E_{max}^2 = \frac{1}{2} \frac{B_{max}^2}{\mu_0} \), \( S_{av} = u_{av} c = I \)
    (intensity, flux)
Momentum Carried By EM Waves

- EM waves: momentum = energy/c

Change of momentum in 100% absorption

\[ \Delta p = \frac{\Delta U}{c} = \frac{uA c \Delta t}{c} = uA \Delta t \]

Change of momentum in 100% reflection

\[ \Delta p = 2 \frac{\Delta U}{c} = 2 \frac{uA c \Delta t}{c} = 2uA \Delta t \]

Radiation Pressure (P):

\[ P = \frac{F}{A} = \frac{\Delta p}{\Delta t} = \frac{S}{c} \]

100% absorption

\[ \Delta p = p \rightarrow P = \frac{S}{c} \]

100% reflection

\[ \Delta p = 2p \rightarrow P = \frac{2S}{c} \]

Mariner 10: “Sail on sunlight”
Example: Solar Radiation (Sun Lights)

The average intensity of the EM radiation from the Sun on Earth is $S \sim 10^3 \text{ W/m}^2$

- What is the average radiation pressure for 100% absorption:

$$P = \frac{S}{c} = \frac{10^3 \text{ W/m}^2}{3 \cdot 10^8 \text{ m/s}} = 3.3 \cdot 10^{-6} \text{ N/m}^2$$

- What is the force exerted by EM radiation by the Sun on a surface of 1 m$^2$ (with 100% absorption)

$$F = PA = 3.3 \cdot 10^{-6} \text{ N/m}^2 \cdot 1 \text{ m}^2 = 3.3 \cdot 10^{-6} \text{ N}$$
Solar Energy

- Sun transmit its energy to the Earth in the form of EM wave (lights, visible or invisible)
- All activities on earth are powered by energy from the Sun, directly or indirectly.
- Solar energy is HUGE: 1 hour of solar energy to earth is sufficient to supply our energy consumption in 1 year! (a ratio of 1:10000)

- Solar to electricity: high tech in renewable energy
  - Solar thermal.
    - Solar energy heats up water/oil/etc → conventional generator
  - Photovoltaic.
    - light to electricity directly (needs quantum theory to understand)
Spherical Waves and Plane Waves

- Spherical EM waves: Radiation from a point source.
  - By laws of energy conservation, intensity (flux) of spherical EM waves goes as \( \sim 1/r^2 \).
  (why? see drawing and board)

- Plane waves: Beams are in parallel.
  - Intensity (flux) of plane EM waves remains as a constant.
About Exam 3

- **When and where**
  - Monday Nov. 23rd 5:30-7:00 pm
  - 125 Ag. Hall and 2103 Ch. (same as exam 1 and 2)

- **Format**
  - Closed book
  - One 8x11 formula sheet allowed, must be self prepared.
  - Four full problems.
  - Bring a calculator (but no computer). Only basic calculation functionality can be used.

- **Special needs:**
  - Please talk to me asap.
  - No early test before Monday Nov 23rd possible.
  - There will be 1 alternative sessions: 4:00pm in our lab rooms, only for approved requests.
Chapters Covered

- Chapter 31: Electromagnetic Induction and Faraday’s Law
  - All sections covered.

- Chapter 32: Inductance
  - All sections covered.

- Chapter 33: AC Circuits
  - Section 33.1-33.7

- Chapter 34: EM Waves
  - All sections covered
  - Displacement Current (34.1) only conceptual level.
  - Solving differential equations (for Maxwell’s eqs.) not required

- Chapter 16: Wave Motion. Not directly, but knowledge helps Ch. 34
- Section 28.4 (RC circuit) will be covered in w.r.t. time constant
Review Sessions

- Two (identical) review sessions are scheduled:
  - Friday, Nov 20\textsuperscript{th}, 5-7pm 2103 Chamberlin
  - Saturday, Nov 21\textsuperscript{st}, 1-3 pm 2103 Chamberlin

- You are free to attend the session that best fit your time.
  - However, to avoid over-crowdedness in a particular session, I am suggesting the following soft guideline:
    - 1:20pm class → Friday session
    - 2:25pm class → Saturday session

- Review slides are to be posted online on 21\textsuperscript{st} after the review session

- Please do your own review before coming to the review sessions.
Sample Review Exercise From My Spring Teaching
(Ampere’s Law will not be tested in exam 3)

- An infinite straight thin wire is at the center of two concentric conducting cylinders of radius R and 2R.

  The currents are \( I \) (into the page), \( 2I \) (out), and \( I \) (in), respectively for the center wire and the two cylinders. (as color coded).

Use Ampere's Law, find \( B \) as function of \( r \).

- **Solution:**

  \[
  \oint \mathbf{B} \cdot d\mathbf{s} = 2\pi r B = \mu_0 I_{\text{enclosed}}
  \]

  \[
  \Rightarrow B = \frac{\mu_0 I_{\text{enclosed}}}{2\pi r}
  \]

Answers:

- \( r < R \), \( B = \mu_0 I/2\pi r \) (Clockwise)
- \( R < r < 2R \), \( B = \mu_0 I/2\pi r \) (counter-clockwise)
- \( r > 2R \), \( B = 0 \)