Physics 202, Lecture 27

Today’s Topics

- Diffraction
  - Single-slit Diffraction
  - Reminder of Two-slit Interference
  - Double-Slit Diffraction
  - Diffraction on Circular Apertures
  - The Rayleigh Criterion
- About the Final Exam
Ray Approximation Again

- When the wavelength of the light is much smaller than the size of the optical objects it encounters, it can be treated as (colored) rays.

Ray approximation is valid when $\lambda \ll d$

Ray approximation is not valid near the gap when $\lambda \approx d$. OK elsewhere.
Single-Slit Interference
(Single-Slit Diffraction)

If lights were just rays
Single-Slit Diffraction Pattern Explained

- The slit is not a point source → Interference

The text also offers a derivation using phasors. Not to be examined but please read.
Where Are the Dark Fringes?

The dark fringes occur at:

\[ I = 0 \rightarrow \sin(\beta/2) = 0 \rightarrow \sin \theta_{\text{dark}} = m\lambda/D, \quad m = \pm 1, \pm 2, \pm 3, \ldots \]

\[ \text{Central bright dot width } \Delta \theta = 2\lambda/D \]

\[ \text{First dark fringes at } = \pm \lambda/D \]
Lights As Rays?

Due to diffraction, light beam of finite size can never travel as perfect straight rays!

θ ∼ λ/a

Numerical example: Estimate the size of laser beam on screen (λ~600nm, L=1m)

a = 1cm, θ ∼ λ/a = 6x10⁻⁵, a’ ~ 1.01cm → +1%
a = 2mm, θ ∼ λ/a = 3x10⁻⁴, a’ ~ 1cm+2.6mm → +30%
a = 1mm, θ ∼ λ/a = 6x10⁻⁴, a’ ~ 1cm+1cm → +100%
a = 0.1mm, θ ∼ λ/a = 6x10⁻³, a’ ~ 1cm+11cm → +12000%
Reminder: Two-slit Interference

\[ \delta = dsin\theta \sim d\theta \sim d\frac{y}{l} \]

\[ I = I_o \cos^2 \left( \frac{\pi d \sin \theta}{\lambda} \right) \]

Separation between minima = \( \frac{\lambda}{d} \)
Two-slit Diffraction

\[ I = I_0 \left( \frac{\sin(\beta/2)}{\beta/2} \right)^2 \]

\[ \beta \equiv \frac{2\pi}{\lambda} D \sin \theta \]

\[ I = I_o \cos^2 \left( \frac{\delta}{2} \right) \]

\[ \delta = \frac{2\pi}{\lambda} d \sin \theta \]

\[ I = I_0 \left[ \frac{\sin(\beta/2)}{\beta/2} \right]^2 \cos^2 \left( \frac{\delta}{2} \right) \]
Light through apertures will produce diffractive patterns depending on their shape. For circular apertures, the diffractive pattern is made of concentric rings. First dark ring at \( \theta = \frac{1.22 \lambda}{D} \).
Resolution of Single-slit and Circular Apparatus

Each smeared due to diffraction

Rayleigh’s Criterion

\( \theta_{\text{min}} = \frac{\lambda}{D} \)

Single slit: \( \theta_{\text{min}} = \frac{\lambda}{D} \)

Circular opening: \( \theta_{\text{min}} = 1.22 \frac{\lambda}{D} \)
Example: Watching HDTV

- Quiz: Human eye has a typical pupil diameter of about 5 mm.

→ What is the minimum distance between two red (λ=700nm) dots human eye can separate at 3 meters?

\[
\theta_{\text{min}} = 1.22 \frac{\lambda}{D} = 1.22 \frac{700\text{nm}}{5\text{mm}} \approx 1.7 \times 10^{-4}\text{ Rad}
\]

\[
\Delta S_{\text{min}} \approx \theta_{\text{min}} \times L \approx 1.7 \times 10^{-4} \times 3000\text{ mm} = 0.51\text{ mm}
\]

- Compare the above resolution to the pixel spacing of a 32” HDTV (720p or 1080p), what conclusion can you get?

<table>
<thead>
<tr>
<th>HDTV Geometry Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Screen Size (inch)</td>
</tr>
<tr>
<td>--------------------------</td>
</tr>
<tr>
<td>32</td>
</tr>
<tr>
<td>47</td>
</tr>
</tbody>
</table>
About the Final Exam

- The exam will be on 5:05-7:05pm, Friday, Dec 18 in:
  - Van Vleck B102 and Humanities 3650
    - It will be exactly 120 minutes.
    - Distribution of tests starts at 5pm.
    - McBurney students: contact me for special arrangements
- Four (3+1) 8½ x 11” double sided sheets are allowed.
  - Put down whatever you like, hand written or printed. You have to prepare it yourself. (no photocopying)
  - No shrink copy/printing of lecture notes/exam solutions/examples.
- Any calculator is fine.
  - Do not use programming functionality.
  - Absolutely no communication functionality.
- Bring a simple ruler for possible ray diagram problem.
About the Final Exam (2)

- The Exam is accumulative.
  - ~50% for new chapters (35, 36, 37, 38)
  - ~50% for old chapters (23-34)

- There will be 5-6 full problems with 20-24 questions.

- New chapters since Exam 3
  - Chapter 35: Reflection and Refraction
    - 35.1-35.8
  - Chapter 36: Lens and optical Instruments (Image)
    - 36.1-36.10 (36.5 Lens aberration: conception only)
  - Chapter 37: Light as Waves, interferences
    - 37.1-37.7
  - Chapter 38: Diffraction
    - 38.1-38.3

(Please read 38.4/5 “grating”, 38.6 Polarization)
Review Lectures

- Review Lecture 1: Lights and optics.
  - Thursday Dec 10, regular lecture time

- Review Lecture 2: More optics & (some) Exam 3 coverage*
  - Tuesday, Dec 15, regular lecture time

- Review Lecture 3: Exam 1 and Exam 2 materials. (ch. 23-30)
  - Two identical sessions
    • Friday Dec 11, 7-9pm, B130 Van Vleck.
    • Saturday Dec 12, 1-3pm, 2103 Chamberlin

* Most Exam 3 materials will not be repeated, please read Review 3 again.