Experiment 5
Filter Circuits

1 Motivation

Filters are fundamental to electronics and the design of well behaved circuits. In this experiment you will measure the gain and the phase shift of three basic but important filter circuits. You will compare your measurements with theoretical expectations derived from frequency-domain analysis of the circuits. You will exercise many of the measurement capabilities of a digital storage oscilloscope.

2 Background

Filters are used extensively in circuit design to control the range of frequencies of the desired measurement bandwidth. Filter design can be very sophisticated, with books dedicated to the subject. This experiment will investigate three basic filters: a low-pass filter, a high-pass filter, and a resonant filter. These basic filters are the building blocks for more advanced filter design.

Filters can also appear as side effects associated with using real-world circuit components. A classic example is "stray" capacitance that can significantly reduce the bandwidth of a high-resistance sensor by creating an undesired lowpass R-C filter. There is always a stray amount of capacitance associated with the geometry and arrangement of conductors. Stray inductance can appear as well. Careful circuit construction can reduce these effects.

The simplest filter circuit is the impedance divider shown in Fig. 1. This circuit can serve as a low-pass and high-pass filter depending on the choice of impedances. More advanced filters add impedances, typically in pairs. Resonant filters are created when one Z is an equivalent impedance of a combination of C and L. In this experiment you will examine the resonant filter where Z_1 is the series combination of C and L.

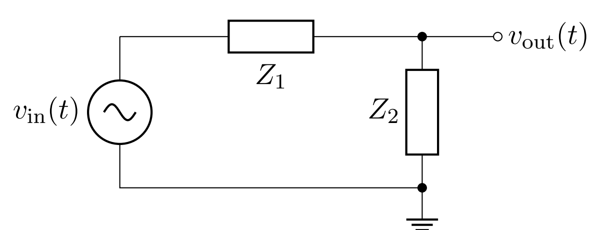

Figure 1: Circuit for a generic impedance filter

Filter circuits are typically analyzed using frequency-domain analysis for a single-frequency AC source. The filter is characterized by a frequency-dependent "transfer function" for the gain, $|v_{\rm out}(\omega)/v_{\rm in}|$, and relative phase, $\phi(\omega)$, between the output and input voltages. You will construct several filters and measure their gain and relative phase by driving the circuits with a fixed-frequency sinusoidal voltage from the function generator. You will adjust the frequency one-by-one to measure the filter's overall frequency response and compare with theoretical expectations.

3 Equipment

For this experiment, you will use:

- One Tektronix MSO 2014B Digital Storage Oscilloscope
- One Tektronix P3010 10X scope probe
- One AFG2021 Arbitrary Function Generator
- One ELC variable resistance box
- One ELC variable capacitance box
- One inductor coil* mounted on a circuit board

* If you select the numbered inductor board you used for the AC Wheatstone Bridge then you will know the value of L to within 2%, but this is not crucial.

4 Procedure

For your high-pass and low-pass filters, choose R and C that give corner (aka cutoff) frequencies $f_c = \omega_c/2\pi = 1/2\pi RC \approx 1\,\text{kHz}$. You are the filter designer! Pick appropriate values for R and C, but choose a resistance $1\,\text{k}\Omega < R < 10\,\text{k}\Omega$. The upper bound limits the impact of stray capacitance, and the lower bound protects the ECL boxes from too much current.

1. Construct the high-pass filter shown in Fig. 2.

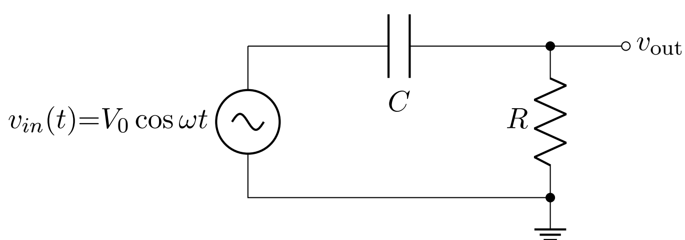

Figure 2: The high-pass R-C filter circuit

   (a) Set up measurements of v_{in} and v_{out} using scope channels CH 1 and CH 2. Use a BNC "T"(or "Y") adapter along with a BNC-to-banana adapter to make the connection from the generator to the filter network. Use a coaxial cable from the other branch of the BNC adapter to make a parallel connection from the generator to the scope to measure v_{in}. You do not need to use a scope probe to measure $v_{\text{out}} = v_R$. Instead, use a second BNC-to-banana adapter on the scope to connect directly across the resistor using patch cables. Set the function generator to create a sine wave and output voltage $10\,\text{V}_{\text{pp}}$ peak-to-peak (zero DC offset). Set up scope display measurements for the rms amplitudes for both CH 1 and CH 2 and the frequency (either channel). Make sure these measurements make sense w.r.t. to the function generator's settings.

   (b) Set the function generator frequency to $f \approx f_c$. You should see $|v_{\text{out}}| \approx |v_{\text{in}}|/\sqrt{2}$, and the two waveforms should have a clear phase shift. If not, double check your calculation of f_c and your measurement setup. Use the scope's cursors to measure the time lead/lag between v_{in} and v_{out}. Convert the lead/lag to relative phase, ϕ, in degrees. What ϕ do you expect when $f = f_c$?

(c) The scope is able to measure the relative phase between two channels much like the manual method with cursors. Use the scope menus to set up its measurement of the relative phase between CH 1 and CH 2. Your measurement and the scope's measurement of the relative phase should agree to within a few percent and have the same sign. Do not proceed until you are sure you understand how the relative phase is measured and what it means.

(d) Now scan the frequency and measure $|v_{\text{in}}|$, $|v_{\text{out}}| = |v_R|$, and ϕ. Take measurements for around 15 different frequencies from $f = 50\,\text{Hz}$ to $50\,\text{kHz}$ in steps differing by roughly a factor of 1.5. Use the scope's measurement of ϕ, but repeat the manual cursor measurement at least twice, once for $f \sim f_c/5$ and once for $f \sim 5f_c$. Create a table and record your data in your lab notebook. Your table will have additional entries as the procedure continues below. Appendix B gives guidance on how to record and present your data. Identify the frequencies for which you used cursors to measure ϕ and compare with the scope's measurement of ϕ at those frequencies. Make sure it is clear how the various quantities in your table were measured.

(e) For each frequency, calculate the filter gain in decibels, $A_{\text{dB}} = 20\log_{10}|v_R/v_{\text{in}}|$. (Attenuation is $A_{\text{dB}} < 0$.) Add the results to your table in your lab notebook. What value for A_{dB} is expected at $f = f_c$? Estimate f_c for your filter from your measurements of ϕ and/or A_{dB}.

(f) Now measure f_c "directly" by adjusting the function generator's frequency until one of the conditions for the corner frequency gain and relative phase is satisfied.

Helpful tip: *With "frequency" selected in the generator's menu, you can use the round knob to quickly make step changes up and down in frequency. The left-right arrows select which significant digit the knob adjusts for fine tuning the frequency. This works for other adjustable parameters, like the output amplitude.*

Using the "direct" value for f_c, calculate the ratio f/f_c for each frequency in your dataset and include this in your table. How well do your estimated and "direct" values of $f_c = \omega_c/2\pi$ compare with the expected value for your circuit, $\omega_c = 1/RC$?

(g) Plot your measured values of A_{dB} and ϕ versus ω/ω_c. Make a semi-log plot, with ω/ω_c the logarithmic axis following the example in Appendix B. Since dB is logarithmic, this is really a log-log plot.

(h) Compare your measurements with the theoretical expectations

$$A_{\text{dB}} = 20\log_{10}\left[\frac{\omega/\omega_c}{\sqrt{1+(\omega/\omega_c)^2}}\right] \qquad \text{and} \qquad \phi = \pi/2 - \tan^{-1}(\omega/\omega_c) \qquad (1)$$

If you have been using Excel to make your data table, you can easily add "theory" columns to your table using Eqs. 1. Otherwise make a comparison at one or two frequencies below the corner, $f < f_c$, and one or two above the corner, $f > f_c$.

(i) Straight lines in log-log plots show power-law relationships. Determine the asymptotic behavior of A_{dB}, i.e., the slope of A_{dB} versus $\log(\omega/\omega_c)$ in decibels per decade for $\omega \ll \omega_c$ and $\omega \gg \omega_c$. If you made your graphs using Excel, you can print the graph and draw asymptotes using a ruler.

(j) Set the function generator to $f \approx 2f_c$. Use the scope's "$-$" math function to display the voltage across C. At this frequency, measure the amplitude and phase of v_{in}, v_R, and v_C. Draw a phasor diagram for $\vec{v}_{\text{in}}$, $\vec{v}_R$, $\vec{v}_C$, and current $\vec{I}$.

2. Convert the circuit to a low-pass filter by interchanging the positions of C and R. Repeat steps (a)-(g) but measure the filter response at about 8 (half as many) frequencies from $f = 50\,\mathrm{Hz} - 50\,\mathrm{kHz}$. How are Eqs. 1 modified for the low-pass R-C filter? (If you have time, you can add plots for the theoretical gain and frequency for the low-pass filter to your table and plots.) Your analysis of the low-pass filter is likely to go more quickly, but when you have about 45 minutes left, move on to Step 3 so that you see how a resonant filter works.

3. Construct the resonant filter circuit shown in Fig. 3. Use $C = 50\,\mathrm{nF}$ and $R = 100\,\Omega$ ("10×" decade, full scale). The inductance is $L \approx 20\,\mathrm{mH}$. (If you use the same inductor as for the AC Wheatstone Bridge experiment, you know L to within 2%.) Calculate the circuit's predicted resonance frequency, $f_0 = \omega_0/2\pi = 1/2\pi\sqrt{LC}$.

   CAUTION! Use the "10×" decade resistor set to full scale to make $R = 100\,\Omega$. This decade can handle current up to 200 mA. Do **NOT** use the "100×" decade.

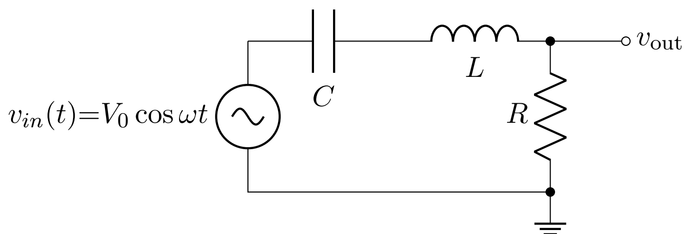

Figure 3: A resonant L-C-R filter

   (a) Set up scope measurements for v_{in} and v_R. (You do not need a scope probe.) Set the function generator to make a sine wave at $f \approx f_0$ and output voltage $10\,\mathrm{V_{pp}}$ (zero DC offset). Vary the generator frequency plus-and-minus relative to f_0 (using the knob and arrows) and observe how the two signals behave. You should see a clear peak in $|v_R|$ and a substantial relative phase shift if you calculated f_0 correctly.
   (b) Measure $|v_{\mathrm{in}}|$, $|v_R|$, and ϕ for frequencies from 200 Hz to 25 kHz. You will need to take most of your measurements near the resonant frequency to resolve the shape of the resonance, so you may want to start your data collection near f_0 and then scan f above and below f_0.
   (c) Calculate A_{dB} for your data, and then plot A_{dB} and ϕ versus ω/ω_0.
   (d) Determine the quality factor, Q, of the resonance by measuring $|v_R|$ and $|v_L|$ at the resonant frequency. Be careful, you now need to use a scope probe connected to CH 3 (or CH 4) to measure the voltage at the point between C and L ! Use the scope's math mode to measure v_L. How does your measured value $Q = |v_L|/|v_R|$ compare with the theoretical value, $Q = \omega_0 L/R$? (Reminder: Q is the full-width at half-maximum (FWHM) of the peak in A_{dB} against f.)
   (e) Set the function generator to a frequency about 500 Hz below the resonance. Measure the amplitudes and relative phases of v_{in}, v_C, v_L and v_R. (Use math mode to measure both v_L and v_C.)
   (f) Draw a phasor diagram for these measurements. If your measurements are consistent, $\vec{v}_{\mathrm{in}}$ should be the vector sum of $\vec{v}_C$ and $\vec{v}_L$. How close did you get?