Problem 1. Find the commutator $\hat{K}_0 = [\hat{K}_+; \hat{K}_-]/2$ of the operators $\hat{K}_\pm = \hat{a}_\pm \hat{a}_\pm/2$, where the operators $\hat{a}_\pm$ are the raising and lowering operators of a harmonic oscillator, their commutator is $[\hat{a}_-; \hat{a}_+] = 1$.
Calculate the commutation relations $[\hat{K}_\pm; \hat{K}_0]$.

Problem 2. Problem 4.31.

Problem 3. Problem 4.49.


Problem 5. Calculate energies $E_n$ of stationary states of a two-dimensional particle moving in a potential
$$U(r) = \frac{\hbar^2}{2m} \left[ \alpha^2 |r|^2 + \beta^2 \right].$$
Use the cylindrical coordinates and look for the wave function in the form
$$\psi_n(|r|, \varphi) = e^{im\varphi} e^{-\alpha|\varphi|^2/2} \sqrt{\beta^2 + m^2} w(|r|),$$
where $w(|r|)$ is a finite order polynomial, $w(0) \neq 0$. Explain the choice of the above equation for the wave function $\psi_n(|r|, \varphi)$.

Problem 6. Problem 7.2.

Problem 7. Problem 7.7.