B) A small scale, of mass 1.0 kg, is mounted on top of a massless vertically mounted Hooke's Law spring with spring constant 200 N/m. A 1.0 kg mass is placed on the scale and now the assembly moves to the new equilibrium position.

a) If the spring is now displaced downwards 0.200 m further and then released from rest, what is the initial frequency of oscillation?

\[ \omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{200}{2.0}} = 10 \text{ rad/sec} \]

\[ f = \frac{\omega}{2\pi} = 3.2 \text{ Hz} \]

b) What is the initial acceleration of the scale/mass assembly?

\[ a = \omega^2 A \cos(\omega t) \]
\[ a_{\text{MAX}} = \omega^2 A = 100 \cdot 0.2 \text{ m/s}^2 \]
\[ a_{\text{MAX}} = 20 \text{ m/s}^2 \]

At max. displacement

c) What does the scale read after T/4 where T is the period?

\[ a \text{ goes from } +20 \text{ m/s}^2 \text{ to } -20 \text{ m/s}^2 \text{ in } T/2 \]

\[ a_{\text{SHM}} = a_{\text{MIN}} = 0 \]

\[ \sum F_y = m \cdot a_{\text{MIN}} = N - mg = 0 \]

Just the mass

\[ N = mg = 10 \text{ N} \]

d) Does the mass always remain in contact with the spring? If yes, then then how long from release before the scale reads a minimum in the weight? If no, then how long after the release [i.e., from part (a)] does the 1.0 kg mass remain in contact with the scale?

If \( a \leq -10 \text{ m/s}^2 \) then mass goes into freefall starting at \( t = 0 \)

\[ a = \omega^2 A \cos(\omega t) \]
\[ -10 = \omega^2 \cdot 0.20 \cos(\omega t) \]
\[ -\frac{1}{2} = \cos(\omega t) \]

\[ \omega \cdot t = \frac{2\pi}{3} \]
\[ t = 0.21 \text{ sec} \]
C) It has recently become possible to "weigh" DNA molecules by measuring the influence of their mass on a nano-oscillator. The figure shows a thin rectangular cantilever etched out of silicon (density 2300 kg/m³) with a small gold dot at the end. If pulled down and released, the end of the cantilever vibrates with simple harmonic motion, moving up and down like a diving board after a jump. When bathed with DNA molecules whose ends have been modified to bind with gold, one or more molecules may attach to the gold dot. The addition of their mass causes a very slight-but measurable-decrease in the oscillation frequency. A vibrating cantilever of mass M can be modeled as a block of mass M/3 attached to a spring. (The factor of 1/3 arises from the moment of inertia of a bar pivoted at one end.) Neither the mass nor the spring constant can be determined very accurately—perhaps to only two significant figures—but the oscillation frequency can be measured with very high precision simply by counting the oscillations. In one experiment, the cantilever was initially vibrating at exactly 12 MHz (1 MHz = 10⁶ Hz). Attachment of a DNA molecule caused the frequency to decrease by 58 Hz.

1) What was the mass of the DNA?

\[
\omega = \left(\frac{k}{M/3}\right)^{1/2}
\]

\[
\frac{\omega}{\omega'} = \left(\frac{M'}{M}\right)^{1/2}
\]

\[
M \frac{\omega^2}{\omega'^2} = M' = M + \Delta M
\]

\[
M \left(\frac{\omega^2}{\omega'^2} - 1\right) = \Delta M = M \left(\frac{\omega^2 - \omega'^2}{\omega'^2}\right)
\]

Notice \(\Delta \omega = \omega' - \omega\)

So \(\Delta \omega \ll \omega\) and \(\Delta \omega \ll \Delta \omega'^2\)

So \(\Delta M = M \left(\frac{\omega^2 - (\omega + \Delta \omega)^2}{\omega'^2}\right) = M \left(-\frac{2 \omega \Delta \omega - \Delta \omega^2}{\omega'^2}\right)\)

Approximate! \(\omega \approx \omega'\)

\[\Delta M \approx -2M \omega \Delta \omega \frac{\omega}{\omega'^2} = -2M \frac{\Delta \omega}{\omega} = -\frac{2M \Delta f}{f}\]

\[M = \int LWH \quad L = 4000 \times 10^{-9} \text{ m} \quad W = 400 \times 10^{-9} \text{ m} \quad \rho = 2300 \text{ kg/m}^3 \quad \Delta f = -58 \text{ Hz} \quad \Delta f = 12 \times 10^6 \text{ Hz} \quad f = \frac{2M \Delta f}{f} \]