1. A ladder of length $L$ and mass $M$ is propped against a frictionless wall at an angle $\theta$ from horizontal. The ladder/ground interface has coefficient of static friction $\mu_s$. Find an expression for the maximum distance $d$ that a man of mass $m$ can climb the ladder before it slips (assuming that it will slip).

2. An object of mass $m$ is dropped from altitude $h$ above the surface of a planet of mass $M$ and radius $R$. Find an expression for the object's speed $v$ as it hits the ground.

3. A cylinder of height $h$ and density $\rho$ floats in water of density $\rho_w$ with its long axis vertical. Find an expression for the ratio $z/h$ of the exposed height to the total height.

4. A large water tank has a hole at height $y$. The tank is kept full of water at height $h$. Find an expression for the value of $y$ at which the range of the water stream $x$ is maximized.

5. A mass $m$ is held by two stretched rubber bands of length $L$ on a frictionless surface. (The diagram shows the view looking down at the surface.) At equilibrium, the bands have tension $T$. Find an expression for the frequency $\omega$ of small oscillations perpendicular to the bands. Assume that the magnitude of $T$ is constant throughout the oscillations.
1. No friction

\[ y: N_g = M_g + mg = (M+m)g \]

\[ x: N_w = f_s = \mu_s N_g = \mu_s (M+m)g \]

When \( f_s = \mu_s N_g \),

\[ \Rightarrow N_w = \mu_s (M+m)g \]

Torque: use ladder/ground as pivot

\[ N_w L \cos \theta = M_g \frac{L}{2} \sin \theta + mgd \sin \theta \]

\[ \mu_s (M+m)g L \cos \theta = (\frac{M}{2} + md) g \sin \theta \]

\[ \mu_s (M+m) L \cot \theta = \frac{M}{2} + md \]

\[ (\mu_s (M+m) \cot \theta - \frac{M}{2}) L = md \]

\[ \max d = L \left( \mu_s \left( 1 + \frac{M}{2m} \right) \cot \theta - \frac{M}{2m} \right) \]

2. \( U_1 = -GM \frac{m}{R+h} \quad U_2 = -GM \frac{m}{R} \)

\[ K = U_1 - U_2 = -GMm \left( \frac{1}{R+h} - \frac{1}{R} \right) \]

\[ = -GMm \left( \frac{R - R - h}{R(R+h)} \right) \]

\[ \frac{1}{2}mv^2 = +GMm \frac{h}{R(R+h)} \]

\[ v^2 = 2GM \frac{h}{R(R+h)} \]
3. \[ w = Ahp \quad W = mg = Ahpg \]
\[ F_b = A (h-z) \rho w g \]
\[ W = F_b \rightarrow Ahpg = A (h-z) \rho w g \]
\[ \rho p = \frac{\rho w - \rho}{\rho w} = 1 - \frac{\rho}{\rho w} \]
\[
\frac{z}{h} = \frac{\rho w - \rho}{\rho w} = 1 - \frac{\rho}{\rho w}
\]

4. \[ p = \rho g (h-y) \text{ at hole height} \]
\[ \frac{1}{2} \rho w^2 = \rho g (h-y) \]
\[ v^2 = 2g (h-y) \]

Range: \[ x = ut \quad t = x/u \]
\[ x = \frac{u^2 t^2}{2g} = \frac{u^2 (x/u)^2}{2g} \]
\[ x = \frac{2u^2 v^2}{g} = x^2 \]

\[ x^2 = \frac{2y}{g} g (h-y) = 4y (h-y) \]
\[ x=0 \quad y = h \]
\[ x = 0 \quad \frac{dy}{dx} = 0 \]
\[ x \text{ max when } y (h-y) \text{ max} \]
\[ \frac{dy}{dx} y (h-y) = 0 \]
\[ h - y + y = 0 \quad \Rightarrow \quad y = \frac{1}{2} h \]
\[ F_y = -2T \sin \theta \]

\[ y = l \sin \theta \]

\[ \frac{m \ddot{y}}{l} = -2T \sin \theta \]

\[ m \ddot{y} = -2T \frac{y}{l} \Rightarrow \ddot{y} + \frac{2T}{mL} y = 0 \]

(Small angles because otherwise \( T \) would increase due to stretching, which is a higher-order term. Try it.)